Alberta Conference for Young Researchers in Mathematics
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Abstracts

Some history of analysis and France at the turn of the 19th century
Garth Boucher, University of Calgary

The turn of the 19th century was a vibrant time in Europe and paricularly in France. The Enlightenment and industrial revolution were in full swing, the scientific revolution was still accelerating smoothly and social unrest was in the air.
Popular revolutions in America and France were setting the stage for a new world political order. Indeed, many historians use the French revolution and World War I to delineate an age known as the "Pre-modern era". Intellectuals such as those part of the "Philosophe" movement fuelled every facet of these changing times. And mathematics was certainly not exempt from the impact of this electric environment.
Mathematical rigor in analysis can arguably trace its roots to this period. The pivot of these talks is Joseph Fourier and his 1807 submission of an infinite trig series modelling heat flow. The physical implications of this paper pale in comparison to the rippling effect Fourier's series had in redifining such analytical notions as functions, continuity and convergence.
These aspects of analysis will be explored in the backdrop of a remarkable era.

Non commutative function spaces
Shawn Desaulniers, University of Alberta

In this talk first I will review some background from abstract harmonic analysis. Then I will discuss as class of Dunford-Pettis operators associated to a locally compact group G.

A geometric interpretation of the trace of a matrix
Lynn Dover, University of Alberta

Matrices represent linear transformations. As such, any property of a matrix which is invariant under a change of basis, must represent some quality intrinsic to the transformation. For instance, the determinant of a matrix gives the (signed) volume of the image of the unit n-cube. (Sign representing orientation.) It also happens to be the product of the eigenvalues which makes geometric sense. Now the trace of a matrix is also change of basis invariant, but it is the sum of the eigenvalues. Geometrically, what does this tell us about the transformation? I will discuss a geometric interpretation of the trace and, if time allows, outline some ways in which it can make proofs and definitions more intuitively clear.

Strange Attractor of the Henon Map
Matthew Emmett, University of Calgary

In this talk we will define what a "strange attractor" is, and explore an example of one through the Henon map. We will begin by defining terms such as fixed point, stability, Lyapunov exponent, chaos, trapping region, attractor, and, finally, strange attractor. We will then define the Henon map and explore its properties both analytically and numerically.

Lyapunov Functions and Global Dynamics of A Mathematical Model of Tuberculosis (TB)
Hongbin Guo, University of Alberta

Lyapunov's Direct Method has gained increasing significance and has given decisive impetus for modern development of stability theory of dynamic systems during the past century. In this talk a mathematical model for the intrinsic transmission population dynamics of Tuberculosis (TB) is considered. Global dynamics of the model are rigorously established by using global Lyapunov functions. Mathematical analysis establishes that the global dynamics of the model are completely determined by a basic reproduction number R_0. If $R_0 \leq 1$, the TB always dies out. If $R_0 > 1$, the TB becomes endemic, and a unique endemic equilibrium is globally asymptotically stable in the interior of the feasible region.

Estimating sensitivity and Specificity for Binocular Data: Application in Ophthalmology
Meijie Guo, University of Calgary

Binocular data typically arise in ophthalmology, where pairs of eyes are screened, through some diagnostic procedure, for the presence of certain abnormalities or pathologies. Treating the eyes as independent and adopting the usual approach in estimating the sensitivity and specificity of a diagnostic test ignores the correlation between the eyes, and may consequently yield incorrect estimates, especially of the standard errors. This talk proposes a likelihood-based method of estimating sensitivity and specificity using a parametric model for paired binary outcomes. Estimation of model parameters via MLE is outlined and approximate tests are provided. The efficiency of the model was assessed by simulation study. Data from a diabetic retinopathy are analysed to illustrate the method.

Relation Algebras
Csaba Henk, University of Calgary

Relation algebras are a fascinating crossroad beetwen many important branches of algebra and logics.
A set S gives rise to a (representable) relation algebra R as follows: the bset of R is the set of all binary relations on S, and we supply it with all the natural operations which can be executed on binary relations.
That is, we take the Boolean operations, composition, converse and the identical relation as a constant. Thus R is a Boolean algebra and a monoid (semigroup with unit element) at the same time. The set of permutations forms a full symmetric subgroup, and much of the expressive power of cylindric algrebras are captured by relation algebras.
The axiomatization of the class of such algrebras is know to be hard. There were many failed attempts for giving a "convetional" axiom scheme. It just happened recently that Hodkinsdon has given an elegant axiomatization using games.

Pricing on a two interest rates jump diffusion financial market
Selly Kane, University of Alberta

This paper deals with the problem of hedging contingent claims in the framework of a two factors jump–diffusion model with different credit and deposit rates. We derive the upper and lower hedging for European options.

On Quasiuniform convergence
Damir Kinzebulatov, University of Calgary

The notion of quasiuniform convergence, originally introduced by A.Arzela is discussed. One of the properties of quasiuniform convergence is that for a given pointwise convergent sequence of continuous functions defined on a closed interval, quasiuniform convergence (unlike uniform convergence) is not only sufficient but also necessary condition of continuity of limit function.

On the Relative Distances of Seven Points in a Plane Convex Body
Dr. Zsolt Langi, University of Calgary

Let C be a convex body in the Euclidean plane. The relative distance of points p and q is twice the Euclidean distance of p and q divided by the Euclidean length of a longest chord in C with the direction, say, from p to q. We prove that, among any seven points of a plane convex body, there are two points at relative distance at most one, and one cannot be replaced by a smaller value. We apply our result to determine the diameter of point sets in normed planes.

Derivation of Prey–taxis and Its Application
Jungmin Lee, University of Alberta

Prey-predator models have been studied by numerous people. Spatial homogeneous models consider the temporal variations of each species where the effects of movement are included only implicitly. On the other hand, diffusion–reaction models explicitly describe spatial structures due to motility of species and heterogeneous environments. However, the modelling of predator movement by simple diffusion fails to consider a characteristic feature of living organisms is that they respond to the environment to search for food, mate, reproduction etc. In this talk, I derive a model of spatial predator dynamics in response to prey and its density, which includes diffusion, prey–taxis, and reaction terms. In the end of talking, I will consider the effects of prey–taxis terms for predators to slow down prey spread in the comparison with diffusion–only by analyzing travelling wave solutions in the interaction between prey and predators.

Algebraic Cryptography
Dr. Stephane Lemieux, University of Calgary

Algebraic cryptography attempts to apply the concepts of classical, number theoretic cryptosystems in a more abstract algebraic setting. Often the underlying algorithms are dependent on operations in non-abelian, and possibly infinite, groups, semigroups and rings. It is hoped that the added complexity required for computation is offset by additional security against attacks on the resulting cryptosystems.
We give an overview of the justification for the recent rise in interest in the field of algebraic cryptography as well as the challenges and criticisms facing algebraic cryptographers. "A new public key cryptosystem over the braid groups" (Ko et. al.) and the current research of the speaker into cryptosystems based on shortlex automatic groups are used to exemplify these challenges.

Optimal Two-Period Two-Treatment Repeated Measurement Designs With Or Without Baseline Measurements
Yuanyuan Liang, University of Alberta

We consider the problem of finding optimal two-period two-treatment repeated measurements designs with or without baseline observations based on the traditional model (proposed by Hedayat and Afsarinejad 1978) and the self and mixed carryover effects model (proposed by Afsarinejad and Hedayat 2002), respectively. With the equicorrelated covariance structure, we found that baseline observations improve efficiency considerably under both models.

Antipodal sets in Hyperbolic spaces
Marton Naszodi, University of Calgary

An antipodal set in Euclidean n-space is a set of points with the property that through any two of them there is a pair of parallel hyperplanes supporting the set. Translating this notion to hyperbolic n-space is not obvious. In this talk I will discuss the various possible ways to do this and find the maximal cardinality of a hyperbolic antipodal set (according to the different definitions).

An Introduction to Ball-Polytopes and 1-Convexity.
Peter Papez, University of Calgary

The study of polytopes is one of the oldest and most well researched areas in all of mathematics. One way of looking at polytopes is to interpret them as the region bounded by intersecting hyperplanes. These hyperplanes are just surfaces of zero curvature. Suppose that we use surfaces of non-zero curvature, say of curvature one. What do we obtain by doing this? With some care we obtain ball-polytopes. Intuitively, we can think of these as fattened polytopes, but the concept is more delicate than may first appear. Furthermore, in this setting we obtain a new, more general, notion of convexity which we call 1-convexity. A set is 1-convex if it satisfies the following two properties. First, the distance between any two points in the set is at most two. Second, given any two points in the set, the intersection of all unit balls containing those two points is also contained in the set. The aim of this talk is to survey the results obtained in the study of ball-polytopes and 1-convexity. Most results pass to higher dimensions, but we will focus on the two- and three-dimensional cases to provide insight regarding the techniques used in this field of study.

Banach-Mazur Distance and Random Convex Bodies
Peter Pivovarov, University of Alberta

Banach-Mazur distance is an important tool used in studying symmetric convex bodies in $\mathbb{R}^n$. In 1981, Gluskin presented random symmetric convex bodies that have maximal Banach-Mazur distance. A non-symmetric analog of these bodies was recently used by Gluskin, Litvak and Tomczak-Jaegermann to prove the existence of a convex body lacking symmetric projections. I will discuss the basic methods and tools involved in studying such bodies and briefly outline a proof of this theorem.

Pseudospectrum
Bryan Quaife, University of Calgary

It is well known that for any given initial condition, a discrete linear dynamical system will converge to zero if and only if the spectrum of the system lies strictly inside the unit circle of the complex plane.
However, this tells us nothing about the behaviour of the system at any given finite time. Using the pseudospectrum of the system, one can find a nice lower bound for the norm of any term in the sequence of time steps.
I will show what this lower bound is, and a nice algorithm that converges to this lower bound.

Using History to Teach Mathematics
Tina Rapke, University of Calgary

This talk will look at using Math History to approach some of the possible problems one might experience while learning or teaching mathematics. Examples of using history to teach aspects of mathematics will be given and demonstrated. A look will also be taken at different teaching techniques and learning styles.

Uniformly distributed sequences: some examples and conjectures
Omar Rivasplata, University of Alberta

The theory of uniform distribution modulo one, a modern branch of number theory, had its roots in diophantine approximations and became an intense area of research early in the 20th century, especially after the seminal work of H. Weyl. This theory is concerned with the distribution of fractional parts of real numbers on the unit interval [0,1). In this talk I will present the basics of uniform distribution modulo one of real sequences, including Weyl's criterion and the results of Fejr and van der Corput. Finally, I will discuss some examples of uniformly distributed sequences and some conjectures about the distribution of other sequences.

An Axiomatic Approach to Primary Degree
Haibo Ruan, University of Alberta

Many mathematical models of natural phenomena exibit symmetric properties related to some physical or geometric regularities. These models have been studied using different topological techniques. The equivariant degree is an important alternative to those approaches. The construction of the equivariant degree usually involves the equivariant topology jargon, which makes the actual computation of the equivariant degree more subtle and delicated. In my talk, I would like to present an axiomatic approach to the primary equivariant degree(the primary part of the equivariant degree, which presents, from the practical point of view, the most important portion of the complete degree), which gives us certain freedom to carry on the computations without refering to the complicated technical construction. Also, I would like to mention, as a paricular case, when we are dealing with the S1-primary degree, under the axioms, the S1-degree is completely computable, which, at the same ti me, gives us a "basis" to do all other computations for a compact Lie group G, with one free parameter.

Art and Geometry
Bouchra Sabbagh, University of Calgary

In this talk I will give a brief introduction to Frieze groups and classification of Wall-Pattern groups. Also, I will analyse some artwork if time permit.

A Game of Cops and Robber
Adrian Tang, University of Calgary

The game Cops and Robber is a pursuit game played on the vertices of a graph. There are k cops, for a fixed positive integer k and one robber. The k cops and the robber alternate moves. A move is defined as either moving from one vertex to an adjacent vertex or staying on the same vertex. This talk will first introduce basic graph theory definitions. The game of Cops and Robber will then be described. The question of interest is finding the minimum number of cops required for the cops to capture the robber on a given graph G. This number is denoted by s(G). The talk will present some basic results for finding lower and upper bounds of s(G) for certain classes of graphs and will give a brief outline of how to find s(G) if G is a tree.

Introduction to Seifert manifolds
Satoshi Tomoda, University of Calgary

In 1904, Poincar posed a question: if a closed 3-dimensional manifold X has the same homology groups as the 3-sphere, is X homeomorphic to the 3-sphere? He soon discovered a counterexample to his question and revised it to: if a closed 3-dimensional manifold X has the same homotopy groups as the 3-sphere, is X homeomorphic to the 3-sphere? This question is now referred to as Poincar Conjecture and is one of the seven Millennium Problems selected by the Clay Mathematics Institute.
The counterexample to the first question is known as Poincar homology sphere, it has a fundamental group of order 120. In 1932, Seifert introduced a large class of 3-manifolds, now called Seifert fibred spaces, or simply Seifert manifolds. In his paper, he determined the fundamental groups and homology groups of Seifert manifolds. It appears that one of his objectives may have been to find a counterexample to the Poincar Conjecture, since he mentions the Poincar homology sphere as an example of a Seifert manifold with finite fundamental group that is also a homology sphere. In fact, he defined Poincar (fibred) spaces. In this talk, we will define what Seifert manifolds are and describe some of the properties. Time permitting, we will demonstrate how to construct the Poincar homology sphere.

-1 = 1 + 2 + 4 + 8 + 16 + 32 + ...
Tracy Walker, University of Calgary

In this talk I will start by defining and providing examples of valuations. I will then construct the p-adic fields by completing the rationals with respect to the p-adic valuation. Interesting properties of p-adic fields will be discussed.

On Milloux's Theorem
Qian Wang, University of Alberta

A theorem of H. Milloux(1934) states that if $a : (0, \infty) \to \mathbb{R}$ is positive and nondecreasing, then the scalar equation $u'' + a(t)u = 0$ has a nontrivial solution $u = u_0(t)$ such that $\lim_{t\to\infty} u_0(t) = 0$ if and only if $\lim_{t\to\infty} a(t) = \infty$. This led us to a conjecture that for the hyperbolic equation

$$u_{tt} = u_{xx} - a(t)u, \quad 0 < x < 1,\ t > 0,$$
$$u(0, t) = u(1, t) = 0, \quad t \geq 0,$$

if $\lim_{t\to\infty} a(t) = \infty$, then

$$\operatorname{codim}\left\{u : \int_0^1 u^2 dx \xrightarrow{t\to\infty} 0\right\} < 2$$

We show that this conjecture is false.

Asymptotic Behavior of Solutions to Dissipative Nonlinear Evolutions with Ellipticity

Zhian Wang, University of Alberta

In this talk, we examine the global existence and the asymptotic behavior of the solutions to the Cauchy problem for the following nonlinear evolution equations with ellipticity and dissipative effects

$$\begin{cases} \psi_t = -(1-\alpha)\psi - \theta_x + \alpha\psi_{xx}, \\ \theta_t = -(1-\alpha)\theta + \nu\psi_x + 2\psi\theta_x + \alpha\theta_{xx}, \end{cases} \tag{E}$$

with initial data

$$(\psi,\theta)(x,0) = (\psi_0(x),\theta_0(x)) \to (\psi_\pm,\theta_\pm) \quad \text{as} \quad x \to \pm\infty, \tag{I}$$

where α and ν are positive constants such that $\alpha < 1$, $\nu < \alpha(1-\alpha)$. By constructing a correct function $\hat{\theta}(x,t)$ defined by (2.13) and using the energy method, we show $\sup_{x\in\mathbb{R}}(|(\psi,\theta)(x,t)| + |(\psi_x,\theta_x)(x,t)|) \to 0$ as $t \to \infty$ and the solutions decay with exponential rates. Finally, we will touch a littile bit about the optimal decay estimates of the solution if time permits.

Calculus on Subcartesian Spaces

Jordan Watts, University of Calgary

Subcartesian spaces were first introduced by Aronszajn in 1967, a result from his study of Bessel potentials. Since then the theory of these mathematical entities has developed further, finding applications in many places, such as Hamiltonian mechanics and geometric quantisation. In this presentation, we shall examine some basic notions of calculus on subcartesian spaces, such as structurally regular points, tangent spaces and (time permitting) a brief discussion on forms and their evaluation at structurally singular points.

Redundant Number Systems in Cryptography

Kjell Wooding, University of Calgary

Cryptography is a funny business. Much of the time, it concerns itself with tedious calculations involving squaring and multiplying in finite fields. This talk will focus on making all these finite field calculations a little less tedious by incorporating redundancy. Specificaly, this talk will focus on a two-dimensional integer representation called the Double-Base Number System, and explain how it can make the business of modular exponentation—or even Elliptic Curve point addition—just a little bit more managable.

Minimum Hellinger Distance Estimators for Mixing Proportions: Finite Support Case

Jingjing Wu, University of Alberta

Mixture models are widely used in many fields. After Hosmer introduced this model, many methods have been developed, e.g. maximum likelihood, Bayesian parametric techniques and various distribution-free kernel methods. In this talk, we will consider the minimum Hellinger distance estimator (MHD) of the mixing proportions, since it has both good efficiency and excellent robustness properties. Several MHD estimators will be suggested, based on either single data set or combined data sets. The consistency and asymptotic normality of the MHD estimators for finite-supported densities will be addressed.

Is there a m-th root for y in G?
Qingquan Wu, University of Illinois

Given a finite cyclic group G with known order, an element y in G, and an positive integer m, determine whether or not y has a m-th root in G. That is, if there is some element x in G, such that mx=y. We present an efficient algorithm to solve the above problem. Also, if the answer is yes, we compute the number of such x's. Our contribution is not to show how to compute y, which is hard even if G has very particular structure, but to provide a way for the 'primality test' problem of finding such y. We successfully solve this 'primality test' problem in linear time both in the number of group operations needed and the size of m. This problem comes from the presenter's research in function fields so it has immediate applications.

A group sequential test with longitudinal data
Eunha Yang, University of Alberta

In many clinical trials, responses of patients are measured repeatedly over time. For example, a heart rate or a blood pressure would be monitored repeatedly over time. When the objective of a clinical trial is to compare effects of two treatments, interim analyses are required to assess performance of the treatments. This presentaiton will give an overview of the group sequential methods on longitudinal data.

An analysis of chaotic behavor in a networked control systems
Guofeng Zhang, University of Alberta

In this talk, a new network transmission strategy is proposed to reduce netowork traffic in network communications. The resulting system under this transmission strategy exhibits highly nonlinear behariors. We will investigate the following:
1) Construct first-return maps of this nonlinear system and derive the existence conditions of periodic orbits and their properties.
2) Formulate the system as a hybrid system which will ease the succeeding analysis.
3) Give two proofs that this type of hybrid systems is not structurally stable; the first proof is a direct one based on homomorphisms, the second one is from the viewpoint of phase transition which provides much more insight of the behavior of this type of hybrid systems.
4) A glance at higher dimension models with emphasis on the existence of periodic orbits.
5) The study of a class of continuous-time hybrid systems which is the counterpart of the discrete-time one mentioned above.
6) Propose new controller design methods based on this network transmission strategy to improve performance of the individual systems and the whole networks.
Keywords: stability, hybrid systems, first-return maps, bifurcation, tracking, networked control systems.

Financing Conditions in Small Canadian Businesses – An Introduction to Boosting Method
Shali Zhang, University of Calgary

Boosting is a kind of Classification and Regression Tree method developed in recently years. It takes bootstrap samples without replacement to generate weak learners. After combining weak learners into a strong one, boosting can reduce the prediction error dramatically.