Tensor Tomography and Boundary Rigidity

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This minicourse will cover topics in integral geometry of tensors and the boundary rigidity problem. Let $(M, g, \partial M)$ be a compact Riemannian manifold with smooth boundary. The first main problem we study is the following: given a symmetric 2-tensor f_{ij}, is it possible to recover the tensor f from the associated geodesic X-ray transform $I_g f$, i.e., from integrals along geodesics? It is known that one can hope to recover the *solenoidal part* f^s of the tensor only, since all *potential* tensors are in the kernel of that linear operator I_g. The second main problem is the boundary rigidity problem: given the distance function $\rho(x, y)$ between any pair (x, y) on the boundary ∂M, can we recover the metric g? If so, this can be done only up to a pull-back of a diffeomorphism fixing the boundary ∂M. It turns out that the linearization of the boundary rigidity problem is given by the X-ray transform I_g.

Those two problems are related in a natural way (through propagation of singularities) to other inverse problems, for example the problem of recovering of g given the hyperbolic Dirichlet-to-Neumann map, the inverse scattering problem related to a anisotropic medium, etc. They are motivated also by application in geophysics, medical imaging, etc. On the other hand, they are problems of independent interest in geometry.

We will emphasize on microlocal methods in attacking those problems. We will show that for *simple* metrics, $I_g^* I_g$ is a pseudo-differential operator, we will find its principal symbol, analyze its kernel, and show how to construct a parametrix. As a result, we will show how to recover the solenoidal part f^s of f up to a smoothing operator. We will also show that for generic simple metrics, I_g recovers uniquely f^s and there is a stability estimate. Next, we will apply this to treat the boundary rigidity problem. We will explain how one can prove generic local and global uniqueness and Hölder stability. Applications to other problems will be discussed.

References

[Sh] V. Sharafutdinov, Integral Geometry of Tensor Fields, VSP, Utrech, the Netherlands, 1994.

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