FPSAC PROCEEDINGS 2004

ACTES SFCA 2004

Vancouver CANADA
Abstracts

16th Annual International Conference on
FORMAL POWER SERIES AND ALGEBRAIC COMBINATORICS
June 28 - July 2, 2004
University of British Columbia
(Vancouver B.C., Canada)

16ième Conférence Internationale sur
SÉRIES FORMELLES ET COMBINATOIRE ALGÉBRIQUE
28 juin - 2 juillet 2004
University of British Columbia
(Vancouver C.B., Canada)

Programme Committee – Comité de programme

Jonathan Borwein *(Simon Fraser, Dalhousie)*, chair/président
Mike Atkinson *(Otago)*
Nantel Bergeron *(CRC, York)*
Luisa Carini *(Messina)*
William Chen *(Nankai)*
Sylvie Corteel *(CNRS U.V.S.Q.)*
Ira Gessel *(Brandeis)*
Ian Goulden *(Waterloo)*
Michael Joswig *(T.U. Berlin)*
Martin Klazar *(Prague)*
Christian Krattenthaler *(Lyon)*
Gilbert Labelle *(UQAM)*
Soichi Okada *(Nagoya)*
Margaret Readdy *(Kentucky)*
Frank Sottile *(Massachusetts)*
Einar Steingrimsson *(Göteborg)*
Julian West *(Malaspina, Victoria)*, co-chair/co-président
Stephanie van Willigenburg *(UBC)*, co-chair/co-président

Organizing Committee – Comité d'organisation

Julian West *(Malaspina, Victoria)*, chair/président
Nantel Bergeron *(CRC at York)*
Luis Goddyn *(Simon Fraser)*
Marni Mishna *(Bordeaux)*
Tom Roby *(Cal State Hayward)*
Frank Ruskey *(Victoria)*
Stephanie van Willigenburg *(UBC)*

Sponsors – Commanditaires

TABLE OF CONTENTS

PART 1

Invited Speakers

Chromatic Polynomials and Representations of the Symmetric Group

Norman Biggs

The chromatic polynomial $P(G;k)$ is the function which gives the number of ways of colouring a graph G when k colours are available. The fact that it is a polynomial function of k is essentially a consequence of the fact that, when k exceeds the number of vertices of G, not all the colours can be used. Another quite trivial property of the construction is that the names of the k colours are immaterial; in other words, if we are given a colouring, then any permutation of the colours produces another colouring. In this talk I shall outline some theoretical developments, based on these simple facts and some experimental observations about the complex roots of chromatic polynomials of 'bracelets'.

A 'bracelet' $G_n = G_n(B, L)$ is formed by taking n copies of a graph B and joining each copy to the next by a set of links L (with $n+1 = 1$ by convention). The chromatic polynomial of G_n can be expressed in the form

$$P(G_n;k) \;=\; \sum_{\pi} m_{B,\pi}(k)\,\mathrm{tr}(N_L^{\pi})^n.$$

The sum is taken over all partitions π such that $0 \le |\pi| \le b$, where b is the number of vertices of B. The terms $m_{B,\pi}(k)$ are polynomials in k, and they are independent of L. When B is the complete graph K_b the relevant polynomials $m_\pi(k)$ are given by a remarkably simple formula, and when B is incomplete they can be expressed in terms of the $m_\pi(k)$ with $|\pi| \le b$. The entries of the matrices N_L^{π} are also polynomials in k, but they do depend on L. In order to calculate these entries we construct explicit bases for certain irreducible modules, corresponding to the Specht modules of representation theory.

References

(1) N.L.Biggs: Equimodular curves, *Discrete Math.* 259 (2002) 37-57.

(2) N.L. Biggs, M.H. Klin, P. Reinfeld, Algebraic methods for chromatic polynomials, *European J. Combinatorics* 25 (2004) 147-160.

(3) N.L. Biggs, Specht modules and chromatic polynomials, *CDAM Research Report Series* LSE-CDAM-2003-06.

London School of Economics

Decomposable Compositions and Ribbon Schur Functions

Louis J. Billera

Abstract. We describe recent results, obtained in collaboration with Hugh Thomas and Stephanie van Willigenburg [**1**], which provide a complete description of when two ribbon Schur functions are identical.
Résumé. Nous présentons des résultats récents, obtenus en collaboration avec Hugh Thomas et Stephanie van Willigenburg [**1**], qui permettent de déterminer légalité de deux fonctions Schur ruban.

Extended Abstract

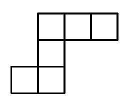
An important basis for the space of symmetric functions of degree n is the set of classical Schur functions s_λ, where λ runs over all partitions of n. For example, the skew Schur functions $s_{\lambda/\mu}$ can be expressed in terms of these by means of the Littlewood-Richardson coefficients $c^\lambda_{\mu\nu}$ by

$$s_{\lambda/\mu} = \sum_\nu c^\lambda_{\mu\nu} s_\nu.$$

These same coefficients give the expressions for the product of two Schur functions, as well as the multiplicity of irreducible representations of the symmetric group in the tensor product of two irreducibles. Thus there is some interest in determining relations among the $c^\lambda_{\mu\nu}$.

A particular type of skew Schur functions are those corresponding to *ribbon* or *border strip* shapes λ/μ. These are connected shapes with no 2×2 square. The resulting skew Schur functions $s_{\lambda/\mu}$ are called *ribbon Schur functions*.

Ribbons of size n are in one-to-one correspondence with compositions β of size n by setting β_i equal to the number of boxes in the i-th row from the bottom. For example, the skew diagram 422/11



is a ribbon, corresponding to the composition 213. We will henceforth indicate ribbon Schur functions by means of the compositions corresponding to their ribbon shapes. Thus $s_{422/11}$ will be denoted s_{213}.

We address the question of when two ribbon Schur functions are identical; that is, for compositions β and γ of n, when is it true that $s_\beta = s_\gamma$? When equality holds, we automatically get Littlewood-Richardson coefficient identities of the form

$$c^\lambda_{\mu,\nu} = c^\rho_{\eta,\nu}$$

for all partitions ν of n, whenever ribbon skew shapes λ/μ and ρ/η correspond to β and γ.

To this end, we define an equivalence relation on the set of compositions of n. For $\beta \vDash n$, let F_β denote the fundamental quasisymmetric function indexed by β. (In [**2**], these functions are denoted L_β.) Since the F_β, $\beta \vDash n$, form a basis for the quasisymmetric functions of degree n, any *symmetric* function, in particular, can be written $F = \sum_{\beta \vDash n} c_\beta F_\beta$.

Definition 1.1. For compositions $\beta, \gamma \vDash n$, we say β and γ are *equivalent*, denoted $\beta \sim \gamma$, if for all symmetric functions $F = \sum c_\alpha F_\alpha$, $c_\beta = c_\gamma$.

For a composition α, we denote by $\lambda(\alpha)$ the unique partition whose parts are the components of β in weakly decreasing order. We write $\alpha \geq \beta$ if α is a *coarsening* of β, that is, α is obtained from β by adding consecutive components. If $\beta \vDash n$ then always $n \geq \beta$. Let $\mathcal{M}(\beta)$ be the *multiset* of partitions determined by all coarsenings of β, that is,

$$\mathcal{M}(\beta) = \{\lambda(\alpha) \mid \alpha \geq \beta\}.$$

It is easy to see that $\mathcal{M}(\beta) = \mathcal{M}(\beta^*)$, where β^* is the reversal of the composition β.

Finally, we define a way of composing two compositions as follows. If $\alpha \models n$ and $\beta \models m$, then we wish to define $\alpha \circ \beta \models nm$. Let $\beta = \beta_1\beta_2\cdots\beta_k$. For $\alpha = n$, then $\alpha \circ \beta = n \circ \beta$ is the composition

$$\underbrace{\beta_1 \cdots \beta_{k-1}(\beta_k + \beta_1)\beta_2 \cdots (\beta_k + \beta_1)\beta_2 \cdots \beta_k}_{n \text{ times}}$$

which is nearly the concatenation of n copies of the composition β, except each pair of adjacent terms $\beta_k\beta_1$ are added. For $\alpha = \alpha_1\alpha_2\cdots\alpha_l$, then $\alpha \circ \beta$ is the usual concatenation of the l compositions $\alpha_i \circ \beta$:

$$\alpha \circ \beta = \alpha_1 \circ \beta \cdot \alpha_2 \circ \beta \cdots \alpha_l \circ \beta.$$

For example $12 \circ 12 = 12132$.

Our main result is

Theorem 1.2. *The following are equivalent for a pair of compositions $\beta, \gamma \models n$:*

(1) $s_\beta = s_\gamma$,
(2) $\beta \sim \gamma$
(3) $\mathcal{M}(\beta) = \mathcal{M}(\gamma)$,
(4) *for some k,*

$$\beta = \beta_1 \circ \beta_2 \circ \cdots \circ \beta_k \quad \text{and} \quad \gamma = \gamma_1 \circ \gamma_2 \circ \cdots \circ \gamma_k,$$

and, for each i, either $\gamma_i = \beta_i$ or $\gamma_i = \beta_i^*$.

Thus, for example, $s_{12132} = s_{13212} = s_{23121} = s_{21231}$, and these four equal no others.

The last condition shows that the size of the equivalence class of β is 2^r, where r is the number of nonpalindromic factors in the unique nontrivial irreducible factorization of β. We always have, for example, that $s_\beta = s_{\beta^*}$.

References

[1] L.J. Billera, H. Thomas and S. van Willigenburg, Decomposable Compositions, Symmetric Quasisymmetric Functions and Equality of Ribbon Schur Functions, preprint, May 2004.

[2] R. Stanley, *Enumerative Combinatorics, Vol. 2*, Cambridge Studies in Advanced Mathematics, Vol. 62, Cambridge University Press, Cambridge, UK, 1999.

Intersecting Schubert Varieties

Sara Billey

In the late 1800's, H. Schubert was studying classical intersections of linear subspace arrangements. A typical *Schubert problem* asks how many lines in $\mathbb{C}^3$ generically meet 4 given lines? The generic answer, 2, can be obtained by doing a computation in the cohomology ring of the Grassmannian variety of 2-dimensional planes in $\mathbb{C}^4$. During the past century, the study of the Grassmannian has been generalized to the flag manifold where one can ask similar questions in enumerative geometry.

The flag manifold $\mathcal{F}_n(\mathbb{C}^n)$ consists of all complete flags $F_i = F_1 \subset F_2 \subset \cdots \subset F_n = \mathbb{C}^n$ where F_i is a vector space of dimension i. A modern Schubert problem asks how many flags have relative position u, v, w to three fixed flags X_i, Y_i and Z_i. The solution to this problem used over the past twenty years, due to Lascoux and Schützenberger, is to compute a product of Schubert polynomials and expand in the basis of Schubert polynomials. The coefficient indexed by u, v, w is the solution. This represents a computation in the cohomology ring of the flag variety. It has been a long standing open problem to give a combinatorial rule for expanding these products proving the coefficients $c_{u,v,w}$ are nonnegative integers. It is known from the geometry that these coefficients are nonnegative because they count the number of points in a triple intersection of Schubert varieties with respect to three generic flags.

The main goal of this talk is to describe a method for directly identifying all flags in $X_u(F_i) \cap X_v(G_i) \cap X_w(H_i)$ when $\ell(u) + \ell(v) + \ell(w) = \binom{n}{2}$ and F_i, G_i, H_i are generic, thereby computing $c_{u,v,w}$ explicitly. In 2000, Eriksson and Linusson have shown that the rank tables of intersecting flags are determined by a combinatorial structure they call *permutation arrays*. We prove there is a unique permutation array for each nonempty 0-dimensional intersection of Schubert varieties with respect to flags in generic position. Then we use the structure of this permutation array to solve a small subset of the rank equations previously needed to identify flags in the given intersection. These equations are also useful for determining monodromy and Galois groups on specified collections of flags. This is joint work with Ravi Vakil at Stanford University.

Department of Mathematics, University of Washington, Seattle, WA 98195-4350

Alexander Duality in Combinatorics

Takayuki Hibi

Alexander duality theorem plays a vital role in [**7**] to show that the second Betti number of the minimal graded resolution of the Stanley–Reisner ring $K[\Delta]$ of a simplicial complex Δ is independent of the base field K. On the other hand, a beautiful theorem by Eagon and Reiner [**2**] guarantees that the Stanley–Reisner ideal I_Δ of Δ has a linear resolution if and only if the Alexander dual $\Delta^\vee$ of Δ is Cohen–Macaulay.

With a survey of the recent papers [**3**], [**4**], [**5**] and [**6**], my talk will demonstrate how Alexander duality is used in algebraic combinatorics. More precisely,

- Let $\mathcal{L}$ be a finite meet-semilattice, P the set of join-irreducible elements of $\mathcal{L}$, and $K[\{x_q, y_q\}_{q \in P}]$ the polynomial ring over a field K. We associate each $\alpha \in \mathcal{L}$ with the squarefree monomial $u_\alpha = \prod_{q \leq \alpha} x_q \prod_{q \not\leq \alpha} y_q$. Let $\Delta_{\mathcal{L}}$ denote the simplicial complex on $\{x_q, y_q\}_{q \in P}$ whose Stanley–Reisner ideal is generated by those monomials u_α with $\alpha \in \mathcal{L}$. In the former part of my talk, combinatorics and algebra on the Alexander dual $\Delta_{\mathcal{L}}^\vee$ of $\Delta_{\mathcal{L}}$ will be discussed.
- One of the fascinating results in classical graph theory is Dirac's theorem on chordal graphs ([**1**]). In the latter part of my talk, it will be shown that, via Hilbert–Burch theorem together with Eagon–Reiner theorem, Alexander duality naturally yields a new and algebraic proof of Dirac's theorem.

No special knowledge is required to enjoy my talk.

References

[1] G. A. Dirac, On rigid circuit graphs, *Abh. Math. Sem. Univ. Hamburg* **25** (1961), 71 – 76.

[2] J. A. Eagon and V. Reiner, Resolutions of Stanley–Reisner rings and Alexander duality, *J. Pure and Appl. Algebra* **130** (1998), 265 – 275.

[3] J. Herzog and T. Hibi, Distributive lattices, bipartite graphs and Alexander duality, preprint, June 2003, math.AC/0307235.

[4] J. Herzog and T. Hibi, Level rings arising from meet-distributive meet-semilattices, preprint, March 2004, math.AC/0403534.

[5] J. Herzog, T. Hibi and X. Zheng, Dirac's theorem on chordal graphs and Alexander duality, *Europ. J. Combin.*, to appear, math.AC/0307224.

[6] J. Herzog, T. Hibi and X. Zheng, The monomial ideal of a finite meet-semilattice, preprint, November 2003, math.AC/0311112.

[7] N. Terai and T. Hibi, Alexander duality theorem and second Betti numbers of Stanley–Reisner rings, *Adv. Math.* **124** (1996), 332 – 333.

Sums of polynomials from degeneration in algebraic geometry

Allen Knutson

Many interesting polynomials with positive coefficients are the "multidegrees" of (irreducible) algebraic varieties. We need to break these unbreakable objects, as we'd like to have formulae for these polynomials as positive sums; this breakage can be achieved through degeneration of the defining polynomials (as of a conic to the union of two lines, giving the degree formula 2=1+1).

I'll give many examples related to Schubert varieties, and explain how the geometry of the degeneration helps control the combinatorics, in suggesting shellings of related simplicial complexes. This work is joint with Ezra Miller and Alex Yong.

Department of Mathematics, University of California at Berkeley
E-mail address: allenk@math.berkeley.edu

Operads and combinatorics

Jean-Louis Loday

In some problems it is necessary to consider series indexed, not by integers, but by some other combinatorial objects like trees for instance. In order to multiply and compose these series one needs to be able to add trees and to multiply them. Most of the time the structure which unravel these constructions is a new type of algebras, or, equivalently, an *operad*.

For instance, consider the algebras equipped with two binary operations $\prec$, called *left*, and $\succ$, called *right*, satisfying the following relations

$$\begin{cases} (x \prec y) \prec z & = & x \prec (y * z), \\ (x \succ y) \prec z & = & x \succ (y \prec z), \\ (x * y) \succ z & = & x \succ (y \succ z), \end{cases}$$

where $x * y := x \prec y + x \succ y$. They are called *dendriform algebras* (cf. [**L1**]). It can be shown that the free dendriform algebra on one generator is the vector space spanned by the planar binary rooted trees. The two products are described by means of grafting. From this result we can construct a product and a composition on the series $\sum_t a(t)x^t$, where the sum runs over the planar binary rooted trees (no constant term). For the product we simply use the fact that the product $*$ in a dendriform algebra is associative (check it !). For the composition we use the explicit description of the free dendriform algebra. In fact these series form a group for composition. This is the *renormalisation group* of Quantum Electro-Dynamics.

There are several examples of this type, few of them have been studied so far.

There are many more problems where operads can help in combinatorics. Let me just mention two of them. A free algebra of some sort is, in general, graded and one can form its generating series. In terms of operads, when the space of n-ary operations $\mathcal{P}(n)$ is finite dimensional one defines

$$f^{\mathcal{P}}(x) := \sum_{n \geq 1} (-1)^n \frac{\dim \mathcal{P}(n)}{n!} x^n \ .$$

An important theory in the operad framework is the Koszul duality. Well-known for associative algebras it has been generalized to operads by Ginzburg and Kapranov (cf. [**G-K**], [**F**], see [**L1**] appendix 2 for a short overview on operads and Koszul duality). One of the consequences is the following. To any quadratic operad $\mathcal{P}$ one can associate its dual $\mathcal{P}^!$ and a certain chain complex, called the Koszul complex. When the Koszul complex is acyclic, then the generating series of $\mathcal{P}$ and $\mathcal{P}^!$ are inverse to each other for composition:

$$f^{\mathcal{P}}(f^{\mathcal{P}^!}(x)) = x \ .$$

This nice theorem has many applications. One of them is, in some instances, to provide a combinatorial interpretation of some integer sequences (cf. [**L2**]).

Here is another application. The partition lattice gives rise to a chain complex whose homology is a representation of the symmetric group. It is not that easy to compute. However there is an operadic way of looking at it, which, by using Koszul duality, permits us to identify this homology group to the space of operations of a dual operad (cf. [**F**]). This interpretation can be generalized to many variations of the classical partition lattice provided that the operad involved is Koszul (cf. [**V1**]).

If, instead of looking only at operations, we want look at operations *and* co-operations, then the notion of operad has to be replaced by the notion of *prop*. At this point there is a need for a Koszul duality theory in the prop framework. This has recently been achieved in [**V2**].

References

[F] B. Fresse, *Koszul duality of operads and homology of partition posets*, Proceedings of the Conference on Algebraic Topology, Northwestern University, Contemporary Mathematics, AMS, to appear. e-print `arXiv:math.AT/0301365` (2003).

[G-K] V. Ginzburg, M.M. Kapranov, *Koszul duality for operads*, Duke Math. J. **76** (1995), 203-272.

[L1] J.-L. Loday, *Dialgebras*, "Dialgebras and related operads", 7–66, Lecture Notes in Math., 1763, Springer, Berlin, 2001.

[L2] J.-L. Loday, *Inversion of integral series enumerating planar trees*, e-print `arXiv:math.CO/0403316`

[V1] B. Vallette, *Homology of generalized partition posets*, preprint IRMA, 2004.

[V2] B. Vallette, *Kosuzl duality for PROPs* e-print `arXiv:math.AT/0402213`

Asymptotics of multivariate generating functions

Robin Pemantle

Let $F(\mathbf{x}) = \sum_{\mathbf{r}} a_{\mathbf{r}} \mathbf{x}^r r$ be the multivariate generating function encoding the coefficients $\mathbf{r} := (r_1, \ldots, r_d)$. We would like to find estimates for the coefficients $\{a_{\mathbf{r}}\}$ that are asymptotically valid as $\mathbf{r} \to \infty$. In the univariate case, there is a well known, powerful, elegant apparatus for deriving such asymptotics from the analytic behavior of F near its minimal modulus singularity. In more than one variable, this problem is nearly untouched. Writing $F = \sum g_n(r_1, \ldots, r_{d-1}) z_d^{r_d}$, if g_n is asymptotically g^n for some g, then theorems by Bender, Richmond, Canfield and Gao yield Gaussian limit laws for $a_{\mathbf{r}}$. No other general results appear to be known.

The present talk will focus on the case of rational generating functions. In the one variable case this class is trivial to analyze, but in the multivariate case even this class poses many unsovled problems. Furthermore, one finds numerous applications within this class. The approach is to write $a_{\mathbf{r}}$ as a multivariate Cauchy integral, and then to use topological techinques to replace this integral with one that is in stationary phase, meaning that it looks locally like $\int_D A(\mathbf{x}) \exp(-|\mathbf{r}| Q(\mathbf{x}))\, d\mathbf{x}$ for some (one hopes postivie definite) quadratic form on a disk-like domain, D. Asymptotics can then be read off in a fairly automated way. It is our extreme good fortune that existing results in Stratified Morse Theory are tailor-made to convert the Cauchy integral to the stationary phase integral. A more complete outline of the steps is as follows. This outline is valid for certain geometries of the pole set of F.

(1) Write $a_{\mathbf{r}}$ as a Cauchy integral

$$a_{\mathbf{r}} = \left(\frac{1}{2\pi i}\right)^d \int_T \mathbf{z}^{-\mathbf{r}} F(\mathbf{z}) \frac{d\mathbf{z}}{\mathbf{z}} \tag{1}$$

where the torus T is a product of sufficiently small circles around the origin in each coordinate.

(2) The torus T may be replaced by an equivalent d-cycle in the homology of $(\mathbb{C}^*)^d$ minus the poles of F. Specifically, we denote by $-\infty$ the set where the integrand in (1) is sufficiently small, and represent T in the homology of $(\mathbb{C}^*)^d$ minus the poles of F, relative to $-\infty$.

(3) Stratified Morse theory identifies the other homology classes with saddles of the gradient $\mathbf{r} \log \mathbf{z}$ of the function $\mathbf{z}^{\mathbf{r}}$. Each such saddle lives in a stratum of dimension $j < d$ and yields a contribution which is an integral over a product of a cycle ßcyc$^{\|}$ in the stratum with a cycle ßcyc$^{\perp}$ in a transversal to the stratum.

(4) A nonzero contribution at a saddle σ occurs when the vector $\mathbf{r}$ is in a certain positive cone determined by the geometry of the pole set of F near σ.

(5) The integral over ßcyc$^{\perp}$ is equal to an easily computed spline, and the integral over ßcyc$^{\|}$ is then asymptotically evaluated by the saddle point method.

References

[1] Pemantle, R. and Wilson, M. (2002). Asymptotics of multivariate sequences, part I: smooth points of the singular variety. *JCT A* **97**, 129–161.
http://www.math.upenn.edu/~pemantle/papers/p1.ps

[2] Pemantle, R. and Wilson, M. (2004). Asymptotics of multivariate sequences, part II: Multiple points of the singular variety. *Comb. Prob. Comput., to appear.*
http://www.math.upenn.edu/~pemantle/papers/Preprints/multiplepoints.ps

[3] Baryshnikov, Y. and Pemantle, R. (2004). Convolutions of inverse linear functions via multivariate residues. *In Preparation.*
http://www.math.upenn.edu/~pemantle/papers/Preprints/arrangements.ps

Virtual Crystals and the $X = M$ Conjecture

Anne Schilling

Abstract. This is an expository talk on virtual crystals and the $X = M$ conjecture.
Résumé. C'est un entretien expositoire sur les cristaux virtuels et la conjecture $X = M$.

2. Extended Abstract

The quantized universal enveloping algebra $U_q(\mathfrak{g})$ associated with a symmetrizable Kac–Moody Lie algebra $\mathfrak{g}$ was introduced independently by Drinfeld [**D**] and Jimbo [**J**] in their study of two dimensional solvable lattice models in statistical mechanics. The parameter q corresponds to the temperature of the underlying model. Kashiwara [**K**] showed that at zero temperature or $q = 0$ the representations of $U_q(\mathfrak{g})$ have bases, which he coined crystal bases, with a beautiful combinatorial structure and favorable properties such as uniqueness and stability under tensor products.

The irreducible finite-dimensional $U'_q(\mathfrak{g})$-modules were classified by Chari and Pressley [**CP1, CP2**] in terms of Drinfeld polynomials. The Kirillov–Reshetikhin modules $W^{r,s}$, labeled by a Dynkin node r and a positive integer s, form a special class of these finite-dimensional modules. They naturally correspond to the weight $s\Lambda_r$, where Λ_r is the r-th fundamental weight of $\mathfrak{g}$. Recently, Hatayama et al. [**HKOTY, HKOTT**] conjectured that the Kirillov–Reshetikhin modules $W^{r,s}$ have a crystal basis denoted by $B^{r,s}$. The existence of such crystals allows the definition of one dimensional configuration sums X, which play an important role in the study of phase transitions of two dimensional exactly solvable lattice models. For $\mathfrak{g}$ of type $A_n^{(1)}$, the existence of the crystal $B^{r,s}$ was settled in [**KKMMNN**], and the one dimensional configuration sums contain the Kostka polynomials, which arise in the theory of symmetric functions, combinatorics, the study of subgroups of finite abelian groups, and Kazhdan–Lusztig theory. In certain limits they are branching functions of integrable highest weight modules.

In [**HKOTY, HKOTT**] fermionic formulas M for the one dimensional configuration sums were conjectured. Fermionic formulas originate in the Bethe Ansatz of the underlying exactly solvable lattice model. The term fermionic formula was coined by the Stony Brook group [**KKMM1, KKMM2**], who interpreted fermionic-type formulas for characters and branching functions of conformal field theory models as partition functions of quasiparticle systems with "fractional" statistics obeying Pauli's exclusion principle. For type $A_n^{(1)}$ the fermionic formulas were proven in [**KSS**] using a generalization of a bijection between crystals and rigged configurations of Kirillov and Reshetikhin [**KR**]. In [**OSS2**] similar bijections were used to prove the fermionic formula for nonexceptional types for crystals $B^{1,1}$. Rigged configurations are combinatorial objects which label the solutions to the Bethe equations. The bijection between crystals and rigged configurations reflects two different methods to solve lattice models in statistical mechanics: the corner-transfer-matrix method and the Bethe Ansatz.

The theory of virtual crystals [**OSS1, OSS3**] provides a realization of crystals of type X as crystals of type Y, based on well-known natural embeddings $X \hookrightarrow Y$ of affine algebras:

$$\begin{array}{ccc} C_n^{(1)}, A_{2n}^{(2)}, A_{2n}^{(2)\dagger}, D_{n+1}^{(2)} & \hookrightarrow & A_{2n-1}^{(1)} \\ A_{2n-1}^{(2)}, B_n^{(1)} & \hookrightarrow & D_{n+1}^{(1)} \\ E_6^{(2)}, F_4^{(1)} & \hookrightarrow & E_6^{(1)} \\ D_4^{(3)}, G_2^{(1)} & \hookrightarrow & D_4^{(1)}. \end{array}$$

Note that under these embeddings every affine Kac–Moody algebra is embedded into one of simply-laced type $A_n^{(1)}$, $D_n^{(1)}$ or $E_6^{(1)}$. Hence, by the virtual crystal method the combinatorial structure of any finite-dimensional affine crystal can be expressed in terms of the combinatorial crystal structure of the simply-laced types. Whereas the affine crystals $B^{r,s}$ of type $A_n^{(1)}$ are already well-understood [**Sh**], this is not the case for $B^{r,s}$ of types $D_n^{(1)}$ and $E_6^{(1)}$.

In this talk we highlight the main results regarding the $X = M$ conjecture of [**HKOTY, HKOTT**] and virtual crystals [**OSS1, OSS3**], which can be summarized as follows:

- Refs. [**HKOTY, HKOTT**] conjecture the existence of $B^{r,s}$ and the identity $X = M$ for general affine Kac-Moody algebras.
- Refs. [**OSS1, OSS3**] introduce the virtual crystal method which yields a description of the combinatorial structure of the crystals $B^{r,s}$ in terms of the combinatorics of $B^{r,s}$ for types $A_n^{(1)}, D_n^{(1)}$ and $E_6^{(1)}$. Similarly, the fermionic formulas and rigged configurations also exhibit this virtual embedding structure. In [**OSS3**] this was used in particular to extend the Kleber algorithm, which provides an efficient algorithm for calculating fermionic formulas, to nonsimply-laced algebras.
- In Ref. [**KSS**] the $X = M$ conjecture was proven for type $A_n^{(1)}$ using a bijection between crystals/tableaux and rigged configurations. This was extended to other nonexceptional types in [**OSS2**] for tensor products of $B^{1,1}$ and in [**SSh**] for tensor products of $B^{1,s}$. Type $D_n^{(1)}$ for tensor products of $B^{r,1}$ was treated in [**S**].
- The combinatorial structure of the crystals $B^{2,s}$ of type $D_n^{(1)}$ is studied in [**SS**]. This work is presented by Philip Sternberg in form of a poster at this conference.

References

[CP1] V. Chari and A. Pressley, *Quantum affine algebras and their representations*, Representations of groups (Banff, AB, 1994), 59–78, CMS Conf. Proc., **16**, Amer. Math. Soc., Providence, RI, 1995.

[CP2] V. Chari and A. Pressley, *Twisted quantum affine algebras*, Comm. Math. Phys. **196** (1998) 461–476.

[D] V. G. Drinfeld, *Hopf algebra and the Yang–Baxter equation*, Soviet. Math. Dokl. **32** (1985) 254–258.

[HKOTT] G. Hatayama, A. Kuniba, M. Okado, T. Takagi and Z. Tsuboi, *Paths, crystals and fermionic formulae*, Prog. Math. Phys. **23** (2002) 205–272, Birkhäuser Boston, Boston, MA.

[HKOTY] G. Hatayama, A. Kuniba, M. Okado, T. Takagi and Y. Yamada, *Remarks on fermionic formula*, Contemp. Math. **248** (1999) 243–291.

[J] M. Jimbo, *A q-difference analogue of $U(\mathcal{G})$ and the Yang–Baxter equation*, Lett. Math. Phys. **10** (1985) 63–69.

[K] M. Kashiwara, *Crystalizing the q-analogue of universal enveloping algebras*, Commun. Math. Phys. **133** (1990) 249–260.

[KKMM1] R. Kedem, T. R. Klassen, B. M. McCoy, E. Melzer, *Fermionic quasi-particle representations for characters of* $(G^{(1)})_1 \times (G^{(1)})_1/(G^{(1)})_2$, Phys. Lett. B **304** (1993), no. 3-4, 263–270.

[KKMM2] R. Kedem, T. R. Klassen, B. M. McCoy, E. Melzer, *Fermionic sum representations for conformal field theory characters*, Phys. Lett. B **307** (1993), no. 1-2, 68–76.

[KKMMNN] S.-J. Kang, M. Kashiwara, K. C. Misra, T. Miwa, T. Nakashima and A. Nakayashiki, *Perfect crystals of quantum affine Lie algebras*, Duke Math. J. **68** (1992) 499–607.

[KR] A. N. Kirillov and N. Y. Reshetikhin, *The Bethe ansatz and the combinatorics of Young tableaux*, (Russian) Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **155** (1986), Differentsialnaya Geometriya, Gruppy Li i Mekh. VIII, 65–115, 194; translation in J. Soviet Math. **41** (1988), no. 2, 925–955.

[KSS] A. N. Kirillov, A. Schilling and M. Shimozono, *A bijection between Littlewood-Richardson tableaux and rigged configurations*, Selecta Mathematica (N.S.) **8** (2002) 67–135.

[OSS1] M. Okado, A. Schilling and M. Shimozono, *Virtual crystals and fermionic formulas of type $D_{n+1}^{(2)}$, $A_{2n}^{(2)}$, and $C_n^{(1)}$*, Representation Theory **7** (2003) 101–163.

[OSS2] M. Okado, A. Schilling and M. Shimozono, *A crystal to rigged configuration bijection for nonexceptional affine algebras*, "Algebraic Combinatorics and Quantum Groups", Edited by N. Jing, World Scientific (2003), 85–124.

[OSS3] M. Okado, A. Schilling and M. Shimozono, *Virtual crystals and Kleber's algorithm*, Commun. Math. Phys. **238** (2003) 187–209.

[S] A. Schilling, *A bijection between type $D_n^{(1)}$ crystals and rigged configurations*, preprint 2004.

[Sh] M. Shimozono, *Affine type A crystal structure on tensor products of rectangles, Demazure characters, and nilpotent varieties*, J. Algebraic Combin. **15** (2002), no. 2, 151–187.

[SS] A. Schilling and P. Sternberg, *Finite-dimensional crystals for quantum affine algebras of type $D_n^{(1)}$*, in preparation.

[SSh] A. Schilling and M. Shimozono, On the $X = M$ conjecture, in preparation.

The Phase Transition for Random Subgraphs of the n-cube

Gordon Slade

Abstract. We describe recent results, obtained in collaborations with C. Borgs, J.T. Chayes, R. van der Hofstad and J. Spencer, which provide a detailed description of the phase transition for random subgraphs of the n-cube.

Résumé. Nous présentons des résultats récents qui donnent une description détaillée de la transition de phase des sous-graphes aléatoires du n-cube. Ces résultats sont obtenus en collaboration avec C. Borgs, J.T. Chayes, R. van der Hofstad et J. Spencer.

Extended Abstract

The phase transition for random subgraphs of the complete graph, or the *random graph* for short, was first studied by Erdős and Rényi [**7**], and has been analyzed in considerable detail since then [**2, 11**]. Let K_V denote the complete graph on V vertices, so that there is an edge joining each of the $\binom{V}{2}$ pairs of vertices. In the random graph, edges of the complete graph are independently occupied with probability p and vacant with probability $1-p$, as in the bond percolation model. The occupied edges naturally determine connected components, called *clusters*. There is a phase transition as p is varied, in the sense that there is an abrupt change in the number of vertices $|\mathcal{C}_{\max}|$ in a cluster $\mathcal{C}_{\max}$ of maximal size, as p is varied through the critical value $p_c = \frac{1}{V}$.

We will say that a sequence of events E_V occurs *with high probability*, denoted w.h.p., if $\mathbb{P}(E_V) \to 1$ as $V \to \infty$. The basic fact of the phase transition is that when p is scaled as $(1+\epsilon)V^{-1}$, there is a phase transition at $\epsilon = 0$ in the sense that w.h.p.

$$(2) \qquad |\mathcal{C}_{\max}| = \begin{cases} \Theta(\log V) & \text{for } \epsilon < 0, \\ \Theta(V^{2/3}) & \text{for } \epsilon = 0, \\ \Theta(V) & \text{for } \epsilon > 0. \end{cases}$$

The asymptotic results of (2) are valid for fixed ϵ, independent of V. These results have been substantially strengthened to show that there is a scaling window of width $V^{-1/3}$, in the sense that if $p = (1+\Lambda_V V^{-1/3})V^{-1}$, then w.h.p.

$$(3) \qquad |\mathcal{C}_{\max}| \begin{cases} \mathbf{l} V^{2/3} & \text{for } \Lambda_V \to -\infty, \\ = \Theta(V^{2/3}) & \text{for } \Lambda_V \text{ uniformly bounded in } V, \\ \gg V^{2/3} & \text{for } \Lambda_V \to -\infty. \end{cases}$$

Here, we are using the notation $f(V)\mathbf{l}g(V)$ to mean that $f(V)/g(V) \to 0$ as $V \to \infty$, while $f(V) \gg g(v)$ means that $f(V)/g(V) \to \infty$ as $V \to \infty$. A great deal more is known, and can be found in [**2, 11**].

Our goal is to understand how these results can be extended to apply to the n-cube $\mathbb{Q}_n$. This graph has vertex set $\{0,1\}^n$, with an edge joining pairs of vertices which differ in exactly one coordinate. It has $V = 2^n$ vertices, each of degree n. Edges are again independently occupied with probability p. If $p = (1+\epsilon)n^{-1}$ with $\epsilon < 0$ independent of n, then $|\mathcal{C}_{\max}|$ turns out to be $\Theta(\log V)$. On the other hand, for $\epsilon > 0$ independent of n, it was shown in [**1**] that $|\mathcal{C}_{\max}| = \Theta(V)$. Thus, a transition takes place at the value $\frac{1}{n}$ of p.

In [**3**], the results of [**1**] were extended to show that w.h.p.

$$(4) \qquad |\mathcal{C}_{\max}| = \begin{cases} 2\epsilon^{-2}\log V\big(1+o(1)\big) & \text{for } \epsilon \le -(\log n)^2(\log\log n)^{-1}n^{-1/2}, \\ 2\epsilon V & \text{for } \epsilon \ge 60(\log n)^3 n^{-1}. \end{cases}$$

Thus, ϵ as in the first line of (4) gives a subcritical p, whereas in the second line p is supercritical. The gap between these ranges of p is much bigger than the $V^{-1/3}$ (here $2^{-n/3}$) seen above as the size of the scaling window for the complete graph.

The following result from [**9, 10**], which builds on results of [**4, 5, 6**], gives bounds for ϵ on an arbitrary scale that is polynomial in n^{-1}.

Theorem 1.1. *For the n-cube, there exists a sequence of rational numbers $a_1, a_2, a_3, \ldots$, with $a_1 = a_2 = 1$ and $a_3 = \frac{7}{2}$, such that for any $M \geq 1$, for $p_c^{(M)} = \sum_{i=1}^{M} a_i n^{-i}$, and for $p = p_c^{(M)} + \delta n^{-M}$ with δ independent of n, the following bounds hold w.h.p.:*

$$|\mathcal{C}_{\max}| \begin{cases} \leq 2(\log 2)\delta^{-2} n^{2M-1}[1+o(1)] & \text{for } \delta < 0, \\ \geq \text{const}\, \delta n^{1-M} 2^n & \text{for } \delta > 0. \end{cases} \tag{5}$$

More is proved in [**9, 10**], but (5) is highlighted here because it shows subcritical behaviour for negative δ and supercritical behaviour for positive δ. Theorem 1.1 suggests that the critical value for the n-cube should be $\sum_{i=1}^{\infty} a_i n^{-i}$, but circumstantial evidence leads us to conjecture that this infinite series is divergent (see [**8**] for a general discussion of such issues). If the conjecture is correct, the critical value cannot be defined in this way. This difficulty was bypassed in [**4**], where the critical value for the phase transition on a "high-dimensional" finite graph $\mathbb{G}$ was defined to be the value $p_c = p_c(\mathbb{G}, \lambda)$ for which

$$\chi(p_c) = \lambda V^{1/3}, \tag{6}$$

where $\chi(p)$ is by definition the expected number of vertices in the component of an arbitrary fixed vertex (e.g., the origin of the n-cube), V is the number of vertices in the graph $\mathbb{G}$, and λ is a fixed positive number. This definition is by analogy with the random graph, where it is known that $\chi(1/V) = \Theta(V^{1/3})$. The parameter λ allows for some flexibility, associated with the fact that criticality corresponds to a scaling window of finite width and not to a single point. The following theorem is proved in [**10**], building on results in [**4, 5, 6**].

Theorem 1.1. *For the n-cube, let $M \geq 1$, fix constants c, c' (independent of n but possibly depending on M), and choose p such that $\chi(p) \in [cn^M, c'n^{-2M}2^n]$. Then for a_i given by Theorem 1.1,*

$$p = \sum_{i=1}^{M} a_i n^{-i} + O(n^{-M-1}) \qquad \text{as } n \to \infty. \tag{7}$$

The constant in the error term depends on M, c, c', but does not depend otherwise on p.

Fix $\lambda > 0$ independent of n. Then $\chi(p_c(\mathbb{Q}_n, \lambda)) = \lambda 2^{n/3}$ is in an interval $[cn^M, c'n^{-2M}2^n]$ for every M, with c, c' dependent on M and λ. By Theorem 1.1, (7) holds for $p = p_c(\mathbb{Q}_n, \lambda)$, for *every* fixed choice of λ and for *every* M. Thus,

$$p_c(\mathbb{Q}_n, \lambda) \sim \sum_{i=1}^{\infty} a_i n^{-i} \tag{8}$$

is an asymptotic expansion for $p_c(\mathbb{Q}_n, \lambda)$, for every positive λ.

By analogy with the complete graph, we would like to prove that the critical scaling window for the n-cube has size $V^{-1/3} = 2^{-n/3}$. This exponential scale is not accessible using the asymptotic expansion of Theorems 1.1–1.1. The following result from [**6**], which builds on the results of [**4, 5**], does not quite prove that the scaling window has size $2^{-n/3}$, but does show that it is smaller than any inverse power of n.

Theorem 1.2. *For the n-cube, let $V = 2^n$, let λ_0 be a fixed sufficiently small constant, and let $p = p_c(\mathbb{Q}_n, \lambda_0) + \epsilon n^{-1}$. If $\epsilon < 0$ and $\epsilon V^{1/3} \to -\infty$ as $V \to \infty$, then w.h.p.*

$$|\mathcal{C}_{\max}| \leq 2\epsilon^{-2} \log V \big(1 + o(1)\big). \tag{9}$$

If $|\epsilon| V^{1/3} \leq B$ for some constant B, then there is a constant b (depending on B and λ_0) such that, for any $\omega \geq 1$,

$$\mathbb{P}\Big(\omega^{-1} V^{2/3} \leq |\mathcal{C}_{\max}| \leq \omega V^{2/3}\Big) \geq 1 - \frac{b}{\omega}. \tag{10}$$

Finally, there are positive constants c, c_1 *such that if* $e^{-cn^{1/3}} \leq \epsilon \leq 1$ *then w.h.p.*

$$|\mathcal{C}_{\max}| \geq c_1 \epsilon V. \tag{11}$$

Additional estimates can be found in [**6**], but those in Theorem 1.2 show that the critical window in ϵ is of size $V^{-1/3} = 2^{-n/3}$ on the subcritical side of $p_c(n)$, and has at most size $e^{-cn^{1/3}}$ on the supercritical side. We expect that the window actually has size $V^{-1/3} = 2^{-n/3}$ on both sides of $p_c(n)$, and that, more generally, the scaling window in high-dimensional graphs has size $V^{-1/3}$.

An interesting consequence of the above theorems is that the approximate critical values $p_c^{(M)} = \sum_{i=1}^{M} a_i n^{-i}$ will lie outside the critical window around $p_c(\mathbb{Q}_n, \lambda)$, for *every* M, unless the sequence a_i is eventually zero and the asymptotic series is actually a polynomial in n^{-1}. We expect the series to be divergent, and not a polynomial. We regard the definition (6) as superior to any definition based on the asymptotic expansion. In particular, the coefficients a_i are obtained from an asymptotic expansion for $p_c(\mathbb{Q}_n, \lambda)$, so the latter contains all information contained in the former.

There are several ingredients in the proof of these theorems, most of which are more familiar in mathematical physics than in combinatorics. These include differential inequalities, the triangle condition, finite-size scaling ideas, and the lace expansion. The method of [**1**], which we call sprinkling, is used in conjunction with estimates obtained via these other methods to prove the lower bound (11). In [**4, 5**], other graphs besides the n-cube are also treated, including finite periodic approximations to $\mathbb{Z}^n$ for n large, with less complete results.

References

[1] M. Ajtai, J. Komlós, and E. Szemerédi. Largest random component of a k-cube. *Combinatorica*, **2**:1–7, (1982).

[2] B. Bollobás. *Random Graphs.* Cambridge University Press, Cambridge, 2nd edition, (2001).

[3] B. Bollobás, Y. Kohayakawa, and T. Łuczak. The evolution of random subgraphs of the cube. *Random Struct. Alg.*, **3**:55–90, (1992).

[4] C. Borgs, J.T. Chayes, R. van der Hofstad, G. Slade, and J. Spencer. Random subgraphs of finite graphs: I. The scaling window under the triangle condition. Preprint, (2003). Available at `http://www.math.ubc.ca/~slade`.

[5] C. Borgs, J.T. Chayes, R. van der Hofstad, G. Slade, and J. Spencer. Random subgraphs of finite graphs: II. The lace expansion and the triangle condition. Preprint, (2003). Available at `http://www.math.ubc.ca/~slade`.

[6] C. Borgs, J.T. Chayes, R. van der Hofstad, G. Slade, and J. Spencer. Random subgraphs of finite graphs: III. The phase transition for the n-cube. Preprint, (2003). Available at `http://www.math.ubc.ca/~slade`.

[7] P. Erdős and A. Rényi. On the evolution of random graphs. *Magyar Tud. Akad. Mat. Kutató Int. Közl.*, **5**:17–61, (1960).

[8] M.E. Fisher and R.R.P. Singh. Critical points, large-dimensionality expansions, and the Ising spin glass. In G.R. Grimmett and D.J.A. Welsh, editors, *Disorder in Physical Systems.* Clarendon Press, Oxford, (1990).

[9] R. van der Hofstad and G. Slade. Expansion in n^{-1} for percolation critical values on the n-cube and $\mathbb{Z}^n$: the first three terms. Preprint, (2003). Available at `http://www.math.ubc.ca/~slade`.

[10] R. van der Hofstad and G. Slade. Asymptotic expansions in n^{-1} for percolation critical values on the n-cube and $\mathbb{Z}^n$. Preprint, (2003). Available at `http://www.math.ubc.ca/~slade`.

[11] S. Janson, T. Łuczak, and A. Ruciński. *Random Graphs.* John Wiley and Sons, New York, (2000).

PART 2

Contributed Presentations

Presentations

Utilizing Relationships Among Linear Systems Generated by Zeilberger's Algorithm

S. A. Abramov and H. Q. Le

Abstract. We show that the sequence of first order linear difference equations generated by Zeilberger's algorithm can be described recursively. Each of these difference equations induces a system of linear algebraic equations and the mentioned recurrent relations can be utilized so that the values computed during the investigation of the J-th system can be used to accelerate the investigation of the $(J+1)$-th system. An implementation of this result and an experimental comparison between this implementation and an implementation of the original Zeilberger's algorithm are also done.

Résumé. Nous montrons que la suite des équations linéaires aux différences du premier ordre produites par l'algorithme de Zeilberger peut être décrite de façon récursive. Chacune de ces équations aux différences induit un système d'équations linéaires algébriques et lesdites relations de récurrence peuvent être employées de façon à ce que les valeurs calculées pendant l'analyse du J-ème système puissent être utilisées pour accélérer l'analyse du $(J+1)$-ème système. Nous faisons aussi une implantation de ce résultat et une comparaison avec l'implantation originale de l'algorithme de Zeilberger.

Bounds For The Growth Rate Of Meander Numbers

M. H. Albert and M. S. Paterson

Abstract. We provide improvements on the best currently known upper and lower bounds for the exponential growth rate of meanders. The method of proof for the upper bounds is to extend the Goulden-Jackson *cluster method*.

The Bergman Complex of a Matroid and Phylogenetic Trees

Federico Ardila and Caroline J. Klivans

Abstract. We study the Bergman complex $\mathcal{B}(M)$ of a matroid M: a polyhedral complex which arises in algebraic geometry, but which we describe purely combinatorially. We prove that a natural subdivision of the Bergman complex of M is a geometric realization of the order complex of its lattice of flats. In addition, we show that the Bergman fan $\widetilde{\mathcal{B}}(K_n)$ of the graphical matroid of the complete graph K_n is homeomorphic to the space of phylogenetic trees $\mathcal{T}_n$.

On a Conjecture Concerning Littlewood-Richardson Coefficients

François Bergeron and Riccardo Biagioli and Mercedes Rosas

Abstract. We prove that a conjecture of Fomin, Fulton, Li, and Poon, associated to ordered pairs of partitions, holds for many infinite families of such pairs. We also show that the generic bounded height case can be reduced to only checking that the conjecture holds for a finite number of pairs, for any given height. Moreover, we propose a natural generalization of the conjecture to the case of skew shapes.

Résumé. Nous démontrons qu'une conjecture de Fomin, Fulton, Li et Poon, associée aux couples de partages, se vérifie pour plusieurs classes infinies de tels couples. Nous montrons aussi que le cas générique, pour des partages de hauteurs bornés, se réduit à la vérification de la conjecture pour un nombre fini de couples, et ce pour chaque hauteur. De plus, nous présentons une généralisation naturelle de la conjecture au cas des couples de partages gauches.

A Four-Parameter Partition Identity

Cilanne E. Boulet

Abstract. We present a new partition identity and give a combinatorial proof of our result. This generalizes a result of Andrews' in which he considers the generating function for partitions with respect to size, number of odd parts, and number of parts of the conjugate.
Résumé. Nous présentons une nouvelle identité sur les partitions ainsi qu'une démonstration combinatoire de notre résultat. Ceci généralise un résultat d'Andrews au sujet de la série généra-trice des partitions relative à trois statistiques : la somme des parts, le nombre de parts impaires et le nombre de parts impaires de la partition conjuguée.

Finite automata and pattern avoidance in words

Petter Brändén and Toufik Mansour

Abstract. We say that a word w on a totally ordered alphabet avoids the word v if there are no subsequences in w order-equivalent to v. In this paper we suggest a new approach to the enumeration of words on at most k letters avoiding a given pattern. By studying an automaton which for fixed k generates the words avoiding a given pattern we derive several previously known results for problems of this kind, as well as many new. In particular, we give a simple proof of the formula [**?**] for the exact asymptotics for the number of words on k letters of length n that avoids the pattern $12\cdots(\ell+1)$. Moreover, we give the first combinatorial proof of the exact formula [**2**] for the number of words on k letters of length n avoiding a three letter permutation pattern.
Résumé. Soient v et w, deux mots sur un alphabet totalement ordonné. Le mot w évite le motif v si aucun sous-mot de w n'est équivalent (au sens de l'ordre) à v. Dans ce papier, nous suggérons une nouvelle approche pour énumérer les mots sur un alphabet d'au plus k lettres qui évitent un motif donné. En étudiant un automate qui engendre, pour un k fixé, tous les mots évitant un motif donné, nous obtenons des résultats nouveaux dans ce domaine, ainsi que d'autres déjà connus. En particulier, nous donnons une preuve simple

de la formule de Regev pour une estimation asymptotique précise du nombre de mots de longueur n sur k lettres qui évitent le motif $12\cdots(\ell+1)$. De plus, nous donnons pour la première fois une preuve combinatoire de la formule close de Burstein pour le calcul du nombre de mots de longueur n sur un alphabet à k lettres qui évitent un motif de permutation de 3 lettres.

References

[1] A. V. Aho, R. Sethi, J.D. Ullman, Compilers: principles, techniques and tools, Addison-Wesley, Reading, Mass., 1986.

[2] A. Burstein, Enumeration of words with forbidden patterns, Ph.D. thesis, University of Pennsylvania, 1998.

Height Arrow Model

Arnaud Dartois and Dominique Rossin

Abstract. We study in this article the characteristics of the so-called Height-Arrow Model (HAM), introduced by physicists as an extension of the Abelian Sandpile Model and the Eulerian Walker. We show that recurrent configurations of this model form an Abelian group and that classical algorithms such as the recurrence criterion or the burning algorithm for the ASM could be extended to the HAM.

Résumé. Dans cet article, nous étudions le modèle dit hauteur-orientation introduit par des physiciens comme une généralisa-tion du modèle du Tas de Sable Abélien et du Marcheur Eulérien. Nous montrons que les configurations récurrentes du système forment un groupe Abélien dont le cardinal est lié aux arbres couvrants du graphe sous-jacent. De plus, nous généralisons quelques algorithmes classiques connus pour le modèle du Tas de Sable Abélien comme le critère de récurrence ou l'algorithme de mise à feu.

A combinatorial approach to jumping particles I: maximal flow regime

Enrica Duchi and Gilles Schaeffer

Abstract. In this paper we consider a model of particles jumping on a row of cells, called in physics the one dimensional totally asymmetric exclusion process (TASEP). More precisely we deal with the TASEP with two or three types of particles, with or without boundaries, in the maximal flow regime. From the point of view of combinatorics a remarkable feature of these Markov chains is that they involve Catalan numbers in several entries of their stationary distribution.

We give a combinatorial interpretation and a simple proof of these observations. In doing this we reveal a second row of cells, which is used by particles to travel backward. As a byproduct we also obtain an interpretation of the occurrence of the Brownian excursion in the description of the density of particles on a long row of cells.

Résumé. Dans cet article, nous étudions un modèle de particules qui sautent le long d'une ligne, appelé en physique le processus d'exclusion totalement asymétrique unidimensionnel (TASEP). Plus précisément, nous traitons le TASEP avec deux ou trois types de particules, avec ou sans bords, dans le régime de flux maximal. D'un point de vue combinatoire, une propriété remarquable de ces chaînes de Markov est qu'elles font intervenir des nombres de Catalan dans plusieurs entrées de leur distribution stationnaire.

Nous donnons une interprétation combinatoire et une preuve simple de ces observations. Ce faisant, nous révélons une deuxième rangée de cases, utilisées par les particules pour retourner en arrière. Nous en déduisons enfin une interprétation de l'apparition d'excursion Brownienne dans la description de la densité des particules le long d'une longue rangée de cases.

Generalised Schur P–Functions and Weyl's Denominator Formula

A. M. Hamel and R. C. King

Abstract. We derive a general identity that relates generalised P–functions to the product of a Schur function and

$$\prod_{1 \leq i < j \leq n} (x_i + y_j).$$

This result generalises a number of well–known results in Robbins and Rumsey, Chapman, Tokuyama, and Macdonald. We also interpret our result in terms of μ–alternating sign matrices.

Résumé. Nous dérivons une identité générale reliant les P–fonctions généralisées et, le produit d'une fonction de Schur et $\prod_{1 \leq i < j \leq n}(x_i + y_j)$. Ce résultat est une généralisation des travaux de Robbins et Rumsey, Chapman, Tokuyama, et Macdonald. Nous en donnons aussi une variante avec des μ–matrices à signes alternants.

The Octahedron Recurrence and $\mathfrak{gl}_n$ Crystals

André Henriques and Joel Kamnitzer

Abstract. We study the hives of Knutson, Tao, and Woodward by means of a modified octahedron recurrence. We define a tensor category where tensor product is given by hives and where the associator and commutor are defined using our recurrence. We then prove that this category is equivalent to the category of crystals for the Lie algebra $\mathfrak{gl}_n$. The proof of this equivalence uses a new connection between the octahedron recurrence and the Jeu de Taquin and Schützenberger involution procedures on Young Tableaux.

Résumé. Nous étudions les hives de Knutson, Tao, et Woodward avec une récurrence octaèdre modifiée. Nous définissons une catégorie tensorielle où le produit tensoriel est donné par les hives et où l'associateur et le commuteur sont définies en termes de notre récurrence. Nous montrons que cette catégorie est équivalente à la catégorie des cristaux pour l'algèbre de Lie $\mathfrak{gl}_\mathfrak{n}$. La preuve de cette équivalence emploie une connection nouvelle entre la récurrence d'octaèdre et, les procédures de Jeu de Taquin et de l'involution de Schützenberger sur les tableaux de Young.

A Signed Analog of the Birkhoff Transform

Samuel K. Hsiao

Abstract. We construct a family of posets, called signed Birkhoff posets, that may be viewed as signed analogs of distributive lattices. Our posets are generally not lattices, but they are shown to posses many combinatorial properties corresponding to well known properties of distributive lattices. They have the additional virtue of being face posets of regular cell decompositions of spheres. We give a combinatorial description the **cd**-index of a signed Birkhoff poset in terms of peak sets of linear extensions of an associated

labeled poset. Our description is closely related to a result of Billera, Ehrenborg, and Readdy's expressing the **cd**-index of an oriented matroid in terms of the flag f-vector of the underlying geometric lattice. As an analog of the Distributive Lattice Conjecture, we conjecture that the chain polynomial of a signed Birkhoff poset has only real zeros.

Pieri and Cauchy Formulae for Ribbon Tableaux

Thomas Lam

Abstract. In [**LLT**] Lascoux, Leclerc and Thibon introduced symmetric functions $\mathcal{G}_\lambda$ which are spin and weight generating functions for ribbon tableaux. This article is aimed at studying these functions in analogy with Schur functions. In particular we will describe:

- a Pieri and dual-Pieri formula for ribbon functions,
- a ribbon Murnaghan-Nakayama formula,
- ribbon Cauchy and dual Cauchy identities,
- and a $\mathbb{C}$-algebra isomorphism $\omega_n : \Lambda(q) \to \Lambda(q)$ which sends each $\mathcal{G}_\lambda$ to $\mathcal{G}_{\lambda'}$.

We will show that the ribbon Pieri and Murnaghan–Nakayama rules are formally equivalent in a purely combinatorial manner. We will also connect the ribbon Cauchy and Pieri formulae to the combinatorics of ribbon insertion as studied by Shimozono and White [**SW2**]. In particular we give complete combinatorial proofs for the domino $n = 2$ case.

Résumé. Dans [**LLT**], Lascoux, Leclerc et Thibon ont introduit des fonctions symétriques $\mathcal{G}_\lambda$ qui sont les series formelles pour tableaux des rubans, selon la rotation et le poids. Notre article étudie l'analogie entre ces fonctions et les fonctions de Schur. En particulier, nous décrirons:

- des formules ruban-Pieri et dual-ruban-Pieri,
- une formule de ruban Murnagham–Nakayama,
- les identités ruban-Cauchy et dual-ruban-Cauchy pour fonctions de ruban,
- et un isomorphisme $\mathbb{C}$-algèbre $\omega_n : \Lambda(q) \to \Lambda(q)$ qui envoie chaque $\mathcal{G}_\lambda$ sur $\mathcal{G}_{\lambda'}$.

Nous montrerons que les règles Pieri de et Murnagham–Nakayama sont formellement équivalentes dans une manière purement combinatoire. Nous connecterons aussi les formules ruban-Cauchy et ruban-Pieri au combinatoire d'insertion des rubans, comme étudié par Shimozono et White [**SW2**]. En particulier, nous donnons les preuves combinatoires complétes pour le cas domino $n = 2$.

References

[LLT] A. Lascoux, B. Leclerc, and J.-Y. Thibon, Ribbon tableaux, Hall-Littlewood symmetric functions, quantum affine algebras, and unipotent varieties, *J. Math. Phys.*. **38**(3) (1997), 1041-1068.

[SW2] M. Shimozono and D.E. White, Color-to-spin ribbon Schensted algorithms, *Discrete Math.*, **246** (2002), 295-316.

q, t-Kostka Polynomials and the Affine Symmetric Group

Luc Lapointe and Jennifer Morse

Abstract. The k-Young lattice Y^k is a partial order on partitions with no part larger than k that originated [**LLM**] from the study of k-Schur functions $s_\lambda^{(k)}$, symmetric functions that form a natural basis of the space spanned by homogeneous functions indexed by k-bounded partitions. The chains in the k-Young lattice are induced by a Pieri-type rule experimentally satisfied by the k-Schur functions. Here, using a natural bijection between k-bounded partitions and $k+1$-cores, we can identify chains in the k-Young lattice with certain tableaux on $k+1$ cores. This identification reveals that the k-Young lattice is isomorphic to the weak order on the quotient of the affine symmetric group $\tilde{S}_{k+1}$ by a maximal parabolic subgroup. From this, the conjectured k-Pieri rule implies that the k-Kostka matrix connecting the homogeneous basis $\{h_\lambda\}_{\lambda \in Y^k}$ to $\{s_\lambda^{(k)}\}_{\lambda \in Y^k}$ may now be obtained by counting appropriate classes of tableaux on $k+1$-cores. This suggests that the conjecturally positive k-Schur expansion coefficients for Macdonald polynomials (reducing to q,t-Kostka polynomials for large k) could be described by a q,t-statistic on these tableaux, or equivalently on reduced words for affine permutations.

Résumé. Un ordre partiel Y^k sur les partitions dont les parties ne dépassent pas un certain entier positif k tire son origine de l'étude de fonctions de Schur généralisées [**LLM**], fonctions symétriques formant une base de l'espace engendré par les fonctions homogènes indicées par des partitions k-bornées. Les chaînes dans le treillis Y^k sont induites par une règle du type Pieri que satisfont expérimentalement les fonctions de k-Schur. En utilisant une bijection naturelle entre les partitions k-bornées et les $k+1$-cores, nous obtenons une correspondance entre les chaînes dans le treillis Y^k et certains remplissages de $k+1$-cores. Cette correspondance révèle que le treillis Y^k est isomorphe à l'ordre faible du groupe symétrique affine $\tilde{S}_{k+1}$ modulo un sous-groupe parabolique maximal. La règle de Pieri experimentale implique ainsi que la matrice de k-Kostka connectant les bases $\{h_\lambda\}_{\lambda \in Y^k}$ et $\{s_\lambda^{(k)}\}_{\lambda \in Y^k}$ peut être obtenue en énumérant certaines classes de tableaux sur les $k+1$-cores, et suggère entre autres que les coefficients de développements, que nous conjecturons positifs, des polynômes de Macdonald en termes de fonctions de k-Schur (se reduisant aux polynômes de q,t-Kostka lorsque k est grand) pourraient être décrits par une q,t-statistique sur ces tableaux, ou de façon équivalente, par une q,t-statistique sur les décompositions réduites de certaines permutations affines.

References

L. Lapointe, A. Lascoux, and J. Morse, *Tableau atoms and a new Macdonald positivity conjecture*, Duke Math. J. **116** (2003), 103–146.

Affine Weyl groups in K-theory and representation theory

Cristian Lenart and Alexander Postnikov

Abstract. We present a simple combinatorial model for the characters of the irreducible representations of complex semisimple Lie groups and, more generally, for Demazure characters. On the other hand, we give an explicit combinatorial Chevalley-type formula for the T-equivariant K-theory of generalized flag manifolds G/B. The construction is given in terms of alcove paths, which correspond to decompositions of affine Weyl group elements, and saturated chains in the Bruhat order on the (nonaffine) Weyl group. A key ingredient is a certain R-matrix, that is, a collection of operators satisfying the Yang-Baxter equation. Our model has several advantages over the Littelmann path model and the LS-galleries of Gaussent and Littelmann. The relationship between our model and the latter ones is yet to be explored.

Résumé. Nous présentons un modèle combinatoire simple pour les caractères des représentations d'un groupe de Lie complexe semisimple et, en général, pour les caractères de Demazure. D'autre part, nous présentons une généralisation combinatoire de la formule de Chevelley pour la K-théorie équivariante des variétés de drapeaux G/B. Notre construction est en termes de chemins sur les alcôves déterminées par le groupe de Weyl affine (qui correspondent aux décompositions réduites dans ce groupe) et de chemins saturés sur le groupe de Weyl (nonaffine). Un ingrédient important est une certaine R-matrice, c'est-à-dire une collection des opérateurs qui vérifient l'équation de Yang-Baxter. Notre modèle a plusieurs avantages par comparaison avec le modèle de chemins de Littelmann et les galeries LS de Gaussent et Littelmann. La relation entre notre modèle et les deux autres n'a pas encore été étudiée.

Littelmann Paths for Affine Lie Algebras

Peter Magyar

Abstract. We give a new combinatorial model for the crystal graphs of an affine Lie algebra $\widehat{\mathfrak{g}}$, unifying Littelmann's path model with the Kyoto path model. The vertices of the crystal graph are represented by certain infinitely looping paths which we call skeins.

We apply this model to the case when the corresponding finite-dimensional algebra $\mathfrak{g}$ has a minuscule representation
(classical type and E_6, E_7). We prove that the basic level-one representation of $\widehat{\mathfrak{g}}$, when considered as a representation of $\mathfrak{g}$, is an infinite tensor product of fundamental representations of $\mathfrak{g}$.

This is the infinite limit of a finer result: that the finite-dimensional Demazure submodules of the basic representation are finite tensor products. The corresponding Demazure characters give generalizations of the Hall-Littlewood polynomials.

This paper is an extended abstract of [**Mag**].

References

[Mag] P. Magyar, *Littelmann paths for the basic representation of an affine Lie algebra*, http://arxiv.org/math.RT/0308156.

Tutte Meets Poincaré

Jeremy L. Martin

Abstract. Let G be a graph and $\mathcal{X}^d(G)$ the space of all "pictures" of G in complex projective d-space. We prove that $\mathcal{X}^d(G)$ has no torsion or odd-dimensional integral homology, and that its Poincaré series is a specialization of the Tutte polynomial of G. As an application to combinatorial rigidity theory, we give a criterion for *d-parallel independence* in terms of the Tutte polynomial. In the case that $\mathcal{X}^d(G)$ is smooth (which is equivalent to the condition that G is an *orchard*), we give a presentation of its cohomology ring, and relate the intersection theory on $\mathcal{X}^d(G)$ to the Schubert calculus on flag varieties.

Résumé. Soient G un graphe et $\mathcal{X}^d(G)$ l'espace de toutes les "figures" de G dans l'espace complexe projectif d-dimensionnel. Nous prouvons que $\mathcal{X}^d(G)$ ne présente ni de torsion, ni d'homologie entière en dimension impaire, et que sa série de Poincaré est une spécialisation du polynôme de Tutte de G. Comme application à la théorie combinatoire de la rigidité, nous développons un critère pour *l'indépendance d-parallel* en termes du polynôme de Tutte. Dans le cas où $\mathcal{X}^d(G)$ est lisse (ce qui est équivalent à la condition que G soit un *verger*), nous donnons une présentation de son anneau de cohomologie, et relions la théorie d'intersection de $\mathcal{X}^d(G)$ au calcul de Schubert sur les variétés de drapeaux.

A Hopf Algebra of Parking Functions

J.-C. Novelli and J.-Y. Thibon

Abstract. If the moments of a probability measure on $\mathbb{R}$ are interpreted as a specialization of complete homogeneous symmetric functions, its free cumulants are, up to sign, the corresponding specializations of a sequence of Schur positive symmetric functions (f_n). We prove that (f_n) is the Frobenius characteristic of the natural permutation representation of $\mathfrak{S}_n$ on the set of prime parking functions. This observation leads us to the construction of a Hopf algebra of parking functions, which we study in some detail.

Résumé. Si on interprète les moments d'une mesure de probabilité sur $\mathbb{R}$ comme une spécialisation de fonctions symétriques complètes, ses cumulants libres sont, au signe près, les spécialisations correspondantes d'une suite de fonctions symétriques (f_n) Schur-positives. Nous montrons que (f_n) est la caractéristique de Frobenius d'une représentation permutationnelle naturelle de $\mathfrak{S}_n$ sur l'ensemble des fonctions de parking primitives. Cette observation nous conduit à construire une algèbre de Hopf des fonctions de parking que nous étudions ensuite en détail.

An Arctic Circle Theorem For Groves

T. Kyle Petersen and David Speyer

Abstract. In earlier work, Jockusch, Propp, and Shor proved a theorem describing the limiting shape of the boundary between the uniformly tiled corners of a random tiling of an Aztec diamond and the more unpredictable 'temperate zone' in the interior of the region. The so-called arctic circle theorem made precise a phenomenon observed in random tilings of large Aztec diamonds.

Here we examine a related combinatorial model called groves. Created by Carroll and Speyer as combinatorial interpretations for Laurent polynomials given by the cube recurrence, groves have observable frozen regions which we describe precisely via asymptotic analysis of generating functions, in the spirit of Pemantle and Wilson. Our methods also provide another way to prove the arctic circle theorem for Aztec diamonds.

Résumé. Dans leurs travaux, Jockusch, Propp, et Shor ont prouvé un théorème décrivant la forme limite de la frontière entre les coins uniformement pavés ("gelés") d'un pavage aléatoire d'un diamant aztèque et la zone "temperee" moins prévisible a l'intérieur du diamant. Le théorème du cercle arctique a rendu précis un phénomène observé dans les pavages aléatoires de grands diamants aztèques.

Nous examinons un modèle combinatoire relie appelé les bosquets. Créé par Carroll et Speyer en tant qu'interprétation combinatoires pour des polynômes de Laurent donnés par la récurrence du cube, les bosquets laissent apparaître des régions gelées que nous décrivons avec précision par l'intermediaire de l'analyse asymptotique de fonctions génératrices, dans l'esprit de Pemantle et de Wilson. Nos méthodes fournissent également une autre manière de prouver le théorème du cercle arctique pour les diamants aztèques.

The Equivariant Orlik-Solomon Algebra

Nicholas Proudfoot

Abstract. Given a real hyperplane arrangement $\mathcal{A}$, the complement $\mathcal{M}(\mathcal{A})$ of the complexification of $\mathcal{A}$ admits an action of $\mathbb{Z}_2$ by complex conjugation. We define the equivariant Orlik-Solomon algebra of $\mathcal{A}$ to be the $\mathbb{Z}_2$-equivariant cohomology ring of $\mathcal{M}(\mathcal{A})$ with coefficients in $\mathbb{Z}_2$. We give a combinatorial presentation of this ring, and interpret it as a deformation of the ordinary Orlik-Solomon algebra into the Varchenko-Gel′fand ring of locally constant $\mathbb{Z}_2$-valued functions on the complement $\mathcal{M}_{\mathbb{R}}(\mathcal{A})$ of $\mathcal{A}$ in $\mathbb{R}^n$. We also show that the $\mathbb{Z}_2$-equivariant homotopy type of $\mathcal{M}(\mathcal{A})$ is determined by the oriented matroid of $\mathcal{A}$. As an application, we give two examples of pairs of arrangements $\mathcal{A}$ and $\mathcal{A}'$ such that $\mathcal{M}(\mathcal{A})$ and $\mathcal{M}(\mathcal{A}')$ have the same nonequivariant homotopy type, but are distinguished by the equivariant Orlik-Solomon algebra.

A Polynomiality Property for Littlewood-Richardson Coefficients

Etienne Rassart

Abstract. We present a polynomiality property of the Littlewood-Richardson coefficients $c_{\lambda\mu}^{\nu}$. The coefficients are shown to be given by polynomials in λ, μ and ν on the cones of the chamber complex of a vector partition function. We give bounds on the degree of the polynomials depending on the maximum allowed number of parts of the partitions λ, μ and ν. We first express the Littlewood-Richardson coefficients as a vector partition function. We then define a hyperplane arrangement from Steinberg's formula, over whose regions the Littlewood-Richardson coefficients are given by polynomials, and relate this arrangement to the chamber complex of the partition function. As an easy consequence, we get a new proof of the fact that $c_{N\lambda\,N\mu}^{N\nu}$ is given by a polynomial in N, which partially establishes the conjecture of King, Tollu and Toumazet [**KTT03**] that $c_{N\lambda\,N\mu}^{N\nu}$ is a polynomial in N with nonnegative rational coefficients.

Résumé. Nous présentons une propriété de polynomialité des coefficients de Littlewood-Richardson $c_{\lambda\mu}^{\nu}$. Nous démontrons que ces coefficients sont donnés par des fonctions polynomiales en λ, μ et ν dans les cônes du complexe d'une fonction de partition vectorielle. Nous donnons des bornes sur les degrés de ces polynômes en termes du nombre de parts des partitions λ, μ and ν. Nous exprimons premièrement les coefficients de Littlewood-Richardson en termes d'une fonction de partition vectorielle. Nous définissons ensuite un arrangement d'hyperplans à partir de la formule de Steinberg, sur les régions duquel les coefficients de Littlewood-Richardson sont donnés par des polynômes, puis faisons le lien entre cet arrangement et le complexe de cônes de la fonction de partition vectorielle. Comme conséquence simple, nous obtenons une preuve élémentaire du fait que $c_{N\lambda\,N\mu}^{N\nu}$ est donné par un polynôme en N, ce qui établit partiellement une conjecture de King, Tollu et Toumazet [**KTT03**], voulant que $c_{N\lambda\,N\mu}^{N\nu}$ soit un polynôme en N avec des coefficients rationnels nonnégatifs.

References

[KTT03] R. C. King, C. Tollu, and F. Toumazet, *Stretched Littlewood-Richardson and Kostka coefficients*, To appear in CRM Proceedings and Lecture Notes, volume 34, 2003.

Cambrian Lattices

Nathan Reading

Abstract. For an arbitrary finite Coxeter group W, we define the family of Cambrian lattices for W as quotients of the weak order on W with respect to certain lattice congruences. We associate to each Cambrian lattice a complete fan, which we conjecture is the normal fan of a polytope combinatorially isomorphic to the generalized associahedron for W. In types A and B, we obtain, by means of a fiber-polytope construction, combinatorial realizations of the Cambrian lattices in terms of triangulations and in terms of permutations. Using this combinatorial information, we prove that in types A and B the Cambrian fans are combinatorially isomorphic to the normal fans of the generalized associahedra, and that one of the Cambrian fans is linearly isomorphic to Fomin and Zelevinsky's construction of the normal fan as a "cluster fan." Our construction does not require a crystallographic Coxeter group and therefore suggests a definition, at least on the level of cellular spheres, of a generalized associahedron for any finite Coxeter group. The Tamari lattice is one of the Cambrian lattices of type A, and two "Tamari" lattices in type B are identified, and characterized in terms of signed pattern avoidance. We also show that intervals in Cambrian lattices are either contractible or homotopy equivalent to spheres.
Résumé. Pour un groupe fini arbitraire de Coxeter W, nous définissons la famille des treillis cambriens pour W comme des quotients de l'ordre faible sur W par certaines congruences de treillis. Nous associons à chaque treillis cambrien un éventail complet et nous conjecturons que cet éventail est l'éventail normal d'un polytope isomorphe, au sens combinatoire, à un associèdre généralisé. Dans le cas des types A et B, nous obtenons, par une construction de fibre-polytope, des réalisations combinatoires des treillis cambriens en termes de triangulations et en termes de permutations. En utilisant cette information combinatoire, nous montrons que, dans le cas des types A et B, les éventails cambriens sont isomorphes, au sens combinatoire, aux éventails normaux des associaèdre généralisés, et qu'un des éventails cambriens est linéairement isomorphe à l'éventail normal construit par Fomin et Zelevinsky sous forme de l'éventail des amas. Notre construction n'exige pas que le groupe de Coxeter soit cristallographique et suggère une définition, du moins au niveau des sphères cellulaires, d'un associaèdre généralisé pour tout groupe fini de Coxeter. Le treillis de Tamari est un des treillis cambriens du type A, et deux "treillis de Tamari" dans le type B sont identifiés, et caractérisés en termes des permutations signées à motifs exclus. Nous prouvons également que les intervalles dans les treillis cambriens sont soit contractibles, soit équivalent aux sphères, par homotopie.

On a Class of Totally Nonnegative f-immanants

Brendon Rhoades and Mark Skandera

Abstract. We define a family of totally nonnegative polynomials of the form $\sum f(\sigma)x_{1,\sigma(1)}\cdots x_{n,\sigma(n)}$ and show that this family generalizes all known totally nonnegative polynomials of the form $\Delta_{J,J'}(x)\Delta_{L,L'}(x) - \Delta_{I,I'}(x)\Delta_{K,K'}(x)$, where $\Delta_{J,J'}(x),\ldots,\Delta_{K,K'}(x)$ are matrix minors. We also give new conditions on the sets $J,\ldots,K'$ which guarantee that the corresponding polynomials are totally nonnegative.

Résumé. Nous donons une famille de polynômes totallement nonnegatifs de la forme $\sum f(\sigma)x_{1,\sigma(1)}\cdots x_{n,\sigma(n)}$ et montrons que cette famille generalise tous les polynômes totalement nonnegatifs de la forme $\Delta_{J,J'}(x)\Delta_{L,L'}(x) -$

$\Delta_{I,I'}(x)\Delta_{K,K'}(x)$, ou $\Delta_{J,J'}(x),\ldots,\Delta_{K,K'}(x)$ sont des mineurs des maitrices. Nous donons aussi des conditions nouvelles sur les ensembles $J,\ldots,K'$ qui guarantisent que les polynômes correspondents sont totalement nonnegatifs.

The weak and Kazhdan-Lusztig orders on standard Young tableaux

Victor Reiner and Muge Taskin

Abstract. Let SYT_n be the set of all standard Young tableaux with n cells. After recalling the definition of a partial order on SYT_n first defined by Melnikov, which we call the weak order, we prove two main results:

- Intervals in the weak order essentially describe the product in a Hopf algebra of tableaux defined by Poirier and Reutenauer.
- The map sending a tableau to its descent set induces a homotopy equivalence of the proper parts of either weak order or Kazhdan-Lusztig order on tableaux with the Boolean algebra $2^{[n-1]}$. In particular, the Möbius function for either of these orders on tableaux is $(-1)^{n-1}$.

The methods use in an essential way the Kazhdan-Lusztig order on SYT_n, and in some cases apply to other orders between the weak order and KL-order.

Deformed Universal Characters for Classical and Affine Algebras and the $X = M = K$ Conjecture

Mark Shimozono and Mike Zabrocki

Abstract. Creation operators are given for three distinguished bases of the type BCD universal character ring of Koike and Terada. Deformed versions of these operators create symmetric functions whose expansion in the universal character basis, has coefficient polynomials $K \in \mathbb{Z}_{\geq 0}[q]$. We conjecture that for every nonexceptional affine root system, these polynomials coincide with the graded tensor product multiplicities for affine characters that occur in the $X = M$ conjecture of Hatayama, Kuniba, Okado, Takagi, Tsuboi, and Yamada, which asserts the equality of an affine crystal theoretic formula X with a rigged configuration fermionic formula M.
Résumé. Nous donnons les opérateurs qui créent trois bases spéciales du type BCD de l'anneau des caractères de Koike et Terada. Les versions difformes de ces opérateurs créent les fonctions symétriques avec les coefficients $K \in \mathbb{Z}_{\geq 0}[q]$. Nous conjecturons que pour tous les systèmes des racines affines et nonexceptionnels, ces polynômes coïncident avec les multiplicités des produit tensoriels des charactres affines qui apparaissent dans le conjecture $X = M$ de Hatayama, Kuniba, Okado, Takagi, Tsuboi, et Yamada. Cette conjecture affirme qu'une formule pour X liée aux crystaux affines, est égale à une formule fermionique des configurations 'gréées' pour M.

Tamari Lattices and Non-crossing Partitions in Types B and D

Hugh Thomas

Abstract. The usual, or type A_n, Tamari lattice is a partial order on T_n^A, the triangulations of an $(n+3)$-gon. We define a partial order on T_n^B, the set of centrally symmetric triangulations of a $(2n+2)$-gon. We show that it is a lattice, and that it shares certain other nice properties of the A_n Tamari lattice; it can therefore be considered the B_n Tamari lattice.

We define a bijection between T_n^B and the non-crossing partitions of type B_n defined by Reiner. Reiner has also defined the noncrossing partitions of type D_n as a subset of those of type B_n. We show that the elements of T_n^B which correspond to the noncrossing partitions of type D_n form a lattice under the order induced from their inclusion in T_n^B, which therefore can be considered the D_n Tamari lattice.

This is a somewhat abridged version of a longer paper with the same title, available at `www.arxiv.org/math.CO/0311334`.

Résumé. Le treillis de Tamari standard (de type A_n) est un ordre partiel sur T_n^A, les triangulations d'un $(n+3)$-gone. Nous définissons un ordre partiel sur T_n^B, l'ensemble des triangulations centralement symétriques d'un $(2n+2)$-gone. Nous montrons que c'est un treillis et qu'il possède aussi d'autres propriétés intéressantes similaires au treillis de Tamari de type A_n. Ce treillis peut donc être considéré comme le treillis de Tamari de type B_n.

Nous définissons une bijection entre T_n^B et les partages non-croisés de type B_n définis par Reiner. Reiner a aussi défini les partages non-croisés de type D_n comme un sous-ensemble de ceux de type B_n. Nous montrons que les éléments de T_n^B qui correspondent aux partages non-croisés de type D_n forment un treillis sous l'ordre induit par leur inclusion dans T_n^B, qui peut donc être considéré comme le treillis de Tamari de type D_n.

Cet exposé est une version plus courte d'un exposé du même titre qui est disponible sur `www.arxiv.org/math.CO/0311334`.

Enumeration of Totally Positive Grassmann Cells

Lauren K. Williams

Abstract. In [**1**], Postnikov gave a combinatorially explicit cell decomposition of the totally nonnegative part of a Grassmannian, denoted $Gr_{k,n}^+$, and showed that this set of cells is isomorphic as a graded poset to many other interesting graded posets, such as the posets of decorated permutations, Ⅎ-diagrams (certain $0-1$ tableau), and positroids. The main result of our work is an explicit generating function which enumerates the cells in $Gr_{k,n}^+$ according to their dimension. Equivalently, we compute rank generating functions for the posets of decorated permutations, Ⅎ-diagrams, and positroids. As a corollary, we give a new proof that the Euler characteristic of $Gr_{k,n}^+$ is 1. Additionally, we use our result to produce a new q-analog of the Eulerian numbers, which interpolates between the Eulerian numbers, the Narayana numbers, and the binomial coefficients.

Résumé. Postnikov a décrit explicitement dans [**1**], en termes combinatoires, la décomposition cellulaire de la partie positive (notée $Gr_{k,n}^+$) d'une variété grassmannienne. Il a montré que cet ensemble de cellules est isomorphe, en tant que treillis gradué, à de nombreux ensembles partiellement ordonnés intéressants, comme les permutations décorées, les Ⅎ-diagrammes (qui sont certains tableaux à coefficients $0, 1$) ou les matroïdes positifs. Le résultat principal de notre travail est une fonction génératrice explicite, qui dénombre les cellules

de $Gr_{k,n}^{+}$ selon leur dimension. De façon équivalente, nous calculons la fonction génératrice, pondérée par le rang, pour le treillis des permutations décorées, des Ⅎ-diagrammes et des matroïdes positifs. Nous en déduisons comme corollaire une nouvelle preuve que la caractéristique d'Euler de $Gr_{k,n}^{+}$ est 1. De plus, nous utilisons notre résultat pour exhiber un nouveau q-analogue des nombres eulériens, qui s'interpole entre les nombres eulériens, les nombres de Narayana et les coefficients binomiaux.

References

[1] A. Postnikov, *Webs in Totally positive Grassman cells*, in preparation.

Posters

Combinatorial Aspects of Abstract Young Representations (Extended Abstract)

Ron M. Adin and Francesco Brenti and Yuval Roichman

Abstract. The goal of this paper is to give a new unified axiomatic approach to the representation theory of Coxeter groups and their Hecke algebras. Building upon fundamental works by Young and Kazhdan-Lusztig, followed by Vershik and Ram, we propose a direct combinatorial construction, avoiding a priori use of external concepts (such as Young tableaux). This is carried out by a natural assumption on the representation matrices. For simply laced Coxeter groups, this assumption yields explicit simple matrices, generalizing the Young forms. For the symmetric groups the resulting representations are completely classified and include the irreducible ones. Analysis involves generalized descent classes and convexity (à la Tits) within the Hasse diagram of the weak Bruhat poset.

Résumé. L'objectif de cet article est de donner une nouvelle approche axiomatique unifiée de la théorie des représentation des groupes de Coxeter et de leurs algèbres de Hecke. En utilisant les travaux de Young, Kazhdan-Lusztig ainsi que de Vershik et Ram, nous proposons une construction combinatoire directe qui évite l'introduction de concepts extérieurs (par exemple les tableaux de Young). Cette construction est faite à partir d'une hypothèse naturelle sur les matrices de représentation. Pour les groupes de Coxeter simplement lacé, cette hypothèse donne des matrices simples explicites, généralisant la forme de Young. Pour les groupes symmétriques les représentations associèes sont complètement classifiées, en particulier celles qui sont irréductibles. Ce travail utilise les classes de descente généralisées et la convexité (à la Tits) dans le diagramme de Hasse de l'ordre de Bruhat faible.

Equi-distribution over Descent Classes of the Hyperoctahedral Group (Extended Abstract)

Ron M. Adin and Francesco Brenti and Yuval Roichman

Abstract. A classical result of MacMahon shows that the length function and the major index are equi-distributed over the symmetric group. Foata and Schützenberger gave a remarkable refinement and proved that these parameters are equi-distributed over inverse descent classes, implying bivariate equi-distribution identities. Type B analogues and further refinements and consequences are given in this paper.

Résumé. Un résultat classique de MacMahon montre l'équi-distribution de l'indice majeur et de la fonction de longueur sur le groupe symétrique. Foata et Schützenberger en ont donné une raffinement remarquable et montré l'équidistribution sur les classes de descentes inverses, impliquant ainsi des équidistribu-tions bivariées. Les analogues pour le type B ainsi que d'autres raffinements et conséquences sont donnés dans cet article.

A New Representation of Formal Power Series

Kostyantyn V. Archangelsky

Abstract. This paper is dedicated to the genesis arising at the boundary between the theory of formal power series (FPS) and combinatorics.

Similarly to combinatorics where any rational sequence of natural numbers $\{r_k\}_{k\geq 0}$ is representable for all k in the form

$$r_{k+n} = \sum_{i=1}^{n} r_{k+n-i} X_{n-i} \tag{1.12}$$

where X_j – are, generally speaking, complex numbers (Berstel,Reutenauer, [**BR**]), we prove that any rational FPS r is representable in the form (1) where $r_s = \sum_{|w|=s}(r,w)w$, and X_j are elements of some special skew field. As a trivial consequence of such a representation were obtained: 1)truthfulness of Eilenberg's Equality Theorem [**E**], decidability of the equivalence problem of finite multitape deterministic automata (Rabin,Scott [**?**]) and decidability of problem of whether two given morphisms are equivalent on regular language, (Culik, Salomaa [**CS**]); 2) more simply formulated and proved the results from monographs on FPS (Salomaa, Soittola [**?**], Berstel, Reutenauer [**BR**], Kuich, Salomaa [**KS**]); 3) solved partial cases of the problem of existence for an inverse element of Hadamard product and others; 4) provided 3 Conjectures and 10 Open problems.

The conclusion contains a complete comparative analysis of the attempts to utilize linear recurrence in theory of FPS by other authors.

RÉSUMÉ.

References

[BR] BerstelJ., Reutenauer C., *Rational Series and Their Languages*, Springer-Verlag(1988).

[E] Eilenberg S., *Automata, Languages, and Machines*, Vol.A, Acad. Press (1974).

[CS] Culik K. II, Salomaa A.,*On the decidability of homomorphism equivalence for languages*, J.Comput.Syst.Sci, 17(1978) , 163-175.

[KS] Kuich W., Salomaa A.*Semirings, Automata, Languages*, 1985.

Sign balance for finite groups of Lie type (Extended Abstract)

Eli Bagno and Yona Cherniavsky

Abstract. A product formula for the parity generating function of the number of $1'$s in invertible matrices over $\mathbb{Z}_2$ is given. The computation is based on algebraic tools such as the Bruhat decomposition. The same technique is used to obtain a parity generating function also for symplectic matrices over $\mathbb{Z}_2$. We present also a generating function for the sum of entries modulo p of matrices over $\mathbb{Z}_p$. This formula is a new appearance of the Mahonian distribution.

Generating Functions For Kernels of Digraphs (Enumeration & Asymptotics for Nim Games)

Cyril Banderier and Vlady Ravelomanana and Jean-Marie Le Bars

Abstract. In this article, we study directed graphs (digraphs) with a coloring constraint due to Von Neumann and related to Nim-type games. This is equivalent to the notion of kernels of digraphs, which appears in numerous fields of research such as game theory, complexity theory, artificial intelligence (default logic, argumentation in multi-agent systems), 0-1 laws in monadic second order logic, combinatorics (perfect graphs)... Kernels of digraphs lead to numerous difficult questions (in the sense of NP-completeness, #P-completeness). However, we show here that it is possible to use a generating function approach to get new informations: we use technique of symbolic and analytic combinatorics (generating functions and their singularities) in order to get exact and asymptotic results, e.g. for the existence of a kernel in a circuit or in a unicircuit digraph. This is a first step toward a generatingfunctionology treatment of kernels, while using, e.g., an approach "à la Wright". Our method could be applied to more general "local coloring constraints" in decomposable combinatorial structures.

Résumé. Nous étudions dans cet article les graphes dirigés (digraphes) avec une contrainte de coloriage introduite par Von Neumann et reliée aux jeux de type Nim. Elle équivaut à la notion de noyaux de digraphes, qui apparaît dans de nombreux domaines, tels la théorie des jeux, la théorie de la complexité, l'intelligence artificielle (logique des défauts, argumentation dans les systèmes multi-agents), les lois 0-1 en logique monadique du second ordre, la combinatoire (graphes parfaits)... Les noyaux des digraphes posent de nombreuses questions difficiles (au sens de la NP-complétude ou de la #P-complétude). Cependant, nous montrons ici qu'il est possible de recourir aux séries génératrices afin d'obtenir de nouvelles informations : nous utilisons les techniques de la combinatoire symbolique et analytique (étude des singularités d'une série) afin d'obtenir des résultats exacts ou asymptotiques, par exemple pour l'existence d'un noyau dans un digraphe unicircuit. Il s'agit là de la première étape vers une série génératrilogie des noyaux. Notre méthode peut être appliquée plus généralement à des "contraintes locales" de coloriage dans des structures combinatoires décomposables.

Negative-descent representations for Weyl groups of type D

Riccardo Biagioli and Fabrizio Caselli

Abstract. We introduce a monomial basis for the coinvariant algebra of type D, that allows us to define a new family of representations of D_n. We decompose the homogeneous components of the coinvariant algebra into a direct sum of these representations and finally we give the decomposition of them into irreducible components. This algebraic setting is then applied to find new, and generalize various, combinatorial identities.

Résumé. On introduit une base monomiale de l'algèbre coinvariante de type D, ce qui nous permet de definir une nouvelle classe de representations de D_n. On decompose les composantes homogènes de l'algèbre coinvariante comme somme directe de ces representations et on decrit leur decomposition en modules irreductibles. Ce contexte algebrique est finalement utilisé pour decouvrir des nouvelles identités combinatoires.

Sharper estimates for the number of permutations avoiding a layered or decomposable pattern

Miklós Bóna

Abstract. We present two methods that for infinitely many patterns q provide better upper bounds for the number $S_n(q)$ of permutations of length n avoiding the pattern q than the recent general result of Marcus and Tardos. While achieving that, we define an apparently new decomposition of permutations .

A Generalization of $su(2)$

Brian Curtin

Abstract. We consider the following generalization of $su(2)$. Let $P(q, x, y, z)$ denote the associative algebra over any field K generated by A_1, A_2, A_3 with relations $[A_1, A_2]_q = xA_3 + yI + z(A_1 + A_2$, $[A_2, A_3]_q = xA_1 + yI + z(A_2 + A_3)$, $[A_3, A_1]_q = xA_2 + yI + z(A_3 + A_1)$ for some q, x, y, $z \in K$. Assume that $q \neq 0$ is either 1 or not a root of unity and that $x \neq 0$. We describe the multiplicity-free finite-dimensional representations of this generalized algebra, and we describe an action of the modular group on this algebra.

Résumé. Nous considérons la généralisation suivante de $su(2)$. Soit $P(q, x, y, z)$ l'algèbre associative avec des générateurs A_1, A_2, A_3 et rélations $[A_1, A_2]_q = xA_3 + yI + z(A_1 + A_2$, $[A_2, A_3]_q = xA_1 + yI + z(A_2 + A_3)$, $[A_3, A_1]_q = xA_2 + yI + z(A_3 + A_1)$ pour q, x, y, $z \in K$. Supposez que $q \neq 0$ est 1 ou pas une racine de l'unité, et supposez que Nous décrivons l $x \neq 0$.es représentations fini-dimensionnelles sans multiplicité de cette algèbre généralisé, et Nous décrivons une action du groupe modulaire sur cette algèbre.

Two New Criteria for Comparison in the Bruhat Order

Brian Drake and Sean Gerrish and Mark Skandera

Abstract. We give two new criteria by which pairs of permutations may be compared in defining the Bruhat order (of type A). One criterion utilizes totally nonnegative polynomials and the other utilizes Schur functions.
RÉSUMÉ. Nous donons deux critères nouveaux avec lesquels on peut comparer couples de permutations en definant l'order de Bruhat (de type A). Un critère utilise les polynômes totallement nonnegatifs et l'autre utilise les fonctions symétriques de Schur.

Restricted Motzkin permutations, Motzkin paths, continued fractions, and Chebyshev polynomials

Sergi Elizalde and Toufik Mansour

Abstract. We say that a permutation π is a *Motzkin permutation* if it avoids 132 and there do not exist $a < b$ such that $\pi_a < \pi_b < \pi_{b+1}$. We study the distribution of several statistics on Motzkin permutations, including the length of the longest increasing and decreasing subsequences and the number of rises and descents. We also enumerate Motzkin permutations with additional restrictions and study the distribution of occurrences of fairly general patterns in this class of permutations.

Dénombrement des classes de symétries des polyominos hexagonaux convexes

Dominique Gouyou-Beauchamps and Pierre Leroux

Abstract. In this paper, we enumerate the symmetry classes of convex polyominoes on the honeycomb lattice (*hexagonal* polyominoes). Here *convexity* is to be understood as convexity along the three main column directions. We deduce the generating series of *free* (i.e. up to reflection and rotation) and of asymmetric convex hexagonal polyominos, according to area and half-perimeter. See [**?**] for a longer version in English.

A characterization of the simply-laced FC-finite Coxeter groups

HAGIWARA Manabu and Masao ISHIKAWA and Hiroyuki TAGAWA

Abstract. We call an element of a Coxeter group fully covering if its length is equal to the number of the elements covered by it. For the Coxeter groups of type A, an element is fully covering if and only if it is 321-avoiding. In this sense it can be regarded as an extended notion of 321-avoiding. It also can be seen from the definition that a fully covering element is always fully commutative. Also, we call a Coxeter group bi-full when an element of the group is fully commutative if and only if it is fully covering. We show that the bi-full Coxeter groups are of type A, D, E. Note that we do not restrict the type E to E_6, E_7, and E_8. In other words, Coxeter groups of type $E_9, E_{10}, \ldots$ are also bi-full. According to a result of Fan, a Coxeter group is a simply-laced FC-finite Coxeter group if and only if it is a bi-full Coxeter group.

Cyclic Resultants

Christopher J. Hillar

Abstract. Let k be a field of characteristic zero and let $f \in k[x]$. The m-th cyclic resultant of f is $r_m = \text{Res}(f, x^m - 1)$. We characterize polynomials having the same set of nonzero cyclic resultants. Generically, for a polynomial f of degree d, there are exactly 2^{d-1} distinct degree d polynomials with the same set of cyclic resultants as f. However, in the generic monic case, degree d polynomials are uniquely determined by their cyclic resultants. Moreover, two reciprocal ("palindromic") polynomials giving rise to the same set of nonzero r_m are equal. The reciprocal case was stated many years ago (for $k = \mathbb{R}$) and has many applications stemming from such disparate fields as dynamics, number theory, and Lagrangian mechanics. In the process, we also prove a unique factorization result in semigroup algebras involving products of binomials.

Rectangular Schur Functions and Fermions

Takeshi Ikeda and Hiroshi Mizukawa and Tatsuhiro Nakjima and Hiro-Fumi Yamada

Abstract. We give an expression of the Schur function $S_{\square(m,n)}$, indexed by the rectangular partition $\square(m,n) = (n^m)$ as a sum of products of certain Schur functions and Schur's Q-functions.
Résumé. Nous donnons une expression de la fonction de Schur $S_{\square(m,n)}$, indexée par la partition rectangulaire $\square(m,n) = (n^m)$, comme une somme de produits de fonctions de Schur et de Q-fonctions de Schur.

Bruhat Order on the Involutions of Classical Weyl Groups

Federico Incitti

Abstract. It is known that a Coxeter group W, partially ordered by the Bruhat order, is a graded poset, with rank function given by the length, and that it is EL-shellable, hence Cohen-Macaulay, and Eulerian. In this work we consider the subposet of W induced by the set of involutions of W, denoted by $Invol(W)$. Our main result is that, if W is a classical Weyl group, then the poset $Invol(W)$ is graded, with rank function given by the average between the length and the absolute length, and that it is EL-shellable, hence Cohen-Macaulay, and Eulerian. In particular we obtain, as new results, a combinatorial description of the covering relation in the Bruhat order of the hyperoctahedral group and the even-signed permutation group, and a combinatorial description of the absolute length of the involutions in classical Weyl groups.

Résumé. Il est bien connu qu'un groupe de Coxeter W, munis de l'ordre de Bruhat, est un poset gradué, avec fonction rang donnée par la longueur, et qu'il est EL-shellable, donc de Cohen-Macaulay, et Eulerien. Dans cet article on considère le sous-poset induit par l'ensemble des involutions de W, noté $Invol(W)$. Nous montrons que, si W est un groupe de Weyl classique, alors le poset $Invol(W)$ est gradué, avec fonction rang égale à la moyenne entre la longueur et la longueur absolue, et qu'il est EL-shellable, donc de Cohen-Macaulay, et Eulerien. Nous obtenons en particulier deux résultats nouveaux: une description combinatoire de la relation de couverture dans l'ordre de Bruhat de B_n et D_n, et une description combinatoire de la longueur absolue des involutions dans les groupes de Weyl classiques.

Alternating Sign Matrices With One -1 Under Vertical Reflection

Pierre Lalonde

Abstract. We define a bijection that transforms an alternating sign matrix A with one -1 into a pair (N, E) where N is a (so called) *neutral* alternating sign matrix (with one -1) and E is an integer. The bijection preserves the classical parameters of Mills, Robbins and Rumsey as well as three new parameters (including E). It translates vertical reflection of A into vertical reflection of N. A hidden symmetry allows the interchange of E with one of the remaining two new parameters. A second bijection transforms (N, E) into a configuration of lattice paths called "mixed configuration".
Résumé. On définit une bijection qui transforme une matrice à signes alternants A ayant un seul -1 en une paire (N, E) constituée d'une matrice à signes alternants dite *neutre* N (elle aussi à un seul -1) et d'un paramètre entier E. La bijection préserve les paramètres classiques de Mills, Robbins et Rumsey ainsi que trois nouveaux paramètres (dont E). Elle transforme la réflexion verticale de A en la réflexion verticale de N. Une symétrie cachée permet l'échange de E avec un des deux autres nouveaux paramètres. Une seconde bijection transforme (N, E) en une configuration de chemins dite "configuration mixte".

Strict Partitions and Discrete Dynamical Systems

LE Minh Ha and PHAN Thi Ha Duong

Abstract. We prove that the set of partitions with distinct parts of a given positive integer under dominance ordering can be considered as a configuration space of a discrete dynamical model with two transition rules and with initial configuration being the singleton partition. This allows us to characterize its lattice structure, fixed point, longest chains as well as their length, using Chip Firing Game theory. Finally, two extensions and their applications are discussed.

Résumé. Nous montrons que l'ensemble des partitions avec differents parts d'un entier donné n muni de l'ordre de dominance peut être considérećomm l'espace de configurations d'un système dynamique discret avec deux règles de transitions et avec la configuration initialle étant la partition (n). Cela nous permet de caractériser sa structure de treillis, son point fixe, les chaînes les plus longues ainsi que leurs longueur, en utilisant la theorie de Chip Firing Game. Enfin, deux extensions et leurs applications sont données.

Counting Unrooted Loopless Planar Maps

Valery A. Liskovets and Timothy R. Walsh

Abstract. We present a formula for the number of n-edge unrooted loopless planar maps considered up to orientation-preserving isomorphism. The only sum contained in this formula is over the divisors of n.
Résumé. Nous présentons une formule pour le nombre de cartes planaires sans boucles avec n arêtes, à isomorphisme près préservant l'orientation. La seule somme contenue dans cette formule est prise parmi les diviseurs de n.

Descents, Major Indices, and Inversions in Permutation Groups

Anthony Mendes and Jeffrey Remmel

Abstract. We give a new proof of a multivariate generating function involving the descent, major index, and inversion statistic due to Gessel. We then show how one can easily modify this proof to give new generating functions involving these three statistics over Young's hyperoctahedral group, the Weyl group of type D, and multiples of permutations. All of our proofs are combinatorial in nature and exploit fundamental relationships between the elementary and homogeneous symmetric functions.

A Solution to the Tennis Ball Problem

Anna de Mier and Marc Noy

Abstract. We present a complete solution to the so-called tennis ball problem, which is equivalent to counting lattice paths in the plane that use North and East steps and lie between certain boundaries. The solution takes the form of explicit expressions for the corresponding generating functions.

Our method is based on the properties of Tutte polynomials of matroids associated to lattice paths. We also show how the same method provides a solution to a wide generalization of the problem.

Résumé. Nous présentons une solution complète au "problème des balles de tennis", pro-blème qui revient à compter des chemins, formés par des pas Nord et Est, dans une région délimitée de $\mathbb{N}^2$. La solution se présente sous la forme d'expressions explicites pour les séries génératrices correspondantes.

Notre méthode repose sur certaines propriétés des polynômes de Tutte des matroïdes associés à des chemins de $\mathbb{N}^2$. Nous montrons aussi comment cette méthode permet de résoudre un problème beaucoup plus général.

Decomposition of Green polynomials of type A and DeConcini-Procesi algebras of hook partitions

Hideaki Morita and Tatsuhiro Nakajima

Abstract. A Kraśkiewicz-Weymann type theorem is obtained for the DeConcini-Procesi algebras of hook partitions. The DeConcini-Procesi algebras are graded modules of the symmetric groups, that generalize the coinvariant algebras. Defining the direct sums of the homogeneous components of the algebra in some natural way, we show that these submodules are induced from representations of the corresponding subgroup of the symmetric group. The Green polynomials of type A play an essential role in our argument.

On Computing the Coefficients of Rational Formal Series

Paolo Massazza and Roberto Radicioni

Abstract. In this work we study the problem of computing the coefficients of rational formal series in two commuting variables. Given a rational formal series $\phi(x,y) = \sum_{n,k\geq 0} c_{nk}x^n y^k = P(x,y)/Q(x,y)$ with $P, Q \in \{x, y\}$ and $Q(0,0) \neq 0$, we show that the coefficient $[x^i y^j]\phi(x,y)$ can be computed in time $O(i+j)$ under the uniform cost criterion.

RÉSUMÉ. Dans cet article, nous étudions le problème du calcul des coefficients de séries formelles rationelles en deux variables commutatives. Etant donné une série formelle rationnelle $\phi(x,y) = \sum_{n,k\geq 0} c_{nk}x^n y^k = P(x,y)/Q(x,y)$ ou $P, Q \in \{x, y\}$ et $Q(0,0) \neq 0$, nous montrons que le coefficient $[x^i y^j]\phi(x,y)$ peut être calculé en un temps $O(i+j)$ sous le critère de coût uniforme.

Ribbon tilings of Ferrers diagrams, flips and the 0-Hecke algebra

Gilles Radenne

Abstract.

In this article we study how the 0-Hecke algebra $H_m(0)$ can be used to give an algebraic structure to ribbon tilings of Ferrers diagrams. The key to this structure is the Stanton-White bijection, which gives a bijection between n-ribbon tableaux and n-upplets of Young tableaux. Restricting to standard ribbon tableaux, we can define a natural action of $H_m(0)$. Thus we can define local actions on ribbon tableaux, which we call flips or pseudo-flips, and which are generalisations of domino flips. Then with some help from the Yang-Baxter relations we prove some properties about minimal flip chains, properties which remain true for ribbon tilings.

Résumé. Le but de cet article est de montrer comment on peut utiliser la 0-algèbre de Hecke $H_m(0)$ afin de donner une structure algébrique aux pavages par rubans d'un diagramme de Ferrers donné. Cette structure découle de la bijection de Stanton-White entre les tableaux de n-rubans et les n-uplets de tableaux de Young. Si on se limite aux tableaux standards de rubans, cela nous donne une action naturelle de $H_m(0)$, qui nous permet alors de définir des modifications locales sur les tableaux de rubans, que nous appellons flips et pseudo-flips. Ce sont des généralisations du flip classique de dominos. Grâce aux relations de Yang-Baxter on peut alors donner des invariants sur les chaînes minimales de flips, qui se conservent quand on passe aux pavages par rubans.

On Inversions in Standard Young Tableaux

Michael Shynar

Abstract. In this work, we present the inversion number of a standard Young tableau, and determine its distribution over certain sets of standard Young tableaux. Specifically, the work determines the distribution of the inversion number over hook-shaped tableaux and over tableaux of shape (n, n). We also study the parity (also referred to as 'sign balance') of the inversion number over hook-shaped tableaux and over

$(n-k,k)$-shaped tableaux. The latter results resemble results in the field of pattern-avoiding permutations, achieved by Adin, Roichman and Reifegerste.

Finite-Dimensional Crystals for Quantum Affine Algebras of type $D_n^{(1)}$

Philip Sternberg

Abstract. The combinatorial structure of the crystal basis $B^{(2,2)}$ for the $U_q'(\widehat{\mathfrak{so}}_{2n})$-module $W^{(2,2)}$ is given, and a conjecture is presented for the combinatorial structure of the crystal basis $B^{(2,s)}$ for the $U_q'(\widehat{\mathfrak{so}}_{2n})$-module $W^{(2,s)}$.
Résumé. Nous donnons la structure combinatoire de la base cristalline $B^{(2,2)}$ pour le $U_q'(\widehat{\mathfrak{so}}_{2n})$-module $W^{(2,2)}$, et nous conjecturons la structure combinatoire de la base cristalline $B^{(2,s)}$ pour le $U_q'(\widehat{\mathfrak{so}}_{2n})$-module $W^{(2,s)}$.

Rook Numbers and the Normal Ordering Problem

Anna Varvak

Abstract. For an element w in the Weyl algebra generated by D and U with relation $DU = UD + 1$, the normally ordered form is $w = \sum c_{i,j} U^i D^j$. We demonstrate that the normal order coefficients $c_{i,j}$ of a word w are rook numbers on a Ferrers board. We use this interpretation to give a new proof of the rook factorization theorem, which we use to provide an explicit formula for the coefficients $c_{i,j}$. We calculate the Weyl binomial coefficients: normal order coefficients of the element $(D+U)^n$ in the Weyl algebra. We extend some of these results to the q-analogue of the Weyl algebra.
Résumé. Pour un élément dans l'algèbre du Weyl généré par D et U avec de relation $DU = UD + 1$, la forme d'ordre normal est $w = \sum c_{i,j} U^i D^j$. Nous démontrons que les coefficients d'ordre normal $c_{i,j}$ sont des nombres de tours sur d'amier de Ferrers. Nous employons cette interprétation pour fournir une nouvelle preuve de théorème de factorisation de tours, à la laquelle nous mène une formule explicite pour les coefficients $c_{i,j}$. Nous calculons les coefficients binomiaux du Weyl: les coefficients d'ordre normal d'élément $(D+U)^n$ de l'algèbre du Weyl. Nous prolongeons quelque de ces résultats à q-analogue d'algèbre du Weyl.

Sheared Tableaux and bases for the symmetric functions

Benjamin Young

Abstract.

We study the operation of *shearing* Schur functions, which yields a new family of bases for the space of symmetric functions. In the course of this study, we derive some interesting combinatorial results and inequalities on the Littlewood-Richardson coefficients of sheared Schur functions.

Résumé. Nous étudions l'opération de trancher les fonctions de Schur, qui nous donne une nouvelle famille de bases pour l'espace des fonctions symétriques. Pendent cette étude, nous dérivons des résultats et des inégalités combinatoires intéressants. Ces résultats décrivant les coefficients Littlewood-Richardson des fonctions Schur tranchées.