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Pacific Institute for the Mathematical Sciences

International Symposium on Analytic Function Theory, Fractional Calculus and Their Applications

in Honour of

Professor H. M. Srivastava

on his 65th Birth Anniversary

At the University of Victoria

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International Symposium on
Analytic Function Theory, Fractional Calculus
and Their Applications
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Contents

On the Dimer Problem and the Multiple Gamma Function **5**
V. S. Adamchik

Certain Subclasses of Meromorphic Functions Defined by Integral Operators **6**
R. Aghalary

The Distributional Products of Particular Distributions **7**
M. A. Aguirre and C.-K. Li

Neighborhoods of Certain p-Valently Analytic Functions with Negative Coefficients **8**
Osman Altintaş

Some Applications of Differential Subordination **9**
A. A. Attiya

Fractional Differential Equations as Alternative Models to the Nonlinear Differential Equations **10**
B. Bonilla, M. Rivero, L. Rodríguez-Germá and J. J. Trujillo

On Systems of Linear Differential Equations with Constant Coefficients **11**
B. Bonilla, M. Rivero and J. J. Trujillo

Laguerre-Type Population Dynamics **12**
Gabriella Bretti and Paolo E. Ricci

A Conjecture About the Twin Primes **13**
M. Aslam Chaudhry

Inclusion Properties for Certain Subclasses of Meromorphic Functions Associated with the Generalized Hypergeometric Function **14**
Nak Eun Cho

Series Involving the Zeta Function and Multiple Gamma Functions **15**
Junesang Choi

On Order of Convexity of Functions Defined by Certain Integral Transforms **16**
M. Anbu Durai and R. Parvatham

Some Analytic Continuations of the Barnes Zeta Function in Two and Higher Dimensions **17**
E. Elizalde

Mathematical View of a Blind Source Separation on a Time Frequency Space **18**
Keiko Fujita

A Maximum Modulus Principle for Non-Analytic Functions Defined in the Open Unit Disk **19**
Maria E. Gageonea, Shigeyoshi Owa, Radu N. Pascu and Mihai N. Pascu

Normal Structure and Pythagorean Approach in Banach Spaces **20**
Ji Gao

The Asymptotic Universality of the Mittag-Leffler Waiting Time Law **21**
Rudolph Gorenflo and Francesco Mainardi

A Generalization on a Certain Class of Sălăgean-Type Harmonic Univalent Functions and Distortion Theorems for Fractional Calculus **22**
H. Ö. Güney

Decompositions for the Triple Hypergeometric Function $H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z)$ **23**
Anvar Hasanov

A Variant of the Stieltjes Transform on Distributions of Compact Support **24**
N. Hayek, B. J. González and E. R. Negrin

Some Applications of Hadamard Convolution to Multivalently Analytic and Multivalently Meromorphic Functions **25**
Hüseyin Irmak

Extensions of Two q**-Series Expansions with Applications to Biorthogonal Rational Functions** **26**
C. M. Joshi and Yashoverdhan Vyas

Observer's Mathematics - Mathematics of Relativity **27**
Boris Khots and Dmitriy Khots

α**-Analytic Solutions of Differential Equations with Variable Coefficients** **28**
A. A. Kilbas, M. Rivero, L. Rogriguez-Germá and J. J. Trujillo

Kronecker Operational Matrices for Fractional Calculus and Some Applications **29**
Adem Kılıçman and Zeyad Abdel Al Ziz Al Zhour

Fourier Analysis Between Hyperfunctions and Distributions **30**
Dohan Kim

Norm Estimates of the Pre-Schwarzian Derivatives in the Univalent Function Theory **31**
Yong Chan Kim

Minimax Programming with Generalized Convexity for Analytic Functions **32**
Hang-Chin Lai

Fractional Integro-Differentiation for *Mathematica* **33**
Oleg I. Marichev

Lebesgue Function for Multivariate Interpolation by RBFs **34**
Bahman Mehri

On the Strong Nörlund Summability of Conjugate Fourier Series **35**
Madan Lal Mittal and Uday Kumar

Subordination for Analytic Functions and Dziok-Srivastava Operators **36**
R. N. Mohapatra

Some Bivariate Bessel Distributions **37**
Saralees Nadarajah

N**-Fractional Calculus of Products of Some Power Functions and Some Doubly Infinite Sums** **38**
Katsuyuki Nishimoto

Certain Classes of Analytic Functions Concerned with Uniformly Starlike and Convex Functions **39**
Junichi Nishiwaki and Shigeyoshi Owa

Fractional Differentiation via the Cardinal Series **40**
Bruce O'Neill

Sakaguchi Type Functions **41**
Shigeyoshi Owa, Tadayuki Sekine and Rikuo Yamakawa

Some Subordination Results on the Classes of Starlike and Convex Functions of Complex Order **42**
Öznur Özkan

Bernoulli Polynomials from a Number-Theoretical Viewpoint **43**
Á. Pintér

Whittaker-Type Derivative Sampling Reconstruction of $L^\alpha(\Omega)$**-Processes** **44**
Tibor K. Pogány

The Generalization of Special Functions **45**
Asghar Qadir

Geometric Properties of Solutions of a Class of Ordinary Differential Equations **46**
Hitoshi Saitoh

Some New Double Sequence Spaces Defined by a Modulus Function **47**
Ekrem Savaş

Integral Means of Analytic Functions for Fractional Calculus **48**
Tadayuki Sekine, Shigeyoshi Owa and Kazuyuki Tsurumi

Hypergeometric Functions in the Geometric Function Theory **49**
T. N. Shanmugam

On Twisted q**-Hurwitz Zeta Function and** q**-Two-Variable** L**-Function** **50**
Yilmaz Simsek

A Non-Linear Differential Subordination and Starlikeness of Analytic Maps in the Unit Disc **51**
Sukhjit Singh and Sushma Gupta

Structure of Julia Sets and Dynamics of Polynomial Semigroups with Bounded Finite Postcritical Set **52**
Richard L. Stankewitz

Random Dynamics of Polynomials and Devil's-Staircase-Like Functions in the Complex Plane **53**
Hiroki Sumi

The Hypergeometric Functions and Some Formulas of Trigonometric Sums **54**
Katsuo Takano

Some Properties of the Kampé de Fériet Hypergeometric Function **55**
Mamasali Touraev

The Use of the Fractional Derivatives to Expand Analytical Functions in Terms of Quadratic Functions with Applications to Special Functions **56**
Richard Tremblay and B.-J. Fugère

A Survey of Fractional-Calculus Approaches to the Solutions of the Bessel Differential Equation of General Order **57**
Pin-Yu Wang

A Relation Among Subclasses Defined by Ruscheweyh Derivative on Analytic Functions with Negative Coefficients **58**
Teruo Yaguchi

A Further Expansion of the Riemann ζ-Function **59**
Sheldon Yang

Theorems Relating the Riemann-Liouville and the Weyl Fractional Integrals to Various Integral Transforms and Their Applications **60**
Osman Yürekli

On the Dimer Problem and the Multiple Gamma Function

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Abstract

The multiple gamma function, defined as a generalization of the Euler gamma function, was originally introduced by Kinkelin, Glaisher, and Barnes around 1900. This function has been related to certain spectral functions in mathematical physics, to the study of functional determinants of Laplacians, in asymptotics of certain Fredholm and Fisher-Hartwig determinants, to the Selberg zeta function, and to the Random Matrix Theory. There is a wide class of definite integrals appearing in statistical physics (the Ising problem and the dimer problem) that can be computed by means of the multiple gamma function. The main topic of the lecture is to present some recent developments on evaluations of such integrals in terms of the multiple gamma function.

2000 Mathematics Subject Classification. Primary 33E20, 33F99, 11M35, 11B73.

Key Words and Phrases. Riemann zeta function, Hurwitz zeta function, multiple gamma function, Barnes function, gamma function, Stirling numbers, harmonic numbers, Glaisher's constant.

Certain Subclasses of Meromorphic Functions Defined by Integral Operators

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Abstract

By making use of a general linear operator $\mathcal{L}^\lambda(a,c)$, we define new subclasses of meromorphic functions in the open unit disk $\mathbb{U}$ and investigate various inclusion relationships, distortion theorems and coefficient inequalities associated with these function classes. We also deduce some well-known results as corollaries of our theorems.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 30D30.

Key Words and Phrases. Analytic functions, meromorphic functions, integral operators, inclusion relationships, distortion theorems, coefficient inequalities.

The Distributional Products of Particular Distributions

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Abstract

Let f be a C^∞ function on R and P be a quadratic form defined by

$$P(x) = P(x_1, \cdots, x_m) = x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - \cdots - x_{p+q}^2 \quad (p + q = m).$$

In this paper, we mainly show that

$$f(P) \cdot \delta^{(k)}(P) = \sum_{i=0}^{k} \binom{k}{i} f^{(i)}(0)\, \delta^{(k-i)}(P),$$

where $\delta^{(k)}(P)$ is given by

$$(\delta^{(k)}(P),\ \phi) = (-1)^k \int_0^\infty \left[\left(\frac{\partial}{2r\partial r} \right)^k \left\{ r^{p-2} \frac{\psi(r,s)}{2} \right\} \right]_{r=s} s^{q-1}\, ds.$$

In particular, we have

$$P^n \cdot \delta^{(k)}(P) = \begin{cases} n! \binom{k}{n}\, \delta^{(k-n)}(P) & (k \geq n) \\ 0 & (k < n), \end{cases}$$

which solves a problem posed by C.-K. Li in 2004.

2000 Mathematics Subject Classification. Primary 46F10.

Key Words and Phrases. Particular distribution, δ-function, product and quadratic forms.

Neighborhoods of Certain p-Valently Analytic Functions with Negative Coefficients

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Abstract

Let $\mathcal{T}_n(p)$ denote the class of functions $f(z)$ *normalized* by

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k \quad (a_k \geq 0;\ n, p \in \mathbf{N} := \{1, 2, 3, \cdots\}),$$

which are *analytic* and *multivalent* in the unit disk $\mathbf{U} = \{z : z \in \mathbf{C} \quad \text{and} \quad |z| < 1\}$. We then define the (n, δ)-neighborhood of a function $f^{(q)}(z)$ when $f \in \mathcal{T}_n(p)$ by $\mathcal{N}_{n,p}^{\delta}(f^{(q)}; g^{(q)})$

$$= \left\{ g \in \mathcal{T}_n(p) : g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k \ \text{ and } \ \sum_{k=n+p}^{\infty} \tfrac{k!}{(k-q)!} k |a_k - b_k| \leq \delta \right\}.$$

We also let $\mathcal{T}_n(p, q, \alpha, \lambda)$ denote the subclass of $\mathcal{T}_n(p)$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\Re \left(\frac{z f^{(1+q)}(z) + \lambda z^2 f^{(2+q)}(z)}{\lambda z f^{(1+q)}(z) + (1-\lambda) f^{(q)}(z)} \right) > \alpha,$$

where $0 \leq \alpha < p - q$, $p > q$, $p \in \mathbf{N}$, $q \in \mathbf{N}_0 := \mathbf{N} \cup \{0\}$, $z \in \mathbf{U}$, and for each $f \in \mathcal{T}_n(p)$,

$$f^{(q)}(z) = \tfrac{p!}{(p-q)!} z^{p-q} - \sum_{k=n+p}^{\infty} \tfrac{k!}{(k-q)!} a_k z^{k-q} \quad (p > q).$$

Finally, $\mathcal{K}_n(p, q, \alpha, \lambda, \mu)$ denotes the subclass of the general class $\mathcal{T}_n(p)$ consisting of functions $f \in \mathcal{T}_n(p)$ satisfying the following nonhomogenous Cauchy-Euler differential equation:

$$z^2 \tfrac{d^{2+q} w}{dz^{2+q}} + 2(1+\mu) z \tfrac{d^{1+q} w}{dz^{1+q}} + \mu(1+\mu) w = (p - q + \mu)(p - q + \mu + 1) \tfrac{d^q g}{dz^q},$$

where $w = f \in \mathcal{T}_n(p)$, $g \in \mathcal{T}_n(p, q, \alpha, \lambda)$, and $\mu > q - p$.

In the present investigation, several results concerning the (n, δ)-neighborhoods, coefficient bounds, distortion inequalities for functions $f \in \mathcal{T}_n(p)$ in both classes $\mathcal{T}_n(p, q, \alpha, \lambda)$ and $\mathcal{K}_n(p, q, \alpha, \lambda, \mu)$ are given. Relevant connections of the various function classes investigated in this work with those considered by earlier authors are also pointed out.

2000 Mathematics Subject Classification. Primary 30C45.

Some Applications of Differential Subordination

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Joint work with H. M. Srivastava (University of Victoria)

Abstract

The authors introduce a two-parameter function $G(p, \lambda; z)$ which is shown to play an important rôle in a unified presentation of many interesting subclasses of analytic and p-valent functions in the open unit disk $\mathbf{U}$. Making use of the principle of differential subordination, some properties and relations involving the function $G(p, \lambda; z)$ are derived. A sufficient condition for p-valently starlikeness is also obtained.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 30A10.

Key Words and Phrases. Analytic functions, differential subordination, p-valent functions, p-valently starlike functions, p-valently convex functions.

Fractional Differential Equations as Alternative Models to the Nonlinear Differential Equations

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Dedicated to Professor H. M. Srivastava on the Occasion of his 65th Birthday

Abstract

It is known that there are several generalized fractional exponential functions and the corresponding generalized fractional sine and cosine functions. Motivated by these generalized fractional sine and cosine functions, we introduce a Weierstrass type function which is non-differentiable anywhere along the real axis. We will first prove formally that this Weierstrass type function is a solution of a certain fractional differential equation.

The main object of of this paper is to show that there are many dynamical processes which cannot be modelled by linear or non-linear ordinary differential equations, but (on the other hand) it is possible to obtain fractional models for that kind of dynamical processes.

2000 Mathematics Subject Classification. Primary 26A33, 34A05; Secondary 34A06.

Key Words and Phrases. Mittag-Leffler type functions, fractional differential equations, Weierstrass type function, fractional models.

On Systems of Linear Fractional Differential Equations with Constant Coefficients

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Dedicated to Professor H. M. Srivastava

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Abstract

This paper deals with the study of linear systems of fractional differential equations such as the following system:

$$\bar{Y}^{(\alpha} = \mathbf{A}(x)\bar{Y} + \bar{\mathbf{B}}(x), \tag{1}$$

where $D^\alpha \bar{Y} \equiv \bar{Y}^{(\alpha}$ is the Riemann-Liouville or the Caputo fractional derivative of order α $(0 < \alpha \leq 1)$, and

$$\mathbf{A}(x) = \begin{pmatrix} a_{11}(x) & . & . & . & a_{1n}(x) \\ \ldots\ldots & . & . & . & \ldots\ldots \\ \ldots\ldots & . & . & . & \ldots\ldots \\ \ldots\ldots & . & . & . & \ldots\ldots \\ a_{n1}(x) & . & . & . & a_{nn}(x) \end{pmatrix} \quad \text{and} \quad \bar{\mathbf{B}}(x) = \begin{pmatrix} b_1(x) \\ \ldots\ldots \\ \ldots\ldots \\ \ldots\ldots \\ b_n(x) \end{pmatrix} \tag{2}$$

are matrices of known real functions. In a way analogous to the usual case, we show how a generalized matrix exponential function and certain fractional Green function, in connection with the Mittag-Leffler type functions, would allow us to obtain an explicit representation of the general solution to the system (1) when $\mathbf{A}$ is a constant matrix.

2000 Mathematics Subject Classification. Primary 26A33, 34A05; Secondary 34A06, 74B10, 74C10.

Key Words and Phrases. Matrix Mittag-Leffler type functions, linear systems of fractional differential equations, fractional Green function.

Laguerre-Type Population Dynamics

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Abstract

In a recent article, the Laguerre derivative $D_L := DxD$, and its iterations $D_{nL} := DxDx \cdots DxD$ (containing $n+1$ derivatives), were used in order to investigate the so-called Laguerre-type Malthus, Verhulst, and Lotka-Volterra models. The Laguerre-type models arise quite naturally by substituting, in classical models, the ordinary derivatives with the Laguerre derivatives and therefore by using the so-called Laguerre-type exponentials (shortly L-exponentials) instead of the ordinary exponential. The nL-exponential function $e_n(x) := \sum_{k=0}^{\infty} x^k/(k!)^{n+1}$ satisfies with respect to the Laguerre derivatives an eigenvalue property generalizing the classical property of e^x, namely $D_{nL}e_n(ax) = ae_n(ax)$. It is worth noting that $\forall n$, $e_n(0) = 1$, and $e_n(x)$ is an increasing convex function for $x \geq 0$. Furthermore, $\forall x > 0$, $e^x = e_0(x) < e_1(x) < e_2(x) < \cdots < e_n(x) < \cdots$. For this reason, the L-exponential functions can be used in order to substitute the exponential function in many frameworks, including the population dynamics models. This technique was already used for introducing new sets of special functions (starting from the corresponding generating functions), including higher-order Laguerre polynomials, Laguerre-type Bessel functions, generalized Appell polynomials, etc. Applications to the solution of Laguerre-type integral, differential or partial differential equations problems were also considered. In this lecture we recall the above-mentioned theory, and show further applications of this approach in the framework of population dynamics.

2000 Mathematics Subject Classification. Primary 33C45, 30D05; Secondary 33B10, 92D25.

Key Words and Phrases. Laguerre-type exponentials, population dynamics.

A Conjecture About the Twin Primes

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Abstract

Let us define

$$N(x) := \frac{\Gamma(x)+1}{x} \qquad (x > 1).$$

Then, according to Wilson's Theorem, $N(x)$ is an integer when $x = p$ is a prime number. Put

$$I_{p,q} := \{n : n \in \mathbf{Z} \quad \text{and} \quad \mathbf{N(p)} \leq \mathbf{n} \leq \mathbf{N(q)}\}.$$

We would like to make the following conjecture. A solution of the conjecture would solve the open problem about the twin primes.

Conjecture. *Let p and q be odd primes such that $p < q$. Then there are at least $q - p$ twin primes in the interval $I_{p,q}$.*

We will discuss some consequences of the above conjecture.

2000 Mathematics Subject Classification. Primary 11-99, 11A41; Secondary 33B15.

Inclusion Properties for Certain Subclasses of Meromorphic Functions Associated with the Generalized Hypergeometric Function

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Abstract

Making use of a linear operator, which is defined by means of the Hadamard product (or convolution), the author introduces some new subclasses of meromorphic functions and investigates their inclusion relationships with the integral preserving properties.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 30D30, 33C20.

Key Words and Phrases. Meromorphic function, Subordination, Generalized hypergeometric function, Integral operator, Hadamard product (or convolution).

Series Involving the Zeta Function and Multiple Gamma Functions

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Abstract

The theory of multiple Gamma functions, which was recently revived in the study of the determinants of the Laplacians, was applied in several earlier works in order to evaluate some families of series involving the Riemann Zeta function as well as to compute the determinants of the Laplacians. We will introduce some properties of the multiple Gamma functions and a historical profile of the series associated with the Zeta functions. We will also address the converse problem and apply various (known or new) formulas for series associated with the Zeta and related functions with a view to developing the corresponding theory of multiple Gamma functions and then using these series to compute the determinants of the Laplacians on the n-dimensional unit sphere $\mathbf{S}^n$ ($n = 5,\ 6,\ 7$) explicitly.

2000 Mathematics Subject Classification. Primary 33B99; Secondary 11M06, 11M99.

Key Words and Phrases. Zeta functions, multiple Gamma functions, determinants of the Laplacians.

On Order of Convexity of Functions Defined by Certain Integral Transforms

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Abstract

For $\Lambda : [0,1] \to \mathbf{R}$, real-valued, monotonically decreasing on $[0,1]$, and satisfying the conditions $\Lambda(1) = 0$, $t\Lambda(t) \to 0$ as $t \to 0^+$, and $\frac{t\Lambda'(t)}{(1+t)(1-t)^{1+2\gamma}}$ increasing on $(0,1)$, we show that $M_\Lambda^\gamma(f) \geqq 0$ for a suitable $f(z)$, where

$$M_\Lambda^\gamma(f) = \inf_{|z|<1} \int_0^1 \Lambda(t) \left\{ \Re[f'(zt)] - \frac{(1-\gamma) - t(1+\gamma)}{(1-\gamma)(1+t)^3} \right\} dt.$$

Using this result, we obtain many more general results. We determine the least value of β so that, for the function g analytic in $P_\beta = \{g \in A : \Re[e^{i\alpha}(g'(z) - \beta)] > 0 \quad (\beta < 1)\}$ and for $0 \leqq \gamma \leqq \frac{1}{2}$, the functions

$$F(z) = V_\lambda(g) = \int_0^1 \lambda(t) \frac{g(tz)}{t} dt,$$

$$F_1(z) = z\, {}_2F_1(1, a, a+b, z) * g(z) \quad (0 < a < 1;\ b > 2 + 2\gamma),$$

and

$$F_\alpha(z) = \frac{(1-\alpha)(3-\alpha)}{2} \int_0^1 t^{-\alpha-1)}(1-t^2) g(tz) dt \quad (0 \leqq \alpha < 1)$$

are convex of order γ. Here ${}_2F_1$ is the Gaussian hypergeometric function. We have extended these results to functions of the form: $\rho z + (1-\rho)V_\lambda(g)$ $(\rho < 1)$. The corresponding starlikeness result is also obtained for such convex combinations.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 33C05.

Key Words and Phrases. Convex functions of order γ, starlike functions of order γ, convolution and integral operators.

Some Analytic Continuations of the Barnes Zeta Function in Two and Higher Dimensions

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Abstract

Formulas for the analytic continuation of the Barnes zeta function (E. W. Barnes, 1903), and some affine extensions thereof, in two and more dimensions, are constructed. The expressions are used to deal with determinants of multidimensional harmonic oscillators. An example is therewith obtained of the multiplicative anomaly (or defect), associated with the most common definition (due to D. B. Ray and I. M. Singer, 1971) of determinant of a pseudodifferential operator admitting a zeta function (M. Kontsevich and S. Vishik, 1995).

2000 Mathematics Subject Classification. Primary 11M35, 11M41; Secondary 30B40, 30B50.

Key Words and Phrases. Barnes Zeta function, Hurwitz (or generalized) Zeta function, determinant, multiplicative anomaly (or defect), quantum harmonic oscillators.

Mathematical View of a Blind Source Separation on a Time Frequency Space

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Abstract

To treat problems on the blind source separation, in many cases, either statistical independence or statistical orthogonality (uncorrelation) on the sources has been assumed. If we have as many observed signals as sources, we can separate sources from the observal signals under the above assumption.

Jourjine, Rickard and Yilmaz [Blind separation of disjoint orthogonal signals: Demixing n sources from 2 mixtures, Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing (Istanbul, Turkey; June 5-9, 2000), IEEE Press, 2000] considered the problem to separate (more than two) sources. To separate sources, they assumed that the windowed Fourier transforms of sources are mutually orthogonal, which is a stronger assumption than the statistical independence assumption. Then Balan and Rosca [Statistical properties of SRTFT ratios for two channel systems and applications to blind source separation Proceedings ICA (Helsinki, Finland; June 19-20, 2000)] relaxed their assumption. Independently, Napoletani, Berenstein and Krishnaprasad [Quotient Signal Decomposition and Order Estimation, Technical Research Report (TR 2002-47), University of Maryland, 2002] treated a similar problem under the linear independence of the windowed Fourier transforms of sources in some time frequency domain and the continuity of some (statistically defined) density functions. The fundamental idea employed by them is to consider the ratio of the windowed Fourier transforms of two observed signals.

In this talk, we will present a mathematical formulation of the method to consider the ratio of the windowed Fourier transforms of observed signals without assuming any statistical conditions.

2000 Mathematics Subject Classification. Primary 94A12; Secondary 93C70.

Key Words and Phrases. Blind source separation, time frequency space.

A Maximum Modulus Principle for Non-Analytic Functions Defined in the Open Unit Disk

(Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary)

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Abstract

Maximum principles are important tools in many areas of mathematics (differential equations, potential theory, complex analysis). Let us consider non-analytic functions $f(z)$ defined in the open unit disk $\mathbf{U}$ and having of the following form:

$$f(z, \bar{z}) = \sum_{n=1}^{\infty} f_n(z, \bar{z}) \qquad (z \in \mathbf{U}),$$

where $f_n(z, \bar{z})$ are complex functions defined for $z = x + iy$. In the present talk, we show that the maximum modulus principle holds for functions $f(z)$ of this class. Moreover, we show that $|f(z, \bar{z})|$ is a radially increasing function in $\mathbf{U}$.

2000 Mathematics Subject Classification. Primary 30C45.

Key Words and Phrases. Non-analytic functions, maximum modulus principle.

Normal Structure and Pythagorean Approach in Banach Spaces

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Abstract

Let X be a real Banach space and $S(X) = \{x \in X : \|x\| = 1\}$ be the unit sphere of X. The parameters $E_\epsilon(X) = \sup\{\alpha_\epsilon(x) : x \in S(X)\}$, $e_\epsilon(X) = \inf\{\alpha_\epsilon(x) : x \in S(X)\}$, $F_\epsilon(X) = \sup\{\beta_\epsilon(x) : x \in S(X)\}$, and $f_\epsilon(X) = \inf\{\beta_\epsilon(x) : x \in S(X)\}$, where $\alpha_\epsilon(x) = \sup\{\|x + \epsilon y\|^2 + \|x - \epsilon y\|^2 : y \in S(X)\}$ and $\beta_\epsilon(x) = \inf\{\|x + \epsilon y\|^2 + \|x - \epsilon y\|^2 : y \in S(X)\}$, are defined and studied. The main result is that a Banach space X with $E_\epsilon(X) < 2 + 2\epsilon + \frac{1}{2}\epsilon^2$ for some $0 \leq \epsilon \leq 1$ has uniform normal structure.

2000 Mathematics Subject Classification. Primary 46B20.

Key Words and Phrases. Normal structure, uniformly nonsquare space, uniform normal structure, ultraproduct space.

The Asymptotic Universality of the Mittag-Leffler Waiting Time Law

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Abstract

It is known that the waiting time law of Mittag-Leffler type can be obtained by properly scaled thinning (rarefaction) from a power law waiting time. We will show that (without thinning) it can be asymptotically obtained from a general power law waiting time by first rescaling time (so that the distant future cames near) and then decelerating an adjoined continuous time random walk (CTRW). The rescaling results in extremely many renewal events in a moderate interval of time and the deceleration reduces the number of events again to the original moderate size (in suitable average sense). Thus every renewal process with (non-pathalogical) power law waiting time is, when properly rescaled, asymptotically equivalent to a Mittag-Leffler renewal process. The Mittag-Leffler process itself is invariant under the described combination of rescaling and deceleration, and hence it is a distinguished renewal process among the power law processes.

2000 Mathematics Subject Classification. Primary 33E12, 60K05.

A Generalization on a Certain Class of Sălăgean-Type Harmonic Univalent Functions and Distortion Theorems for Fractional Calculus

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Abstract

Complex-valued harmonic functions that are univalent and sense-preserving in the unit disc $U = \{z : z \in \mathbf{C} \quad \text{and} \quad |\mathbf{z}| < \mathbf{1}\}$ can be written in the form:

$$f = h + \bar{g},$$

where h and g are analytic in $\mathbf{U}$. In this paper, we consider the class $HP(n, \lambda, \alpha) \quad (0 \leqq \alpha < 1;\ n \in N_0;\ \lambda > \frac{1}{2})$ consisting of Sălăgean-type harmonic univalent functions $f = h + \bar{g}$ for which

$$\Re\left(\frac{D^n f(z) + \lambda(D^{n+1} f(z) - D^n f(z))}{z}\right) \geqq \alpha,$$

where $D^n f$ is defined by $D^n f(z) = D^n h(z) + (-1)^n \overline{D^n g(z)}$. We obtain sufficient coefficient conditions for this class. Furthermore, we give the Hadamard product of several functions and some distortion theorems for fractional calculus of this generalized class.

2000 Mathematics Subject Classification. Primary 30C45, 30C50; Secondary 26A33.

Key Words and Phrases. Univalent functions, starlike functions, Sălăgean operator, fractional derivative.

Decompositions for the Triple Hypergeometric Function

$H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z)$

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Abstract

Over four decades ago, Professor H. M. Srivastava introduced three hypergeometric functions of three variables, one of which is being recalled here as follows:

$$H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z) = \sum_{m,n,p=0}^{\infty} \frac{(\alpha)_{m+p}(\beta_1)_{m+n}(\beta_2)_{n+p}}{(\gamma_1)_m(\gamma_2)_{n+p}\; m!\; n!\; p!} x^m y^n z^p.$$

There are many scientific articles in which various properties of each of these three triple hypergeometric functions are investigated. In this work, we construct 32 operational formulas for the function $H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z)$ by using the method of Burchnall and Chaundy. With the help of these operational formulas, we derive several decomposition formulas involving products of other hypergeometric functions in one, two, and three variables. For example, we obtain

$$H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z) = \sum_{i,j=0}^{\infty} \frac{(\alpha)_{i+j}(\beta_1)_{i+j}(\beta_2)_{i+j}}{(\gamma_1)_{i+j}(\gamma_2)_{i+j}\; i!\; j!} x^{i+j} y^j z^i$$
$$\cdot {}_2F_1(\alpha + i + j, \beta_1 + i + j; \gamma_1 + i + j; x)$$
$$\cdot F_1(\beta_2 + i + j, \beta_1 + i + j, \alpha + i; \gamma_2 + i + j; y, z);$$

$$H_A(\alpha, \beta_1, \beta_2; \gamma_1, \gamma_2; x, y, z) = \sum_{i,j=0}^{\infty} \frac{(\alpha)_{i+j}(\beta_1)_{i+j}(\beta_2)_{i+j}}{(\gamma_1)_i(\gamma_2)_{i+2j}\; i!\; j!} x^i y^{i+j} z^j$$
$$\cdot F_N(\beta_1 + i + j, \beta_1 + i + j, \beta_2 + j, \alpha + i + j, \beta_2 + i + j, \alpha + i + j;$$
$$\gamma_1 + i, \gamma_2 + i + 2j, \gamma_2 + i + 2j; x, y, z);$$

$$H_A(\alpha, \beta_1, \beta_2; \gamma, \gamma; x, y, z)$$
$$= \sum_{i,j,k=0}^{\infty} \frac{(\alpha)_{i+2j+k}(\beta_1)_{2i+j+k}(\beta_2)_{i+j+k}}{(\gamma)_{i+j}(\gamma)_{2i+2j+2k}\; i!\; j!\; k!} x^{i+j+k} y^{i+k} z^j$$
$$\cdot F_T(\beta_1 + 2i + j + k, \beta_2 + i + j + k, \beta_2 + i + j + k,$$
$$\alpha + i + 2j + k, \beta_1 + 2i + j + k, \alpha + i + 2j + k\; ; \gamma + 2i + 2j + 2k; x, y, z),$$

where ${}_2F_1$ is the Gauss hypergeometric function, F_1 is the Appell function of the first kind, and F_N and F_T are the relatively more popular notations for the triple hypergeometric functions F_6 and F_{13} in Lauricella's explicitly-defined set of 14 hypergeometric functions of three variables (see also Srivastava and Karlsson [*Multiple Gaussian Hypergeometric Series* (Wiley, New York, 1985), pp. 41-43 and pp. 74-87] for a readily-accessible table of 205 distinct triple Gaussian hypergeometric series). It is also shown that these decompositions can be deduced, using series as well as integral representations of the hypergeometric functions involved.

2000 Mathematics Subject Classification. Primary 33C20, 33D65, 33C80; Secondary 44A45.

Key Words and Phrases. Double hypergeometric series, multiple hypergeometric function, inverse pairs of symbolic operators, generalized hypergeometric function, Appell and Kampé de Fériet functions, integral representations.

A Variant of the Stieltjes Transform on Distributions of Compact Support

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Dedicated to Professor H. M. Srivastava
on the Occasion of his 65th Birthday

Abstract

In this paper, by means of a generalized Lambert transform and the Möbius numbers, we obtain an inversion formula for a variant of the Stieltjes transform introduced by Goldberg [*Pacific J. Math.* **8** (1958), 213–217] and defined by

$$G(x) = \int_0^\infty f(t)\, \frac{t}{x^2+t^2}\, dt \qquad (x > 0)$$

on the space $\mathcal{E}'\big((0,\infty)\big)$ of distributions of compact support. We also study the analyticity of G.

2000 Mathematics Subject Classification. Primary 46F12; Secondary 44A15.

Key Words and Phrases. Stieltjets transform, distributions of compact support, inversion formula.

Some Applications of Hadamard Convolution to Multivalently Analytic and Multivalently Meromorphic Functions

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Abstract

Let $\mathbf{A}(p)$ denote the class of functions $f(z)$ *normalized* by $f(z) = z^p + a_0 z^{1+p} + a_1 z^{2+p} + \cdots$ $(p \in \mathbf{Z} := \{\mp 1, \mp 2, \mp 3, \cdots\};\ \mathbf{N} := \{1, 2, 3, \cdots\};\ \mathbf{Z}^- := \mathbf{Z} \setminus \mathbf{N})$, which are *analytic* and *p-valent* in the domains $\mathbf{U} := \{z : z \in \mathbf{C} \text{ and } |z| < 1\}$ if $p \in \mathbf{N}$ and $\mathbf{D} := \mathbf{U} \setminus \{0\}$ if $p \in \mathbf{Z}^-$, where $\mathbf{C}$ denotes the complex plane. Also let $\mathcal{T}(p) := \mathcal{A}(p)$ and $\mathcal{M}(p) := \mathcal{A}(-p)$ when $p \in \mathbf{N}$. For the function $f(z)$ and the function $g(z)$ defined by $g(z) = z^p + b_0 z^{1+p} + b_1 z^{2+p} + \cdots$, the Hadamard product (or convolution) of the functions $f(z)$ and $g(z)$ is given, as usual, by $(f * g)(z) = z^p + a_0 b_0 z^{1+p} + a_1 b_1 z^{2+p} + \cdots$. By using this convolution, we define the operator $\mathcal{D}^{\lambda+p-1}$ by

$$\mathcal{D}^{\lambda+p-1}\{f\} := \frac{z^p}{(1-z)^{\lambda+p}} * f(z) \quad \text{if} \quad f \in \mathcal{T}(p)$$

and

$$\mathcal{D}^{\lambda+p-1}\{f\} := \frac{z^{-p}}{(1-z)^{\lambda+p}} * f(z) \quad \text{if} \quad f \in \mathcal{M}(p),$$

where $\lambda > -p$. From the above definition of the Hadamard product, we obtain the following identities:

$$z\left[\mathcal{D}^{\lambda+p-1}\{f\}\right]' = (\lambda+p)\mathcal{D}^{\lambda+p}\{f(z)\} - \lambda\mathcal{D}^{\lambda+p-1}\{f\} \quad \text{if} \quad f \in \mathcal{T}(p)$$

and

$$z\left[\mathcal{D}^{\lambda+p-1}\{f\}\right]' = (\lambda+p)\mathcal{D}^{\lambda+p}\{f(z)\} - (\lambda+2p)\mathcal{D}^{\lambda+p-1}\{f\} \quad \text{if} \quad f \in \mathcal{M}(p).$$

Applying the operators defined in this way to functions in the general class $\mathcal{A}(p)$, we prove several theorems involving various inequalities and then deduce some interesting connections between these inequalities. In our proofs of the main results, we make use of some known results of I. S. Jack [*J. London Math. Soc.* (*Ser.* 2) **3** (1971), 469-474], S. S. Miller and P. T. Mocanu [*J. Math. Anal. Appl.* **65** (1978), 289-305], and M. Nunokawa [*Proc. Japan Acad. Ser. A Math. Sci.* **68** (1992), 152-153].

2000 Mathematics Subject Classification. Primary 30C45.

Extensions of Two q-Series Expansions with Applications to Biorthogonal Rational Functions

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Abstract

In this paper, we derive two most general possible q-hypergeometric expansion formulas for ${}_{12}\Phi_{11}(q)$ and ${}_r\Phi_s(z)$, respectively. The results are unique in the sense that no such results are presumably available in the literature beyond ${}_4\Phi_3(q)$ so far and have the advantage that these results are also applicable to the top-level ${}_{10}\Phi_9(q)$ biorthogonal rational functions.

2000 Mathematics Subject Classification. 33C20, 33D15, 33D45; Secondary 26C15.

Key Words and Phrases. q-Hypergeometric functions, expansion formulas, biorthogonal rational functions.

Observer's Mathematics - Mathematics of Relativity

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Abstract

Observer dependent ascending chain of embedded sets of decimal fractions and their Cartesian products is considered. For every set, arithmetic operations are defined (these operations locally coincide with standard operations), which transform every set into a local ring. The basic problems of Algebra, Geometry, Topology, Logic, and Functional Analysis are solved for this chain. Definition of Dimension of these sets is introduced. In particular, the dimension of each of these sets is greater than or equal to seven. Euclidean, Lobachevsky, and Riemannian Geometries become the particular cases of the developed Geometry, although many others are possible. For example, we proved that two lines in a plane may intersect each other in 0 (without being parallel in the usual sense), 1, 2, 10, or even 100 points. The three classical Geometries depend on a particular neighborhood of a given line. For example, Euclidean Geometry works in a sufficiently small neighborhood of the given line, but when we enlarge the neighborhood, Lobachevsky Geometry takes over. Developed Topology gives birth to Time, and Time becomes a function of Space. Also, the Axiom of Choice becomes invalid in the new model of Mathematics. The application of the new model to Einstein's special theory of relativity is considered. The existence of the Time and Space quantums is proved. We also construct a new system of coordinate transformations that substitute Lorenz transformations. We also consider the application of the new model to data-mining.

2000 Mathematics Subject Classification. Primary 40-99, 51-99, 83-99.

α-Analytic Solutions of Some Linear Fractional Differential Equations with Variable Coefficients

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Dedicated to Professor H. M. Srivastava
on the Occasion of his 65th Birthday

Abstract

This paper investigates the solutions, around an ordinary point $x_0 \in [a, b]$, for fractional linear differential equations of the form:

$$[L_{n\alpha}(y)](x) = g(x, \alpha),$$

where

$$[L_{n\alpha}(y)](x) = y^{(n\alpha}(x) + \sum_{k=0}^{n-1} a_k(x) y^{(k\alpha}(x)$$

with $\alpha \in (0, 1]$. Here $n \in \mathbf{N}$, the real functions $g(x)$ and $a_k(x)$ $(k = 0, 1, \cdots, n-1)$ are defined on the interval $[a, b]$, and $y^{(k\alpha}(x)$ represents sequential fractional derivatives of order $k\alpha$ of the function $y(x)$. This study is an extension of the corresponding works by Al-Bassam.

2000 Mathematics Subject Classification. Primary 26A33, 34A05; Secondary 34A06, 74B10, 74C10.

Key Words and Phrases. α-Analytic solutions, linear fractional differential equations with variable coefficients, Caputo derivatives, Riemann-Liouville derivatives.

Kronecker Operational Matrices for Fractional Calculus and Some Applications

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Abstract

The problems of systems identification, analysis and optimal control have been recently studied by using orthogonal functions. The specific orthogonal functions used up to now are the Walsh, the block-pulse, the Laguerre, the Legendre, Haar and many other functions. In the present paper, the Kronecker operational matrices for fractional calculus are derived. These matrices are utilized in order to reduce the solution of systems to the solution of algebraic equations. The algorithms proposed here are similar to those already developed for the orthogonal functions; however, several problems might be solved by this approach.

2000 Mathematics Subject Classification. Primary 26A33, 33C45; Secondary 33C47.

Key Words and Phrases. Kronecker product, convolution product, operational matrix, special functions, fractional calculus.

Fourier Analysis Between Hyperfunctions and Distributions

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(Joint work with J. Chung)

Abstract

In this talk we will show the naturalness of hyperfunctions by comparing our results in the theory of hyperfunctions and the corresponding results in the Schwartz theory of distributions in such areas as the characterization of test function spaces in terms of Fourier transformations, Bochner-Schwartz theorem for (conditionally) positive definite (Fourier) hyperfunctions and (almost) periodic hyperfunctions.

To obtain the above theorems of global nature in hyperfunctions, we make use of the heat kernel method of Matsuzawa effectively, which represents various generalized functions as initial values of smooth solutions of the heat equation satisfying suitable growth condition.

2000 Mathematics Subject Classification. Primary 46F15; Secondary 46F05, 35K05.

Norm Estimates of the Pre-Schwarzian Derivatives in the Univalent Function Theory

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Abstract

The hyperbolic sup norm of the pre-Schwarzian derivative of a locally univalent function on the unit disk measures the deviation of the function from similarities. We present sharp norm estimates of the pre-Schwarzian derivatives for typical subclasses of univalent functions. We also consider the Alexander transforms in connection with the pre-Schwarzian derivatives.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 30C80, 33C05.

Key Words and Phrases. Pre-Schwarzian derivative, Alexander transform, univalent function, hypergeometric function.

Minimax Programming with Generalized Convexity for Analytic Functions

(Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary)

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Abstract

Consider a minimax programming problem in complex space in the following form:

$$(P_c) \quad \underset{\xi \in X}{\text{Min}} \ \underset{\eta \in Y}{\text{Max}} \ \Re\big(\varphi(\xi, \eta)\big)$$
$$\text{subject to } -g(\xi) \in S \subset \mathbf{C^p},$$

where Y is a compact subset of

$$\{\eta : \eta = (w, \bar{w}) \in \mathbf{C^{2m}}\}$$

in $\mathbf{C^{2n}}$, S is a polyhedral cone in $\mathbf{C^p}$; and, for each $\eta \in Y$, the mappings

$$\varphi(\cdot, \eta) : \mathbf{C^{2n}} \to \mathbf{C} \text{ and } \mathbf{g} : \mathbf{C^{2n}} \to \mathbf{C^p}$$

are analytic over the manifold

$$X = \{\xi = (z, \bar{z}) \in \mathbf{C^{2n}}, \ \mathbf{z} \in \mathbf{C^{2n}}\},$$

We employ generalized convexity of analytic functions to establish several sufficient optimality conditions for Problem (P_C), and using such criteria to constitute a parametric dual and establish the weak, strong, and strict converse duality theorems in our framework.

2000 Mathematics Subject Classification. Primary 20A51, 49A50; Secondary 90C25.
Key Words and Phrases. Complex minimax programming, $(\mathcal{F}, \rho, \theta)$-convex, -pseudo/-quasi convex, duality theorems.

Fractional Integro-Differentiation for *Mathematica*

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Abstract

Our 976-page monograph [S. G. Samko, A. A. Kilbas and O. I. Marichev, *Fractional Integrals and Derivatives*: *Theory and Applications*, Gordon and Breach Science Publishers, Reading and Langhorne (Pennsylvania), 1993] described practically all existing approaches to ideas of extension of differentiation and integration from integer order to arbitrary fractional or complex orders. In spite of this, the very important aspects of this theory for computer implementation were not developed as well as it is necessary for recent times. This report describes the easiest and the most natural way of the building of fractional integro-differentiation for computer systems like *Mathematica*. The corresponding basic formulas for it were developed and published at our functions Web Site:

`http://functions.wolfram.com`

This site currently comprises about 100,000 formulas and graphics. Its sections on "Fractional Integro-Differentiation" contain large collection of the corresponding fractional derivatives. For instance, the formula 01.02.20.0016.01 accessible via the following Web Site:

`http://functions.wolfram.com/01.02.20.0016.01`

defines fractional derivative for the simple power function z^a as analytic function of two variables supporting the logarithmic cases when

$$a = -1, -2, -3, \cdots .$$

2000 Mathematics Subject Classification. Primary 26A33; Secondary 33-99.

Key Words and Phrases. Fractional integrals, fractional integro-differentiation, analytic functions.

Lebesgue Function for Multivariate Interpolation by RBFs

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Abstract

Multivariate interpolation and approximation are powerful tools for intuition of the real world. In this talk we explain about the Lebesgue function and the Lebesgue constant for multivariate interpolation by conditionally positive definite RBFs. Also we explain about similar Bernstein and Erdös problems in the corresponding univariate cases.

2000 Mathematics Subject Classification. Primary 41A05, 41A10; Secondary 26A45.

Key Words and Phrases. Multivariate interpolation, approximation, Lebesgue function, Lebesgue constant.

On the Strong Nörlund Summability of Conjugate Fourier Series

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Abstract

In this talk, a sufficient condition for the summability $[N, p_n^{(1)}, 2]$ of conjugate Fourier series will be investigated. Thus, in conjunction with the known Tauberian theorem [M. L. Mittal, A Tauberian theorem on strong Nörlund summability, *J. Indian Math. Soc.* (*N. S.*) **44** (1980), 369-277] on strong Nörlund summability, gives a sufficient condition for the summability $[C, 1, 2]$ of conjugate Fourier series. The result presented here generalizes those due to Prasad [G. Prasad, *On Nörlund Summability of Fourier Series*, Ph. D. thesis, University of Roorkee, 1967] and Singh [U. N. Singh, On the strong Nörlund summability of Fourier series and its conjugate series, *Proc. Nat. Inst. Sci. India Part A* **13** (1947), 319-325].

2000 Mathematics Subject Classification. Primary 42A24; Secondary 40F05.

Subordination for Analytic Functions and Dziok-Srivastava Operators

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Abstract

In this talk the general problem of subordination for analytic functions will be discussed. Application to Approximation Theory will be demonstrated. After discussing the work of S. B. Joshi and H. M. Srivastava, the Dziok-Srivastava operators will be defined. Results concerning these operators will be given.

2000 Mathematics Subject Classification. Primary 30C45, 33C20; Secondary 40A30, 41-99.

Key Words and Phrases. Analytic functions, hypergeometric functions, subordination, Dziok-Srivastava operators, convergence of operators.

Some Bivariate Bessel Distributions

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Abstract

Four bivariate Bessel distributions are introduced; these distributions are based on a characterizing property involving linear combinations of chi-squared random variables. Some of these distributions turn out to be multi-modal. Various representations are derived for their joint densities and product moments. Ways to construct multivariate generalizations are also discussed.

2000 Mathematics Subject Classification. Primary 62E99; Secondary 33B15, 33C10.

Key Words and Phrases. Gamma function, Modified Bessel function of the first kind, Modified Bessel function of the third kind.

N-Fractional Calculus of Products of Some Power Functions and Some Doubly Infinite Sums

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Abstract

In this article, the N-fractional calculus of products of power functions

$$\left((z-c)^{\alpha}\cdot(z-c)^{\beta}\right)_{\gamma}, \qquad \left((z-c)^{\beta}\cdot(z-c)^{\alpha}\right)_{\gamma},$$

and

$$\left((z-c)^{\alpha+\beta}\right)_{\gamma} \qquad (z-c\neq 0)$$

are discussed. Moreover, an application of the N-fractional calculus

$$\left((z-c)^{\beta}\cdot(z-c)^{\alpha}\right)_{\gamma}$$

to the following doubly infinite sum:

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} L(\alpha,\beta,\gamma;k,m)\Big(-\frac{c}{z}\Big)^{k}\Big(\frac{z-c}{z}\Big)^{m},$$

where

$$L(\alpha,\beta,\gamma;k,m) := \frac{\Gamma(\alpha+1)\Gamma(\gamma+1)\Gamma(k-\alpha+m)\Gamma(\gamma-\beta-m)}{k!\; m!\;\Gamma(\alpha-k+1)\Gamma(\gamma-m+1)\Gamma(k-\alpha)\Gamma(-\beta)}$$

$$(\alpha,\beta,\gamma\notin \mathbf{Z}),$$

is reported.

2000 Mathematics Subject Classification. Primary 26A33; Secondary 33B15.

Key Words and Phrases. Fractional calculus, N-Fractional calculus operator, N-Fractional calculus of products.

Certain Classes of Analytic Functions Concerned with Uniformly Starlike and Convex Functions

(Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary)

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Abstract

Let $\mathcal{A}$ be the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are analytic in the open unit disk $\mathbf{U}$. In 2004, S. Shams, S. R. Kulkarni and J. M. Jahangiri studied the subclass $\mathcal{SD}(\alpha,\beta)$ of $\mathcal{A}$ consisting of functions $f(z)$ which satisfy the following inequality:

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha\left|\frac{zf'(z)}{f(z)} - 1\right| + \beta \qquad (z \in \mathbf{U})$$

for some α $(\alpha \geqq 0)$ and β $(0 \leqq \beta < 1)$. The subclass $\mathcal{KD}(\alpha,\beta)$ is defined by requiring that $f(z) \in \mathcal{KD}(\alpha,\beta)$ if and only if $zf'(z) \in \mathcal{SD}(\alpha,\beta)$. In the present talk, we introduce the subclass $\mathcal{M}(\alpha,\beta)$ consisting of all functions $f(z) \in \mathcal{A}$ which satisfy the following inequality:

$$\Re\left(\frac{zf'(z)}{f(z)}\right) < \alpha\left|\frac{zf'(z)}{f(z)} - 1\right| + \beta \qquad (z \in \mathbf{U})$$

for some α $(\alpha \leqq 0)$ and β $(\beta > 1)$. Also the class $\mathcal{N}(\alpha,\beta)$ is considered such that $f(z) \in \mathcal{N}(\alpha,\beta)$ if and only if $zf'(z) \in \mathcal{M}(\alpha,\beta)$. We discuss some properties of functions $f(z)$ belonging to the classes $\mathcal{M}(\alpha,\beta)$ and $\mathcal{N}(\alpha,\beta)$.

2000 Mathematics Subject Classification. Primary 30C45.
Key Words and Phrases. Analytic functions, uniformly starlike functions, uniformly convex functions.

Fractional Differentiation via the Cardinal Series

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Abstract

Let f be a function in the Hardy Space H^2, that is, f is analytic in the unit disc and the quantity $\sup\limits_{0<r<1}\left\{\frac{1}{2\pi}\int_0^{2\pi}\left|f\left(re^{i\theta}\right)\right|^2 d\theta\right\}^{\frac{1}{2}}$, the H^2 norm of f, is finite. It is well known that, if f is given by $f(z)=\sum_{n=0}^{\infty}a_n z^n$, then $\{a_n\}_{n=0}^{\infty}\in l^2$. The Cardinal Series of such an l^2 sequence is the function A defined by $A(t):=\sum_{n=0}^{\infty}a_n\,S(t-n)$, where $S(t)=\left\{\begin{array}{cc}\frac{\sin\pi t}{\pi t} & t\neq 0\\ 1 & t=0\end{array}\right\}$.
Important properties of A are that it is an entire function of exponential type π, is in $L^2(\mathbf{R})$ and satisfies $A(n)=a_n$. It can be shown that $\{A(n+\alpha)\}_{n=0}^{\infty}\in l^2$ for every $\alpha\in\mathbf{R}$. The cardinal series figures prominently in signal processing, most notably in the Shannon Sampling Theorem.

We define the operator $L^{\alpha}f(z):=\sum_{n=0}^{\infty}A(n+\alpha)\frac{\Gamma(n+\alpha+1)}{n!}z^n$. We see that $L^k f(z)=f^{[k]}(z)$ for all non-negative integers k, so that L^{α} interpolates the differentiation operator.

By considering the example in which f is a single term, say $f(z)=f_p(z)=z^p$ for some non-negative integer p, we recognize $L^{\alpha}f_p(z)$ as a ${}_2F_1$ hypergeometric function. After a chain of hypergeometric and gamma function identities it is seen that the operator L^{α} agrees with a version of the classical Riemann-Liouville definition for suitably restricted α and f. Since this interpolation method and the Riemann-Liouville definition coincide where their domains overlap, each can be viewed as an extension of the other to a wider class of functions.

Upon restriction of this operator to polynomials, analogues of some classical theorems on roots of polynomials are studied. The local mapping properties of analytic functions show the motion of the m roots of a polynomial of degree m towards the $m-1$ roots of its derivative, and account for the disappearance of the other root.

2000 Mathematics Subject Classification. Primary 26A33, 30-99, 30D55, 33C05; Secondary 30D20, 44-99.

Sakaguchi Type Functions

(Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary)

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Abstract

Let $\mathcal{A}$ be the class of functions of the form: $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, that are analytic in the open unit disk $\mathbf{U} = \{\mathbf{z} : \mathbf{z} \in \mathbf{C} \quad \text{and} \quad |\mathbf{z}| < \mathbf{1}\}$. For a function $f(z) \in \mathcal{A}$, we introduce a class $\mathcal{S}(\alpha, t)$ defined by

$$\mathcal{S}(\alpha, t) := \left\{ f : f(z) \in \mathcal{A} \quad \text{and} \quad \Re\left(\frac{(1-t)zf'(z)}{f(z) - f(tz)} \right) > \alpha \ (0 < \alpha < 1;\ |t| \leqq 1; t \neq 1) \right\}.$$

The class $\mathcal{S}(0, -1)$ was introduced by Sakaguchi. Therefore, a function $f(z) \in \mathcal{S}(0, -1)$ is called a Sakaguchi function. We also denote by $\mathcal{T}(\alpha, t)$ the subclass of $\mathcal{A}$ consisting of all functions $f(z)$ such that $zf'(z) \in \mathcal{S}(\alpha, t)$.

Incidentally. the class of uniformly starlike functions introduced by Goodman is defined as follows:

$$\mathcal{UST} := \left\{ f : f(z) \in \mathcal{A} \quad \text{and} \quad \Re\left(\frac{(z-\zeta)f'(z)}{f(z) - f(\zeta)} \right) > 0 \quad \big((z, \zeta) \in \mathbf{U} \times \mathbf{U}\big) \right\}.$$

Recently, we investigated the class $\mathcal{S}(\alpha, -1)$. Here, in this paper, we present some results for functions belonging to the general classes $\mathcal{S}(\alpha, t)$ and $\mathcal{T}(\alpha, t)$.

2000 Mathematics Subject Classification. Primary 30C45.

Key Words and Phrases. Analytic functions, univalent functions, starlike functions.

Some Subordination Results on the Classes of Starlike and Convex Functions of Complex Order

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Abstract

Let $\mathcal{H}(\mathbf{U})$ be the class of analytic functions defined on the unit disc $\mathbf{U} = \{\mathbf{z} : \mathbf{z} \in \mathbf{C} \text{ and } |\mathbf{z}| < \mathbf{1}\}$ and let $\mathcal{A}$ denote the class of functions $f \in \mathcal{H}(\mathbf{U})$ normalized by $f(0) = f'(0) - 1 = 0$. Also let $\mathcal{S}^*(b)$, $\mathcal{K}(b)$, and $\mathcal{C}(b)$ denote, respectively, the subclasses of $\mathcal{A}$ consisting of functions that are starlike of complex order $b \quad (b \in \mathbf{C} \setminus \{\mathbf{0}\})$, convex of complex order $b \quad (b \in \mathbf{C} \setminus \{\mathbf{0}\})$, and close-to-convex of complex order $b \quad (b \in \mathbf{C} \setminus \{\mathbf{0}\})$ in $\mathbf{U}$. In particular, the classes $\mathcal{S}^* := \mathcal{S}^*(1)$, $\mathcal{K} := \mathcal{K}(1)$, and $\mathcal{C} := \mathcal{C}(1)$ are the familiar classes of starlike, convex, and close-to-convex functions in $\mathbf{U}$, respectively. In the light of these definitions, a function $f \in \mathcal{A}$ is said to be in the class $\mathcal{P}(\lambda, b)$ if it satisfies the following inequaility:

$$\Re\left\{1 + \frac{1}{b}\left(\frac{zf'(z) + \lambda z^2 f''(z)}{(1-\lambda) f(z) + \lambda z f'(z)} - 1\right)\right\} > 0 \qquad (z \in \mathbf{U};\ \mathbf{0} \leqq \lambda \leqq \mathbf{1}).$$

Also, a function $f \in \mathcal{A}$ is said to be in the class $\mathcal{R}(\lambda, b)$ if it satisfies the following inequaility:

$$\Re\left\{1 + \frac{1}{b}\left(f'(z) + \lambda z f''(z) - 1\right)\right\} > 0 \qquad (z \in \mathbf{U};\ \mathbf{0} \leqq \lambda \leqq \mathbf{1}).$$

In 1999, the classes $\mathcal{P}(\lambda, b)$ and $\mathcal{R}(\lambda, b)$ were studied by Altintas and Özkan. In this work, we investigate several subordination results involving the Hadamard products of functions in the above-defined classes.

2000 Mathematics Subject Classification. Primary 30C45.

Key Words and Phrases. Starlike functions, convex functions, close-to-convex functions, complex order, Hadamard product, subordination.

Bernoulli Polynomials from a Number-Theoretical Viewpoint

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Abstract

The Bernoulli polynomials play an important role in mathematics. In this survey talk, we yield some old and recent results on the structure of zeros of the Bernoulli polynomials and the shifted Bernoulli polynomials. For example, we give a lower bound for the number of simple zeros and for the number of zeros of odd multiplicities, respectively. Using these estimates, we also deduce certain effective and ineffective finiteness statements for the number of solutions to the classical diophantine equations related to the following power sum:

$$S_k(x) = 1^k + 2^k + \ldots + x^k.$$

2000 Mathematics Subject Classification. Primary 11B68, 11D41; Secondary 11D45.

Key Words and Phrases. Diophantine equations, Bernoulli numbers, zeros of Bernoulli polynomials.

Whittaker-Type Derivative Sampling Reconstruction of $L^{\alpha}(\Omega)$-Processes

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Abstract

Certain mean square and almost sure Whittaker-type general derivative sampling theorems are obtained for the class $L^{\alpha}(\Omega, \mathfrak{F}, \mathsf{P})$ $(0 \leqq \alpha \leqq 2)$ of stochastic processes having spectral representation, with the aid of the Weierstrass σ-function. Functions of this class are represented by interpolatory series, viewing them as sums of residues. The results are valid for harmonizable and stationary processes ($\alpha = 2$) as well. The formulas are interpreted in the α-mean sense and also in the almost sure P sense when the initial signal function and its derivatives (up to some fixed order) are sampled at the points of the integer lattice $\mathbf{Z}^2$. A variant of the truncation error, the so-called *circular truncation error*, is introduced and used in the truncation error analysis. A sampling sum convergence rate is also provided.

2000 Mathematics Subject Classification. Primary 41A05, 60G35, 94A20; Secondary 30D15, 60G99, 94A12.

Key Words and Phrases. Almost sure P convergence, α-mean convergence, α-mean derivatives, Catalan's constant, circular truncation error, derivative sampling, Karhunen processes, $L^{\alpha}(\Omega, \mathfrak{F}, \mathsf{P})$-processes, Piranashvili α-processes, plane sampling reconstruction, sampling truncation error upper bounds, Weierstrass sigma-function, Whittaker-type sampling formula.

The Generalization of Special Functions

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Abstract

It is not clear exactly what "special functions" are. Yet there are books and courses on them and they are heavily used in analytical number theory, engineering mathematics and in mathematical physics. Whatever they may be, it seems contradictory to talk of generalizing them. If they are special, they are not general, and vice versa. Nevertheless, there has been a lot of work done on their generalization, and those generalizations have been heavily used. It is worthwhile to step back and consider what this enterprise of generalization is and what it amounts to. In particular, there can be infinitely many generalizations of any function that introduce extra parameters and extend the domain of the generalized function (or, in some cases, limit it). Most of them would appear to be futile, but some may lead to new insights and have far-reaching applications. How can we know that we are following up a useful generalization or, on the other hand, not throwing away a valuable generalization? Using examples of generalizations that have proved useful, with special reference to those developed by Chaudhry and me, in this paper an attempt is made to formulate criteria for guiding us in the search for worthwhile generalizations of special functions. First the question of what special functions are will be addressed. Then the significance of generalizations will be dwelt upon. Finally, some concluding remarks will be made.

2000 Mathematics Subject Classification. Primary 11M06, 33B15; Secondary 33-99.

Geometric Properties of Solutions of a Class of Ordinary Differential Equations

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Abstract

The main object of this talk is to investigate several geometric properties of the solutions of some ordinary linear differential equations. Relevant connections of the results presented in this talk with those given earlier by Robertson, Miller, and Saitoh are also considered.

2000 Mathematics Subject Classification. Primary 30C45.

Key Words and Phrases. Differential equations, analytic functions, univalent functions, starlike functions.

Some New Double Sequence Spaces Defined by a Modulus Function

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Abstract

In this paper our goal is to extend a few results known in the literature for ordinary (single) sequences to multiple sequences of real and complex numbers. This will be accomplished by presenting the following sequence spaces:

$$\{x \in s'' : P - \lim_{m,n} \sum_{k,l=0}^{\infty} a_{m,n,k,l} f(|x_{\sigma^k(p),\sigma^l(q)}|) = 0\},$$

$$\{x \in s'' : P - \lim_{m,n} \sum_{k,l=0}^{\infty} a_{m,n,k,l} f(|x_{\sigma^k(p),\sigma^l(q)} - Le|) = 0 \quad \text{for some} \quad L\},$$

and

$$\{x \in s'' : \sup_{m,n,p,q} \sum_{k,l=0}^{\infty} a_{m,n,k,l} f(|x_{\sigma^k(p),\sigma^l(q)}|) < \infty\},$$

where f is a modulus function and A is a nonnegative RH-regular summability matrix method. In addition, we shall establish inclusion theorems between these spaces and other sequence spaces.

2000 Mathematics Subject Classification. Primary 42B15; Secondary 40C05.

Key Words and Phrases. Double sequences, P-convergent, modulus functions.

Integral Means of Analytic Functions for Fractional Calculus

Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary

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Abstract

Let $\mathcal{A}_n$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k \quad (n \in \mathbf{N} := \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots\}),$$

which are analytic in the open unit disk $\mathbf{U} = \{\mathbf{z} : \mathbf{z} \in \mathbf{C} \text{ and } |\mathbf{z}| < \mathbf{1}\}$. We denote by $p(z)$ the analytic function in $\mathbf{U}$ defined by

$$p(z) = z + \sum_{s=1}^{m} b_{sj-s+1} z^{sj-s+1} \quad (j \geq n+1;\ n \in \mathbf{N}).$$

Sekine, Owa and Yamakawa have studied the integral means with coefficient inequalities of the analytic functions $f(z) \in \mathcal{A}_n$ and $p(z)$, and $f'(z)$ and $p'(z)$, by means of the subordination theorem of J. E. Littlewood. In the present talk, we aim at investigating the integral means with coefficient inequalities of the analytic functions $f(z) \in \mathcal{A}_n$ and $p(z)$ for the fractional derivatives and the fractional integrals.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 26A33, 30C80.

Key Words and Phrases. Integral means, analytic functions, Hölder inequality, subordination, fractional calculus, fractional derivatives, fractional integrals.

Hypergeometric Functions in the Geometric Function Theory

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Abstract

The study of univalent functions is a fascinating area of research with continued interest in the recent times also. This is classified under the broader area of Geometric Function Theory due to the interplay between Analysis and Geometry. The Bieberbach conjecture (which remained open for a long time) was positively settled by de Branges in the year 1984. The study based on this problem yielded so many new results. But, on the other hand, further interest on hypergeometric function theory was developed after the surprising use of hypergeometric functions in the proof of the Bieberbach conjecture by de Branges. In this paper, we discuss the uses and influences of the Gaussian hypergeometric function ${}_2F_1(a, b; c; z)$ and the Kummerian confluent hypergeometric function ${}_1F_1(a; c; z)$, and of their generalizations, in the study of Geometric Function Theory.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 33C05, 33C15, 33C20.

Key Words and Phrases. Univalent functions, hypergeometric functions, integral transforms, coefficient inequalities.

On Twisted q-Hurwitz Zeta Function and q-Two-Variable L-Function

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Abstract

In this study, by applying the Mellin transformation to the generating function of the (h, q)-Bernoulli polynomials, we construct integral representation of the twisted (h, q)-Hurwitz function and the twisted (h, q)-two-variable L-function. These functions interpolate the twisted (h, q)-Bernoulli polynomials and the generalized twisted (h, q)-Bernoulli numbers at non-positive integers, respectively. We also give the relation between the twisted (h, q)-Hurwitz zeta function and the twisted (h, q)-two-variable L-function.

2000 Mathematics Subject Classification. Primary 11S40, 11S80; Secondary 11B68.

Key Words and Phrases. q-Bernoulli numbers and polynomials, twisted q-Bernoulli numbers and polynomials, q- zeta function, L-function, twisted q-zeta function, twisted q-L-functions, q-Volkenborn integral.

A Non-Linear Differential Subordination and Starlikeness of Analytic Maps in the Unit Disc

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Abstract

Let α be a complex number with $\mathrm{Re}(\alpha) > 0$. Let the functions p and q be analytic in the unit disc $E = \{z : |z| < 1\}$ and normalized by the conditions $p(0) = q(0) = 1$. In the present article, we study a non-linear differential subordination of the following type:

$$(1-\alpha)p(z) + \alpha(p(z))^2 + \alpha\lambda z p'(z) \prec (1-\alpha)q(z) + \alpha(q(z))^2 + \alpha\lambda z q'(z),$$

where $\lambda > 0$ is some real number and the symbol $\prec$ stands for subordination. We find conditions that the function q must satisfy so that it becomes the best dominant of the above differential subordiantion. By carefully selecting q, we obtain, as consequences, a number of sufficient conditions for starlikeness of analytic maps in the unit disc. Most of the previously known results in this direction either get extended or follow as corollaries to our results.

2000 Mathematics Subject Classification. Primary 30C45; Secondary 30C50.

Structure of Julia Sets and Dynamics of Polynomial Semigroups with Bounded Finite Postcritical Set

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Joint work with Hiroki Sumi (Osaka University)

Abstract

The maps $f_c(z) = z^2 + c$ for c in the Mandelbrot set are such that the critical orbit $\{f_c^n(0)\}$ is bounded, which (in turn) leads to many important dynamic and structural properties. We look at the more general situation of polynomial semigroups with bounded postcritical set. More precisely, let G be a semigroup of complex polynomials (under the operation of composition of functions) such that there exists a bounded set in the plane which contains any finite critical value of any map $g \in G$. We discuss the dynamics of such polynomial semigroups as well as the structure of the Julia set of G (the set of points where G fails to be a normal family). In general, the Julia set of such a semigroup G may be disconnected, and each Fatou component of G is either simply connected or doubly connected. In this talk, we present that, for any two doubly connected components of the Fatou set, the boundaries are separated by a Cantor set of quasicircles inside the Julia set of G. Furthermore, we provide results concerning the (semi) hyperbolicity of such semigroups as well as discuss various topological consequences of the postcritically boundedness condition.

2000 Mathematics Subject Classification. Primary 37F10; Secondary 30D05.

Random Dynamics of Polynomials and Devil's-Staircase-Like Functions in the Complex Plane

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Abstract

We consider the dynamics of polynomial semigroups with bounded postcritical set and random dynamics of complex polynomials in the complex plane.

A polynomial semigroup G is a semigroup generated by polynomials in one variable with the semigroup operation being functional composition. We show that if the postcritical set of G, that is the closure of the G-orbit of the union of any critical values of any generators of G, is bounded in the complex plane, then the space of components of the Julia set of G (Julia set is the set of points in the Riemann sphere $\overline{\mathbf{C}}$ in which G is not normal) has a total order " $\leq$ ", where for two compact connected sets K_1, K_2 in $\overline{\mathbf{C}}$, "$K_1 \leq K_2$" indicates that $K_1 = K_2$, or K_1 is included in a bounded component of $\overline{\mathbf{C}} \setminus K_2$.

Using the above result and combining it with the theory of random dynamics of complex polynomials, we consider the following situation: Let τ be a Borel probability measure in the space $\{g \in \mathbf{C}[\mathbf{z}] \mid \deg(\mathbf{g}) \geq \mathbf{2}\}$ with topology induced by the uniform convergence on the Riemann sphere $\overline{\mathbf{C}}$. We consider the i. i. d. random dynamics in $\overline{\mathbf{C}}$ such that at every step we choose a polynomial according to the distribution τ. Let $T_\infty(z)$ be the probability of tending to $\infty \in \overline{\mathbf{C}}$ starting from the initial value $z \in \overline{\mathbf{C}}$ and let G_τ be the polynomial semigroup generated by the support of τ. Suppose that the support of τ is compact, the postcritical set of G_τ is bounded in the complex plane and the Julia set of G_τ is disconnected. Then, we show the following results:

1. In each component U of the complement of the Julia set of G_τ, $T_\infty|_U$ equals a constant C_U.
2. $T_\infty : \overline{\mathbf{C}} \to [0,1]$ is a continuous function in the whole $\overline{\mathbf{C}}$.
3. If J_1, J_2 are two components of the Julia set of G_τ with $J_1 \leq J_2$, then $\max_{z \in J_1} T_\infty(z) \leq \min_{z \in J_2} T_\infty(z)$.

Hence T_∞ is similar to the devil's staircase function.

2000 Mathematics Subject Classification. Primary 37F10; Secondary 30D05.
Key Words and Phrases. Complex dynamical systems, polynomial semigroups, random dynamical systems, devil's-staircase-like function.

The Hypergeometric Functions and Some Formulas of Trigonometric Sums

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Abstract

For a positive constant m and a positive integer n, we use the the following notations for the Gauss hypergeometric function $F(-n, 2m; 2m+n+1; z)$:

$$R(-n, 2m; 2m+n+1; t) := \mathrm{Re}\big\{F(-n, 2m; 2m+n+1; e^{it})\big\}$$

and

$$I(-n, 2m; 2m+n+1; t) := \Im\big\{F(-n, 2m; 2m+n+1; e^{it})\big\}.$$

Here we derive a formula of trigonometric sums given by

$$\begin{aligned} n\Big(& I(-n+1, 2m; 2m+n+1; t)R(-n, 2m; 2m+n+1; t) \\ &-R(-n+1, 2m; 2m+n+1; t)I(-n, 2m; 2m+n+1; t)\Big) \\ =& \frac{(n+1)_n}{(2m+n+1)_n} \sum_{s=1}^{n} \frac{(-n)_s (n+1-s)_{n-s} (2m)_s \; s \; y^{s-1}}{(-2n)_s (2m+n+1)_{n-s} \; s!} \; \sin t \\ =& \sum_{s=1}^{n} \frac{(2m)_s}{(2m+n+1)_n (2m+n+1)_{n-s}} \binom{n}{s} \binom{2n-s}{n} \big(2(n-s)\big)! \; s \; y^{s-1} \\ & \cdot \sin t \qquad \big(y = 2(1-\cos t)\big). \end{aligned}$$

2000 Mathematics Subject Classification. Primary 33B10, 33C05; Secondary 60E05.

Key Words and Phrases. Gauss hypergeometric function, trigonometric sums.

Some Properties of the Kampé de Fériet Hypergeometric Function
$F^{2:1;1}_{1:1;1}\left[x,y\right]$

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Abstract

Many of the potentially useful properties of the Kampé de Fériet hypergeometric function have been investigated systematically in several earlier works as detailed in the monographs by (for example) P. Appell and J. Kampé de Fériet (1926), H. M. Srivastava and H. L. Manocha (1984), and H. M. Srivastava and P. W. Karlsson (1985). In this article we continue these earlier investigations of properties of the following *special* Kampé de Fériet hypergeometric function of two variables:

$$F^{2:1;1}_{1:1;1}\left[\begin{matrix} a_1, a_2 : b_1; c_1; \\ e : f_1 ; g_1; \end{matrix}\ x, y\right] = \sum_{m,n=0}^{\infty} \frac{(a_1)_{m+n}(a_2)_{m+n}(b_1)_m (c_1)_n}{(e)_{m+n}(f_1)_m (g_1)_n m!n!} x^m y^n. \tag{1}$$

By using the familiar operator method of J. L. Burchnall and T. W. Chaundy [*Quart. J. Math. Oxford Ser.* **11** (1940), 249-270], we derive 11 pairs of decompositions for the special Kampé de Fériet hypergeometric function of two variables, defined by (1). These decompositions are expressed through the products of the Gaussian and Clausenian hypergeometric functions. Here are some of the decomposition formulas proved by us:

$$\begin{aligned} &F^{2:1;1}_{1:1;1}\left[\begin{matrix} a_1, a_2 : & b_1; & c_1; \\ e : & f_1; & g_1; \end{matrix}\ x, y\right] \\ &= \sum_{i,j,k=0}^{\infty} \frac{(-1)^i (e)_{2i} (a_1)_{2i+2j+k} (a_2)^2_{2i+j+k} (b_1)_{i+j+k} (c_1)_{i+j+k}}{(e+i-1)_i (a_2)_{2i+j} (e)^2_{2i+j+k} (f_1)_{i+j+k} (g_1)_{i+j+k} i!j!k!} x^{i+j+k} y^{i+j+k} \\ &\quad \cdot {}_3F_2\left(\begin{matrix} a_1+2i+2j+k, a_2+2i+j+k, b_1+i+j+k; \\ f_1+i+j+k, e+2i+j+k; \end{matrix}\ x\right) \\ &\quad \cdot {}_3F_2\left(\begin{matrix} a_1+2i+2j+k, a_2+2i+j+k, c_1+i+j+k; \\ g_1+i+j+k, e+2i+j+k; \end{matrix}\ y\right); \end{aligned} \tag{2}$$

$$\begin{aligned} &F^{2:1;1}_{1:1;1}\left[\begin{matrix} a_1, a_2 : & b_1; & c_1; \\ e : & f_1; & g_1; \end{matrix}\ x, y\right] = \sum_{i,j=0}^{\infty} \frac{(a_1)_{2i+j} (a_2)^2_{i+j} (b_1)_{i+j} (c_1)_{i+j}}{(a_2)_i (e)_{2i+2j} (f_1)_{i+j} (g_1)_{i+j} i!j!} x^{i+j} y^{i+j} \\ &\cdot F^{0:3;3}_{1:1;1}\left[\begin{matrix} - : & b_1+i+j, a_1+2i+j, a_2+i+j; & c_1+i+j, a_1+2i+j, a_2+i+j; \\ e+2i+2j : & f_1+i+j; & g_1+i+j; \end{matrix}\ x, y\right]. \end{aligned}$$

From the decomposition formula (2) follow the decompositions of the Appell hypergeometric function $F_4(\alpha, \beta; \gamma_1, \gamma_2; x, y)$ that are already well-known. With the help of the above-mentioned operator method, we can similarly derive analogous decompositions for other Kampé de Fériet hypergeometric functions of two variables (see also a closely-related work by H. M. Srivastava [*J. Reine Angew. Math.* **245** (1970), 47-54]).

2000 Mathematics Subject Classification. Primary 33C20, 33C65; Secondary 33C05.
Key Words and Phrases. Kampé de Fériet hypergeometric functions of two variables, Gaussian and Clausenian hypergeometric functions, Appell functions.

The Use of the Fractional Derivatives to Expand Analytical Functions in Terms of Quadratic Functions with Applications to Special Functions

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Abstract

In 1971, T. J. OsIer proposed a generalization of Taylor's series of $f(z)$ in which the general term is

$$\left[D_{z_0-b}^{an+\gamma} f(z_0)\right] (z-z_0)^{an+\gamma}/\Gamma(an+\gamma+1),$$

where $0 < a \leq 1$, $b \neq z_0$, γ is an arbitrary complex number, and D_z^α is the fractional derivative of order α. In this paper, we present a new expansion of an analytic function $f(z)$ in $\mathbf{R}$ in terms of a power series $\theta(t) = tq(t)$, where $q(t)$ is any regular function and t is equal to the quadratic function $[(z-z_1)(z-z_2)]$ $(z_1 \neq z_2)$, where z_1 and z_2 are two points in $\mathbf{R}$ and the region of validity of this formula is also deduced.

To illustrate the concept, if $q(t) = 1$, the coefficient of $(z-z_1)^n(z-z_2)^n$ in the power series of the function $(z-z_1)^\alpha(z-z_2)^\beta f(z)$ is

$$D_{z_1-z_2}^{-\alpha+n}\left[f(z_1)(z_1-z_2)^{\beta-n-1}(z_1-z_2+z-w)\right]|_{w=z_1}/\Gamma(1-\alpha+n),$$

where α and β are arbitrary complex numbers. Many special forms are examined and some new identities involving special functions and integrals are obtained.

2000 Mathematics Subject Classification. Primary 26A33, 41A58; Secondary 33E20, 33C20, 33C65, 32A05.

Key Words and Phrases. Fractional derivatives, Taylor's theorem, Laurent's series, Power series, Quadratic functions, Special functions, H-functions.

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A Survey of Fractional-Calculus Approaches to the Solutions of the Bessel Differential Equation of General Order

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Abstract

In a remarkably large number of recent works, one can find the emphasis upon (and demonstrations of) the usefulness of fractional calculus operators in the derivation of (explicit) particular solutions of significantly general families of linear ordinary and partial differential equations of the second and higher orders. The main object of this presentation is to survey some earlier investigations of this simple fractional-calculus approach to the solutions of the classical Bessel differential equation of general order and to show how it would lead naturally to several interesting consequences which include (for example) an alternative derivation of the complete power-series solutions obtainable usually by the Frobenius method. The underlying analysis presented here is based chiefly upon some of the general theorems on (explicit) particular solutions of a certain family of linear ordinary fractional differintegral equations with polynomial coefficients.

2000 Mathematics Subject Classification. Primary 26A33; Secondary 33C10, 34A05.

Key Words and Phrases. Operators of fractional calculus, Bessel differential equation, Fuchsian (and non-Fuchsian) differential equations, (ordinary and partial) linear differential equations, Frobenius method, Bessel functions, trigonometric function, hypergeometric representations.

A Relation Among Subclasses Defined by Ruscheweyh Derivative on Analytic Functions with Negative Coefficients

(Dedicated to Professor H. M. Srivastava on his 65th Birth Anniversary)

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Abstract

We consider the Ruscheweyh derivative D^n, which is defined by

$$D^0 f(z) = f(z),\ D^1 f(z) = Df(z) = zf'(z)$$

and

$$D^n f(z) = \frac{z\left(z^{n-1}f(z)\right)^{(n)}}{n!} \quad (n = 2, 3, 4, \cdots),$$

on the following class of analytic functions:

$$\mathcal{T} := \left\{ f : f(z) = z - \sum_{n=2}^{\infty} a_n\, z^n\ (a_n \geq 0;\ n = 2, 3, 4, \cdots) \text{ is analytic in } \mathbf{U} \right\}.$$

In this talk we discuss a relation among the subclasses $\mathcal{TR}(n, m; \alpha)$ of the class $\mathcal{T}$, where

$$\mathcal{TR}(n, m; \alpha) := \left\{ f : f \in \mathcal{T} \quad \text{and} \quad \Re\left(\frac{D^{n+m} f(z)}{D^n f(z)}\right) > \alpha \quad (z \in \mathbf{U}) \right\}$$

for $n \in \{0, 1, 2, \cdots\}$, $m \in \{1, 2, 3, \cdots\}$ and $\alpha \in [0, 1)$, $\mathbf{U}$ being the open unit disk: $|z| < 1$.

2000 Mathematics Subject Classification. Primary 30C45.

Key Words and Phrases. Analytic functions with negative coefficients, Ruscheweyh derivative.

A Further Expansion of the Riemann ζ-Function

(Dedicated to Professor H. M. Srivastava in Honour of his 65th Birth Anniversary)

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Abstract

A further explicit expansion is presented for the Riemann ζ-function in terms of the basis-set involving the Digamma (or ψ-) function via the expansion of Tricomi's Ψ-function.

2000 Mathematics Subject Classification. Primary 11M06, 33B15; Secondary 33C15.

Key Words and Phrases. Riemann Zeta function, Hurwitz (or generalized) Zeta function, Digamma (or ψ-) function, Tricomi's Ψ-function.

Theorems Relating the Riemann-Liouville and the Weyl Fractional Integrals to Various Integral Transforms and Their Applications

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Abstract

In this work we will introduce theorems relating the Riemann-Liouville fractional integrals and the Weyl fractional integrals to some well-known integral transforms including Laplace transforms, Stieltjes transforms, and generalized Stieltes transforms. As applications of the theorems and their consequences, a number of infinite integrals of elementary functions and special functions are evaluated, and solution techniques for integral equations are introduced. Some illustrative examples will also be presented.

2000 Mathematics Subject Classification. Primary 44A10, 44A15; Secondary 33C10, 44A35.

Key Words and Phrases. Riemann-Liouville fractional integrals, Weyl fractional integrals, Laplace transforms, Stieltjes transforms, generalized Stieltjes transforms.