

Some Families of Series Associated with the Zeta and Related Functions

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Abstract

A fascinatingly large number of seemingly independent solutions of the so-called *Basler problem* of evaluating the Riemann Zeta function $\zeta(s)$ when $s = 2$, which was of vital importance to Leonhard Euler (1707-1783) and the Bernoulli brothers [Jakob Bernoulli (1654-1705) and Johann Bernoulli (1667-1748)], have appeared in the mathematical literature ever since Euler first solved this problem in the year 1736. The main object of this lecture is to present some recent developments on the evaluations and representations of $\zeta(s)$ when $s \in \mathbb{N} \setminus \{1\}$, \mathbb{N} being the set of natural numbers. We emphasize upon several interesting classes of rapidly convergent series representations for $\zeta(2n+1)$ ($n \in \mathbb{N}$) which have been developed in recent years. In two of many computationally useful special cases considered here, it is observed that $\zeta(3)$ can be represented by means of series which converge much more rapidly than that in Euler's celebrated formula as well as the series considered recently by Roger Apéry (1916-1994) in his proof of the irrationality of $\zeta(3)$. Symbolic and numerical computations using *Mathematica* (Version 4.0) for Linux show, among other things, that only 50 terms of one of these series are capable of producing an accuracy of seven decimal places.

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