Towards Higher Impact Argumentation

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Abstract

There are a number of frameworks for modelling argumentation in logic. They incorporate a formal representation of individual arguments and techniques for comparing conflicting arguments. An example is the framework by Besnard and Hunter that is based on classical logic and in which an argument (obtained from a knowledgebase) is a pair where the first item is a minimal consistent set of formulae that proves the second item (which is a formula). In the framework, the only counter-arguments (defeaters) that need to be taken into account are canonical arguments (a form of minimal undercut). Argument trees then provide a way of exhaustively collating arguments and counter-arguments. A problem with this set up is that some argument trees may be "too big" to have sufficient impact. In this paper, we address the need to increase the impact of argumentation by using pruned argument trees. We formalize this in terms of how arguments resonate with the intended audience of the arguments. For example, if a politician wants to make a case for raising taxes, the arguments used would depend on what is important to the audience: Arguments based on increased taxes are needed to pay for improved healthcare would resonate better with an audience of pensioners, whereas arguments based on increased taxes are needed to pay for improved transport infrastructure would resonate better with an audience of business executives. By analysing the resonance of arguments, we can prune argument trees to raise their impact.

Introduction

Consider reading an article in a current affairs magazine such as the Economist or Newsweek. Such an article only has a relatively small number of arguments. These arguments are quite a small subset of all possible arguments that either the writer or the reader could construct from their own knowledgebases. In producing an article, a writer regards some arguments as having higher impact with the intended audience than others, and only the higher impact ones are used. In a sense, there is some filter on the argument tree that can be constructed.

This perspective raises two questions: (1) What is this notion of impact? (i.e. what does it mean and how do we measure it in practice?) and (2) Given a measure of impact, how do we use it as a filter on arguments? In this paper, we look at some of the issues surrounding these questions.

To characterize these issues we use an existing framework for logic-based argumentation by Besnard and Hunter (Besnard & Hunter 2001). In the next section, we review this framework. Then in subsequent sections, we show how this can be adapted for higher impact argumentation.

Logic-based argumentation

We consider a propositional language. We use $\alpha, \beta, \gamma, \ldots$ to denote formulae and $\Delta, \Phi, \Psi, \ldots$ to denote sets of formulae. Deduction in classical propositional logic is denoted by the symbol $\vdash$ and deductive closure by Th so that $\mathsf{Th}(\Phi) = \{\alpha \mid \Phi \vdash \alpha\}$.

For the following definitions, we first assume a knowledgebase Δ (a finite set of formulae) and use this Δ throughout. We further assume that every subset of Δ is given an enumeration $\langle \alpha_1, \ldots, \alpha_n \rangle$ of its elements, which is called its canonical enumeration. This really is not a demanding constraint: In particular, the constraint is satisfied whenever we impose an arbitrary total ordering over Δ. Importantly, the order has no meaning and is not meant to represent any respective importance of formulae in Δ is imposed. It is only a convenient way to indicate the order in which it is assumed the formulae in any subset of Δ are conjoined to make a formula logically equivalent to that subset.

The paradigm for the approach is a large repository of information, represented by Δ, from which arguments can be constructed for and against arbitrary claims. Apart from information being understood as declarative statements, there is no a priori restriction on the contents, and the pieces of information in the repository can be as complex as possible. Therefore, Δ is not expected to be consistent. It need even not be the case that every single formula in Δ is consistent.

The framework adopts a very common intuitive notion of an argument. Essentially, an argument is a set of relevant formulae that can be used to classically prove some point, together with that point. Each point is represented by a formula.

Definition 1 *An* **argument** *is a pair $\langle \Phi, \alpha \rangle$ such that: (1) $\Phi \nvdash \bot$; (2) $\Phi \vdash \alpha$; and (3) there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash \alpha$. We say that $\langle \Phi, \alpha \rangle$ is an argument for α. We call α the consequent of the argument and Φ the support of the argument. We also say that Φ is a support for α. For an argument $\langle \Phi, \alpha \rangle$, let* $\mathsf{Support}(\langle \Phi, \alpha \rangle) = \Phi$.

Example 1 *Let $\Delta = \{\alpha, \alpha \rightarrow \beta, \gamma \rightarrow \neg\beta, \gamma, \delta, \delta \rightarrow \beta, \neg\alpha, \neg\gamma\}$. Some arguments are:*

$$\begin{gathered}\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle \\ \langle\{\neg\alpha\}, \neg\alpha\rangle \\ \langle\{\alpha \rightarrow \beta\}, \neg\alpha \vee \beta\rangle \\ \langle\{\neg\gamma\}, \delta \rightarrow \neg\gamma\rangle\end{gathered}$$

Arguments are not independent. In a sense, some encompass others. This is clarified as follows.

Definition 2 *An argument $\langle\Phi, \alpha\rangle$ is* **more conservative** *than an argument $\langle\Psi, \beta\rangle$ iff $\Phi \subseteq \Psi$ and $\beta \vdash \alpha$.*

Example 2 *$\langle\{\alpha\}, \alpha \vee \beta\rangle$ is more conservative than $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. Here, the latter argument can be obtained from the former (using $\alpha \rightarrow \beta$ as an extra hypothesis) but this is not the case in general.*

Some arguments directly oppose the support of others, which amounts to the notion of an undercut.

Definition 3 *An* **undercut** *for an argument $\langle\Phi, \alpha\rangle$ is an argument $\langle\Psi, \neg(\phi_1 \wedge \ldots \wedge \phi_n)\rangle$ where $\{\phi_1, \ldots, \phi_n\} \subseteq \Phi$.*

Example 3 *Let $\Delta = \{\alpha, \alpha \rightarrow \beta, \gamma, \gamma \rightarrow \neg\alpha\}$. Then, $\langle\{\gamma, \gamma \rightarrow \neg\alpha\}, \neg(\alpha \wedge (\alpha \rightarrow \beta))\rangle$ is an undercut for $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$. A less conservative undercut for $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$ is $\langle\{\gamma, \gamma \rightarrow \neg\alpha\}, \neg\alpha\rangle$.*

Definition 4 *$\langle\Psi, \beta\rangle$ is a* **maximally conservative undercut** *of $\langle\Phi, \alpha\rangle$ iff $\langle\Psi, \beta\rangle$ is an undercut of $\langle\Phi, \alpha\rangle$ such that no undercuts of $\langle\Phi, \alpha\rangle$ are strictly more conservative than $\langle\Psi, \beta\rangle$ (that is, for all undercuts $\langle\Psi', \beta'\rangle$ of $\langle\Phi, \alpha\rangle$, if $\Psi' \subseteq \Psi$ and $\beta \vdash \beta'$ then $\Psi \subseteq \Psi'$ and $\beta' \vdash \beta$).*

Definition 5 *An argument $\langle\Psi, \neg(\phi_1 \wedge \ldots \wedge \phi_n)\rangle$ is a* **canonical undercut** *for $\langle\Phi, \alpha\rangle$ iff it is a maximally conservative undercut for $\langle\Phi, \alpha\rangle$ and $\langle\phi_1, \ldots, \phi_n\rangle$ is the canonical enumeration of Φ.*

An argument tree describes the various ways an argument can be challenged, as well as how the counter-arguments to the initial argument can themselves be challenged, and so on recursively.

Definition 6 *An* **argument tree** *for α is a tree T where the nodes are arguments such that*

1. *The root is an argument for α.*
2. *For no node $\langle\Phi, \beta\rangle$ with ancestor nodes $\langle\Phi_1, \beta_1\rangle, \ldots, \langle\Phi_n, \beta_n\rangle$ is Φ a subset of $\Phi_1 \cup \cdots \cup \Phi_n$.*
3. *The children nodes of a node A are canonical undercuts for A that obey 2.*

A **complete argument tree** *for α is an argument tree for α such that the children nodes of a node A consist of* all *canonical undercuts for A that obey 2.*

As a notational convenience, in examples of argument trees we will use the $\diamond$ symbol to denote the consequent of an argument when that argument is a canonical undercut.

Definition 7 *Let T be an argument tree and let $T = (N, E)$ where N is a set of nodes and E is a set of edges. The set of nodes in T is given by $\mathsf{Nodes}(T) = N$.*

Definition 8 *Let T_1 and T_2 be argument trees. T_1 is* **more or equally pruned** *than T_2 iff $\mathsf{Nodes}(T_1) \subseteq \mathsf{Nodes}(T_2)$. For a complete argument tree T, let $\mathsf{Space}(T) = \{T' \mid T'$ is more or equally pruned than $T\}$.*

In $\mathsf{Space}(T)$, the least pruned tree is T, and the most pruned tree is the tree given by the empty argument tree (i.e. the tree with no nodes or arcs).

Whilst the use of canonical arguments simplifies the complete argument trees enormously, they may still be "too big" in the sense that too many of the arguments used do not resonate with the intended audience. In this paper, we formalize how arguments resonate with the intended audience. By analysing the resonance of arguments, we can prune argument trees to raise their impact.

Resonance of arguments

The impact of argumentation depends on what an agent regards as important. Different agents think different things are important. We adopt a simple way, called a desideratabase, of capturing what an agent in the intended audience thinks is important, and then use this to measure how arguments resonante with the agent.

Definition 9 *A* **desideratabase** *for an agent is a tuple (Π, λ) where (1) Π is a set of classical propositional formulae called a set of* **desiderata** *and (2) λ is mapping from Π to $[0, 1]$ called a* **desiderata weighting**. *For $\delta_i, \delta_j \in \Pi$, δ_i is a* **more important desideratum** *than δ_j for the agent iff $\lambda(\delta_j) < \lambda(\delta_i)$. A* **unit desiderata weighting** *λ is a desiderata weighting such that for all $\delta \in \Pi$, $\lambda(\delta) = 1$. For desiderata weighting λ_1 and λ_2, λ_1 is* **stronger than or equal** *to λ_2 iff for all $\delta \in \Pi$, $\lambda_2(\delta) \leq \lambda_1(\delta)$.*

Intuitively, a desideratum (a formula in a desiderata) represents what an agent would like to be true in the world. There is no constraint that it has to be something that the agent can actually make true. It may be something unattainable such as "there will be no more wars" or "bread and milk is free for everyone". There is also no constraint that the desiderata for an agent are consistent.

We can view a desiderata weighting λ as delineating a ranking over possible worlds (where the set of worlds is the set of classical interpretations given by the propositional language of Π). We do not provide details in this paper.

Definition 10 *Let (Π, λ) be a desideratabase, and let $\langle\Phi, \alpha\rangle$ be an argument. $\langle\Phi, \alpha\rangle$ is a* **resonating argument** *iff for some $\delta \in \Pi$, $\Phi \vdash \delta$ or $\Phi \vdash \neg\delta$.*

So for an argument to have an impact on an agent, the support for the argument must imply a desideratum or the negation of a desideratum.

Definition 11 *Let (Π, λ) be a desideratabase, and let $\langle\Phi, \alpha\rangle$ be a argument. The* **echo** *of this argument with respect to the desiderata is defined as follows*

$$\mathsf{Echo}(\Phi, \Pi) = \{\delta \in \Pi \mid (\Phi \vdash \delta \text{ or } \Phi \vdash \neg\delta)\}$$

Note if (Π_1, λ_1) and (Π_2, λ_2) are desideratabases, such that $\mathsf{Th}(\Pi_1) \subseteq \mathsf{Th}(\Pi_2)$, and Φ is the support of an argument, then it is not necessarily the case that $\mathsf{Echo}(\Phi, \Pi_1) \subseteq$

$\mathsf{Echo}(\Phi, \Pi_2)$ or that $\mathsf{Echo}(\Phi, \Pi_2) \subseteq \mathsf{Echo}(\Phi, \Pi_1)$, as illustrated in the next example.

Example 4 *Let* $\Pi_1 = \{\alpha, \beta \wedge \gamma\}$, $\Pi_2 = \{\alpha \wedge \beta, \gamma\}$, *and let* $\Phi = \{\alpha, \gamma\}$. *So* $\mathsf{Echo}(\Phi, \Pi_1) = \{\alpha\}$ *and* $\mathsf{Echo}(\Phi, \Pi_2) = \{\gamma\}$. *Hence* $\mathsf{Echo}(\Phi, \Pi_1) \not\subseteq \mathsf{Echo}(\Phi, \Pi_2)$ *and* $\mathsf{Echo}(\Phi, \Pi_2) \not\subseteq \mathsf{Echo}(\Phi, \Pi_1)$.

Obviously if an agent has no desiderata, $\Pi = \emptyset$, then for any argument $\langle \Phi, \alpha \rangle$, $\mathsf{Echo}(\Phi, \Pi) = \emptyset$. At the other extreme, if every desideratum that an agent has is inconsistent with itself, i.e. for all $\delta \in \Pi$, $\{\delta\} \vdash \bot$, then $\vdash \top \leftrightarrow \neg\delta$, and so for any argument $\langle \Phi, \alpha \rangle$, $\Phi \vdash \neg\delta$, and hence $\mathsf{Echo}(\Phi, \Pi) = \Pi$.

Definition 12 *Let* (Π, λ) *be a desideratabase, and let* $\langle \Phi, \alpha \rangle$ *be a argument. The* **resonance** *of this argument with respect to the desideratabase is defined as follows*

$$\mathsf{Resonance}(\Phi, \Pi, \lambda) = \sum_{\delta \in \mathsf{Echo}(\Phi, \Pi)} \lambda(\delta)$$

The resonance function captures the weighted sum of the echo of the argument. This reflects the importance that the agent attaches to the argument.

Example 5 *Let* $\Pi = \{\beta, \gamma\}$, $\lambda(\beta) = 1$, *and* $\lambda(\gamma) = 0.5$. *Let* A_1 *and* A_2 *be arguments as below where* A_2 *is a canonical undercut for* A_1.

$$A_1 = \langle \{\beta, \beta \rightarrow \alpha\}, \alpha \rangle$$
$$A_2 = \langle \{\neg(\beta \vee \gamma)\}, \neg(\beta \wedge (\beta \rightarrow \alpha)) \rangle$$

Hence we get the following resonance values.

$$\mathsf{Resonance}(\mathsf{Support}(A_1), \Pi, \lambda) = 1.0$$
$$\mathsf{Resonance}(\mathsf{Support}(A_2), \Pi, \lambda) = 1.5$$

The resonance function is monotonic in the membership of the support of the argument.

Proposition 1 *Let* Φ_1 *and* Φ_2 *be supports for arguments and let* (Π, λ) *be a desideratabase. If* $\Phi_1 \subseteq \Phi_2$, *then* $\mathsf{Resonance}(\Phi_1, \Pi, \lambda) \leq \mathsf{Resonance}(\Phi_2, \Pi, \lambda)$.

Similarly, a logically weaker support for an argument has a lower resonance.

Proposition 2 *Let* Φ_1 *and* Φ_2 *be supports for arguments and let* (Π, λ) *be a desideratabase. If* $\mathsf{Th}(\Phi_1) \subseteq \mathsf{Th}(\Phi_2)$, *then* $\mathsf{Resonance}(\Phi_1, \Pi, \lambda) \leq \mathsf{Resonance}(\Phi_2, \Pi, \lambda)$.

With unit desiderata weighting, the resonance function is also monotonic in the membership of the desideratabase.

Proposition 3 *Let* Φ *be a support for an argument and let* (Π_1, λ_1) *and* (Π_2, λ_2) *be desideratabases, where* λ_1 *and* λ_2 *are unit desiderata weightings. If* $\Pi_1 \subseteq \Pi_2$, *then* $\mathsf{Resonance}(\Phi, \Pi_1, \lambda_1) \leq \mathsf{Resonance}(\Phi, \Pi_2, \lambda_2)$.

However, if (Π_1, λ_1) and (Π_2, λ_2) are desideratabases, such that $\mathsf{Th}(\Pi_1) \subseteq \mathsf{Th}(\Pi_2)$ and λ_1 and λ_2 are unit desiderata weightings, and $\langle \Phi, \alpha \rangle$ is an argument, then we see in the next example that it is not necessarily the case that $\mathsf{Resonance}(\Phi, \Pi_1, \lambda_1) \leq \mathsf{Resonance}(\Phi, \Pi_2, \lambda_2)$, and it is not necessarily the case that $\mathsf{Resonance}(\Phi, \Pi_2, \lambda_2) \leq \mathsf{Resonance}(\Phi, \Pi_1, \lambda_1)$.

Example 6 *Let* (Π_1, λ_1) *and* (Π_2, λ_2) *be desideratabases where* $\Pi_1 = \{\alpha, \beta\}$ *and* $\Pi_2 = \{\alpha \wedge \beta, \gamma\}$ *and* λ_1 *and* λ_2 *are unit desiderata weightings. So* $\mathsf{Th}(\Pi_1) \subseteq \mathsf{Th}(\Pi_2)$. *Also let* $\Phi = \{\beta\}$ *and* $\Phi' = \{\gamma\}$. *So* $\mathsf{Echo}(\Phi, \Pi_1) = \{\beta\}$, $\mathsf{Echo}(\Phi, \Pi_2) = \emptyset$, $\mathsf{Echo}(\Phi', \Pi_1) = \emptyset$, *and* $\mathsf{Echo}(\Phi', \Pi_2) = \{\gamma\}$. *So* $\mathsf{Resonance}(\Phi, \Pi_2, \lambda_2) < \mathsf{Resonance}(\Phi, \Pi_1, \lambda_1)$ *and* $\mathsf{Resonance}(\Phi', \Pi_1, \lambda_1) < \mathsf{Resonance}(\Phi', \Pi_2, \lambda_2)$

There are various other ways to define one desideratabase being logically weaker than another. One way, given in the following proposition, implies that the logically weaker desideratabase leads to a higher resonance for an argument.

Proposition 4 *Let* Φ *be a support for an argument and let* (Π_1, λ_1) *and* (Π_2, λ_2) *be desideratabases. If there is a bijection* G *from* Π_1 *to* Π_2 *such that for all* $\delta \in \Pi_1$ $\{\delta\} \vdash G(\delta)$ *and* $\lambda_1(\delta) \leq \lambda_2(G(\delta))$, *then* $\mathsf{Resonance}(\Phi, \Pi_1, \lambda_1) \leq \mathsf{Resonance}(\Phi, \Pi_2, \lambda_2)$.

Desiderata weighting can also have a net effect on resonance. At one extreme, if $\lambda(\delta) = 0$ for all $\delta \in \Pi$, then $\mathsf{Resonance}(\Phi, \Pi, \lambda) = 0$. At the other extreme, if $\lambda(\delta) = 1$ for all $\delta \in \Pi$, (i.e. λ is a unit weighting) then $\mathsf{Resonance}(\Phi, \Pi, \lambda) = |\mathsf{Echo}(\Phi, \Pi)|$. More generally, increasing the strength of the weighting in a desideratabase also increases resonance as formalised in the next proposition.

Proposition 5 *Let* Φ *be a support for an argument and let* (Π, λ_1) *and* (Π, λ_2) *be desideratabases. If* λ_1 *is stronger than or equal to* λ_2, *then* $\mathsf{Resonance}(\Phi, \Pi, \lambda_2) \leq \mathsf{Resonance}(\Phi, \Pi, \lambda_1)$.

Finally in this section, we need to measure the cost of an argument. We will use an obvious and simple measure: The cost of an argument is the number of different propositional letters used in the support of the argument. So for a support Φ, and using the definition of $\mathsf{Propositions}$ given below, the cost is the cardinality of $\mathsf{Propositions}(\Phi)$.

Definition 13 *The set of propositional letters used in* $\phi \in \mathcal{L}$ *is given by* $\mathsf{Propositions}(\phi)$, *and the set of propositional letters used in* $\Phi \subseteq \mathcal{L}$ *is given by*

$$\mathsf{Propositions}(\Phi) = \bigcup_{\phi \in \Phi} \mathsf{Propositions}(\phi)$$

Example 7 $\mathsf{Propositions}(\alpha \vee (\beta \rightarrow (\alpha \vee \neg\delta))) = \{\alpha, \beta, \delta\}$.

For this measure of the cost of an argument, we are assuming that an agent in the audience prefers arguments with fewer propositional letters, since they are simpler to read and analyse. We draw Definitions 13 and 12 together in the next section.

Measuring impact of argument trees

Now we want to consider resonance and cost in the context of an argument tree. For this, we want resonance of a tree to be sensitive to the depth of the arguments in the tree. We formalise this with the following notion of a discount function.

Definition 14 *For any argument tree T, the function* Depth *from* $\mathsf{Nodes}(T)$ *to* $\mathbb{N}$ *is such that for the root* $A_r \in \mathsf{Nodes}(T)$, $\mathsf{Depth}(A_r) = 1$, *and for all nodes* $A_i \in \mathsf{Nodes}(T)$, *if* $\mathsf{Depth}(A_i) = n$, *then for any child* A_j *of* A_i *in* T, $\mathsf{Depth}(A_j) = n + 1$.

Definition 15 *A* **discount function** *is a monotonically decreasing function from* $\mathbb{N}$ *to* $[0, 1]$. *So if* μ *is a discount function, then* $\mu(n) \geq \mu(n + 1)$ *for all* $n \in \mathbb{N}$. *For a discount function* μ, *the* **boundary** *of* μ *is* $n \in \mathbb{N}$ *such that* $\mu(n) > 0$ *and* $\mu(n + 1) = 0$. *A* **unit discount function** *is a discount function* μ *such that for all* $n \in \mathbb{N}$, $\mu(n) = 1$. *A* **unit-step discount function** *is a discount function* μ *such that for all* $n \in \mathbb{N}$, $\mu(n) = 1$ *or* $\mu(n) = 0$. *Let* μ_1 *and* μ_2 *be discount functions.* μ_1 *is* **more or equally discounting** *than* μ_2 *iff for all* $n \in \mathbb{N}$, $\mu_1(n) \leq \mu_2(n)$.

Definition 16 *A* **context** *is a tuple* (Π, λ, μ) *where* (Π, λ) *is a desideratabase, and* μ *is a discount function. A* **unit context** *is a context* (Π, λ', μ') *where* λ' *is a unit desiderata weighting, and* μ' *is a unit discount function.*

A unit context is an extreme context where neither the desiderata weighting nor the discount have any net effect on the outcome. In other words, with a unit context the resonance of an argument tree is only based on the actual desiderata.

We now consider a definition for measuring the resonance of an argument tree. This definition takes a sum of the resonance of the arguments in the tree where the net effect of each argument is scaled by the discount function. The discounting increases going down the tree. So arguments at a greater depth have a reduced net effect on the resonance of the tree.

Definition 17 *The* **resonance** *of the argument tree* T *with respect to the context* (Π, λ, μ), *denoted* $\mathsf{Resonance}(T, \Pi, \lambda, \mu)$, *is defined as follows*

$$\sum_{A \in \mathsf{Nodes}(T)} \mathsf{Resonance}(\mathsf{Support}(A), \Pi, \lambda) \times \mu(\mathsf{Depth}(A))$$

Definition 18 *The* **propositional cost** *of an argument tree* T *denoted* $\mathsf{Propositionalcost}(T)$ *is the cardinality of the following set.*

$$\bigcup_{A \in \mathsf{Nodes}(T)} \mathsf{Propositions}(\mathsf{Support}(A))$$

The above measure of cost is $\log_2$ of the number of classical interpretations for the supports of the arguments in T.

Definition 19 *For a context* (Π, λ, μ), *the* **impact** *of an argument tree* T *with one or more nodes is given by the following ratio,*

$$\mathsf{Impact}(T, \Pi, \lambda, \mu) = \frac{\mathsf{Resonance}(T, \Pi, \lambda, \mu)}{\mathsf{Propositionalcost}(T)}$$

and for the empty argument tree $T_\emptyset$ *(i.e. the tree with no nodes),* $\mathsf{Impact}(T_\emptyset, \Pi, \lambda, \mu) = 0$.

Pruning an argument tree may decrease the cost, but this may also decrease the resonance. In a unit context, $\mathsf{Impact}(T, \Pi, \lambda, \mu)$ gives the ratio of desiderata in the echo of the arguments in T to the propositional letters used in the support of the arguments in T.

When measuring impact with a non-unit context, we are also taking into account the relative interest in the desiderata and the relative depth of the arguments in the argument trees. These orthogonal factors help to better evaluate the impact for a particular audience. We therefore see selection of an appropriate context as an important design decision in developing an argumentation system. In some applications, we want fewer arguments per argument tree and in other applications we want more arguments per argument tree.

In the following example, we use the impact function to compare three argument trees, T_1, T_2 and T_3, where T_3 is a complete argument tree, and T_1 and T_2 are more pruned than T_3.

Example 8 *Let* $\Delta = \{\beta, \beta \rightarrow \alpha, \gamma, \gamma \rightarrow \neg\beta, \neg\gamma \wedge \delta\}$ *From this, we obtain the following (non-exhaustive) list of arguments.*

$$\begin{array}{l} A_1 = \langle\{\beta, \beta \rightarrow \alpha\}, \alpha\rangle \\ A_2 = \langle\{\gamma, \gamma \rightarrow \neg\beta\}, \neg(\beta \wedge (\beta \rightarrow \alpha))\rangle \\ A_3 = \langle\{\neg\gamma \wedge \delta\}, \neg(\gamma \wedge (\gamma \rightarrow \neg\beta))\rangle \end{array}$$

Let $\mathsf{Nodes}(T_1) = \{A_1\}$, $\mathsf{Nodes}(T_2) = \{A_1, A_2\}$, *and* $\mathsf{Nodes}(T_3) = \{A_1, A_2, A_3\}$ *giving the following argument trees, where* T_3 *is a complete argument tree, and* T_2 *is more pruned than* T_3, *and* T_1 *is more pruned than* T_2.

$$\begin{array}{ccc} A_1 & A_1 & A_1 \\ & \uparrow & \uparrow \\ & A_2 & A_2 \\ & & \uparrow \\ & & A_3 \end{array}$$

Let (Π, λ, μ) *be a unit context where* $\Pi = \{\beta, \neg\gamma, \delta\}$. $\mathsf{Echo}(A_1, \Pi) = \{\beta\}$, $\mathsf{Echo}(A_2, \Pi) = \{\beta, \neg\gamma\}$, *and* $\mathsf{Echo}(A_3, \Pi) = \{\neg\gamma, \delta\}$. *Hence*

$$\begin{array}{l} \mathsf{Resonance}(T_1, \Pi, \lambda, \mu) \\ = (\mathsf{Resonance}(\mathsf{Support}(A_1), \Pi, \lambda) \times \mu(1)) \\ = (1 \times 1) = 1 \end{array}$$

$$\begin{array}{l} \mathsf{Resonance}(T_2, \Pi, \lambda, \mu) \\ = (\mathsf{Resonance}(\mathsf{Support}(A_1), \Pi, \lambda) \times \mu(1)) \\ \qquad + (\mathsf{Resonance}(\mathsf{Support}(A_2), \Pi, \lambda) \times \mu(2)) \\ = (1 \times 1) + (2 \times 1) = 3 \end{array}$$

$$\begin{array}{l} \mathsf{Resonance}(T_3, \Pi, \lambda, \mu) \\ = \mathsf{Resonance}(\mathsf{Support}(A_1), \Pi, \lambda) \times \mu(1)) \\ \qquad + \mathsf{Resonance}(\mathsf{Support}(A_2), \Pi, \lambda) \times \mu(2)) \\ \qquad + \mathsf{Resonance}(\mathsf{Support}(A_3), \Pi, \lambda) \times \mu(3)) \\ = (1 \times 1) + (2 \times 1) + (2 \times 1) = 5 \end{array}$$

Since $\mathsf{Propositionalcost}(T_1) = 2$, $\mathsf{Propositionalcost}(T_2) = 3$, *and* $\mathsf{Propositionalcost}(T_3) = 4$, *we get* $\mathsf{Impact}(T_1, \Pi, \lambda, \mu) = \frac{1}{2}$, $\mathsf{Impact}(T_2, \Pi, \lambda, \mu) = \frac{3}{3}$, *and* $\mathsf{Impact}(T_3, \Pi, \lambda, \mu) = \frac{5}{4}$. *If we use an alternative discount function, where* $\mu_2(1) = 1$, $\mu_2(2) = 0.5$, *and* $\mu_2(3) = 0$, *then* $\mathsf{Impact}(T_1, \Pi, \lambda, \mu_2) = \frac{1}{2}$, $\mathsf{Impact}(T_2, \Pi, \lambda, \mu_2) = \frac{2}{3}$, *and* $\mathsf{Impact}(T_3, \Pi, \lambda, \mu_2) = \frac{2}{4}$. *So with the unit discount* *function μ, the complete argument tree T_3 has the highest impact, whereas with the non-unit discount function μ_2, the pruned argument tree T_2 has the highest impact.*

When (Π, λ, μ) is not a unit context, the following result summarizes the conditions for zero impact.

Proposition 6 *Let (Π, λ, μ) be a context, and let T be an argument tree.* $\mathsf{Impact}(T, \Pi, \lambda, \mu) = 0$ *iff for all* $A \in \mathsf{Nodes}(T)$ $(\delta \in \mathsf{Echo}(\mathsf{Support}(A), \Pi)$ *implies* $(\mu(\mathsf{Depth}(A)) = 0$ *or* $\lambda(\delta) = 0)))$.

The following series of propositions are counterparts to propositions given for the behaviour of resonance.

Proposition 7 *Let (Π, λ) be a desideratabase, let T be an argument tree and let μ_1 and μ_2 be discount functions. If μ_1 is more or equally discounting than μ_2 then* $\mathsf{Impact}(T, \Pi, \lambda, \mu_1) \leq \mathsf{Impact}(T, \Pi, \lambda, \mu_2)$.

Proposition 8 *Let T be an argument tree and let (Π_1, λ_1) and (Π_2, λ_2) be desideratabases, where λ_1 and λ_2 are unit desiderata weightings, and μ is a discount function. If $\Pi_1 \subseteq \Pi_2$, then* $\mathsf{Impact}(T, \Pi_1, \lambda_1, \mu) \leq \mathsf{Impact}(T, \Pi_2, \lambda_2, \mu)$.

Proposition 9 *Let Φ be a support for an argument and let (Π_1, λ_1) and (Π_2, λ_2) be desideratabases. If there is a bijection G from Π_1 to Π_2 such that for all $\delta \in \Pi_1$ $\{\delta\} \vdash G(\delta)$ and $\lambda_1(\delta) \leq \lambda_2(G(\delta))$, then* $\mathsf{Impact}(T, \Phi, \Pi_1, \lambda_1) \leq \mathsf{Impact}(T, \Phi, \Pi_2, \lambda_2)$.

Replacing an argument in an argument tree with a logically equivalent argument that does not introduce redundant propositional letters results in the same impact as the original argument tree.

Proposition 10 *Let (Π, λ, μ) be a context. Let T and T' be two isomorphic arguments trees. So $T = (N, E)$ and $T' = (N', E')$ and there is a bijection $F : N \mapsto N'$ such that $(A_i, A_j) \in E$ iff $(F(A_i), F(A_j)) \in E'$. If the bijection F is such that for all arguments $A \in N$,* $\mathsf{Propositions}(\mathsf{Support}(A)) = \mathsf{Propositions}(\mathsf{Support}(F(A)))$ *and* $\mathsf{Th}(\mathsf{Support}(A))) = \mathsf{Th}(\mathsf{Support}(F(A)))$, *then* $\mathsf{Impact}(T, \Pi, \lambda, \mu) = \mathsf{Impact}(T', \Pi, \lambda, \mu)$.

Impact is not a monotonic function (i.e. increasing the size of the tree, can increase or decrease the impact): If T_1 is more pruned than T_2, then it is not necessarily the case that $\mathsf{Impact}(T_1, \Pi, \lambda, \mu) \geq \mathsf{Impact}(T_2, \Pi, \lambda, \mu)$, and it is not necessarily the case that $\mathsf{Impact}(T_1, \Pi, \lambda, \mu) \leq \mathsf{Impact}(T_2, \Pi, \lambda, \mu)$. This is illustrated in the next example.

Example 9 *Let (Π, λ, μ) be a unit context where $\Pi = \{\beta\}$. Let $A_1 = \langle\{\alpha \wedge \beta\}, \alpha\rangle$ and $A_2 = \langle\{\neg\alpha \wedge \gamma \wedge \beta\}, \neg(\alpha \wedge \beta)\rangle$. Let T_1 be a tree consisting of nodes A_1 and A_2, let T_2 be a tree consisting of node A_1, and let T_3 be the empty tree. Therefore T_3 is more pruned than T_2 and T_2 is more pruned than T_1. Yet* $\mathsf{Impact}(T_1, \Pi, \lambda, \mu) \leq \mathsf{Impact}(T_2, \Pi, \lambda, \mu)$, *and* $\mathsf{Impact}(T_3, \Pi, \lambda, \mu) \leq \mathsf{Impact}(T_1, \Pi, \lambda, \mu)$.

By definition, the minimum possible value for impact is 0. At the other extreme, we can set up a knowledgebase and desideratabase to create a complete argument tree with maximum impact. We explore this in the next two propositions.

Proposition 11 *For a set of m propositional letters, the knowledgebase Δ and a unit context (Π, λ, μ) are defined as follows: For all $\phi, \psi \in \Delta$, $\{\phi, \psi\} \vdash \perp$, and $\Delta = \Pi$ and $|\Delta| = |\Pi| = 2^m$. Now let T be a complete argument tree where the arguments in* $\mathsf{Nodes}(T)$ *are obtained from Δ. If* $A \in \mathsf{Nodes}(T)$, *then*

$$\mathsf{Echo}(\mathsf{Support}(A), \Pi) = \Pi$$
$$\mathsf{Resonance}(\mathsf{Support}(A), \Pi, \lambda, \mu) = |\Pi| = 2^m$$
$$|\mathsf{Propositions}(\mathsf{Support}(A))| = m$$

In the above proposition, the knowledgebase Δ has the maximum number of pairwise conflicts. This fact, together with $\Delta = \Pi$ holding, leads to a complete argument tree with maximum possible impact as follows.

Proposition 12 *For a knowledgebase Δ composed of m propositional letters, the highest possible impact for an argument tree T composed of arguments from Δ, where* $|\mathsf{Nodes}(T)| = k$, *and (Π, λ, μ) is a unit context, is*

$$\mathsf{Impact}(T, \Pi, \lambda, \mu) = \frac{\mathsf{Resonance}(T, \Pi, \lambda, \mu)}{\mathsf{Propositionalcost}(T)} = \frac{k \times 2^m}{m}$$

Example 10 *For the propositional letters α and β, let $\Delta = \{\alpha \wedge \beta, \neg\alpha \wedge \beta, \alpha \wedge \neg\beta, \neg\alpha \wedge \neg\beta\}$. This knowledgebase can be used to construct a complete argument tree T. Each of these trees has 16 nodes. For $A \in$* $\mathsf{Nodes}(T)$, *if $\Delta = \Pi$, then* $\mathsf{Resonance}(\mathsf{Support}(A), \Pi, \lambda, \mu) = 4$ *and* $\mathsf{Propositionalcost}(T) = 2$. *Hence,* $\mathsf{Impact}(T, \Pi, \lambda, \mu) = 32$.

In general, given some knowledgebase Δ, we can design a context (Π, λ, μ) that allows us to tailor the argumentation for the intended audience. In general, a non-unit discount function tends to favour wider and shallower trees rather than deeper and narrower trees. Dense desiderata (i.e. a desideratabase (Π, λ) where there are many items in Π) with unit desiderata weighting can favour wider trees, less dense desiderata with non-unit desiderata weighting can favour either wide and shallow or deep and narrow trees. The argument trees with higher impact are therefore obtained by considering the interplay between these dimensions.

Optimizing impact

To optimize the impact of argumentation, given a complete argument tree, and a context, we need to select a pruned argument tree that maximizes resonance and minimizes propositional cost.

Definition 20 *Let T be a complete argument tree. An argument tree $T_i \in$* $\mathsf{Space}(T)$ *is an* **optimal tree** *in a context (Π, λ, μ) iff for all $T_j \in$* $\mathsf{Space}(T)$ $\mathsf{Impact}(T_i, \Pi, \lambda, \mu) \geq \mathsf{Impact}(T_j, \Pi, \lambda, \mu)$.

We can view optimization of impact as a decision problem.

Theorem 1 *Let T be a complete argument tree and let $k \in \mathbb{R}$. Determining whether there is a $T_i \in$* $\mathsf{Space}(T)$ *such that* $\mathsf{Impact}(T_i, \Pi, \lambda, \mu) \leq k$ *is an NP-complete decision problem.*

The proof is based on 3-SAT. Whilst in general the search for optimal trees can be expensive, there are some results like the following that can simplify the search space: If an argument in an optimal tree is not a resonating argument, and it adds cost to the tree, then it has an offspring argument in the tree that is a resonating argument. We see this in the following example, and more generally in Proposition 13.

Example 11 *Let* (Π, λ, μ) *be a unit context where* $\Pi = \{\beta, \gamma\}$. *Let* T *be an optimal tree composed of nodes* $\{A_1, A_2, A_3\}$ *where* $\mathsf{Support}(A_1) = \{\alpha\}$, $\mathsf{Support}(A_2) = \{\neg\delta \wedge \beta \wedge \neg\alpha\}$, *and* $\mathsf{Support}(A_3) = \{\beta \wedge \gamma \wedge \neg\alpha\}$. *Hence* T *has the following form.*

$$\begin{tikzcd} & A_1 & \\ A_2 \arrow[ur] & & A_3 \arrow[ul] \end{tikzcd}$$

So $\mathsf{Impact}(T, \Pi, \lambda, \mu) = 1/2$. *Now consider* T' *which is* T *extended with* A_4 *and* $\mathsf{Support}(A_4) = \{\delta \wedge \epsilon\}$. *So* A_4 *is a canonical undercut for* A_2. *Since* $\mathsf{Echo}(\mathsf{Support}(A_4), \Pi) = \emptyset$, *and* $\mathsf{Propositionalcost}(T) < \mathsf{Propositionalcost}(T')$, *we see that* T' *is not an optimal tree.*

Proposition 13 *Let* $T_i \in \mathsf{Space}(T)$ *be an optimal tree in the context* (Π, λ, μ). *For all* $A \in \mathsf{Nodes}(T_i)$, *if* A *is a leaf, then* $\mathsf{Echo}(\mathsf{Support}(A), \Pi) \neq \emptyset$, *or* $\mathsf{Propositionalcost}(T_i) = \mathsf{Propositionalcost}(T_i')$, *where* T_i' *is* T_i *without* A.

The other substantial overhead in high impact argumentation is in constructing the actual arguments. This is the same for many forms of argumentation (e.g. (Brewka 1989; Benferhat, Dubois, & Prade 1993)) and it is essentially that of finding minimally consistent proofs of a formula (Cadoli 1992). One approach to addressing this complexity is to adopt approximate entailment (Schaerf & Cadoli 1995) for approximate coherence-based reasoning (Koriche 2001).

Discussion

In the philosophical study of argumentation, the relationship between the person presenting arguments and the audience has been considered, and in particular the need for arguments to be aposite for an audience has been identified (see for example (Walton 1989; Cockcroft & Cockcroft 1992)). The proposal in this paper is, as far as we know, the first attempt to provide a formal logic-based account of maximising the apositeness of augments for an audience. It is clear that aspects of individual arguments in natural langauge can be represented and analysed using classical logic (see for example (Fisher 1988)). Furthermore, a set of such arguments can be related using argument trees (Besnard & Hunter 2001). Now with the framework in the paper, these arguments can be analysed to select an optimal subset that would have the maximum impact for the audience. This analysis could be useful in a variety of professional domains.

An application where wide but shallow argument trees are desirable is in decision-support for drug prescribing. Capsule is a system for identifying pros and cons for each possible drug that can be prescribed for a particular condition for a particular patient (Fox & Das 2000). Arguments are based on information about known benefits and side-effects of each drug, relative cost of each drug, other drugs the patient is using, the patient's known drug allergies, and any preferences the patient may have. The doctors who use the system want the key pros and cons but they do not want a deep structure to the arguments and counter-arguments. As a result, the form of argumentation used in the Capsule system can be formalized in our framework by using a unit-step discount function with a boundary of 2, so that the highest impact trees are of height 2 but with arbitrary width.

An application where deeper argument trees are desirable is in trial by jury. Here, arguments need to be analysed from various angles, leading to deeper branches. However, normally not all argument trees are presented to the jury since this would lead to too much information being presented. Rather a selection of branches is made on the basis of what is likely to resonate with the jury. For some court trials, lawyers are commissioning research based on focus groups for shaping the presentation of cases (see for example www.trialtech.com). This information may be formalisable with a desideratabase. Finally, discounting as formalised in this paper, may be regarded as a formalization of the notion of serial weakening seen in legal argumentation (Verheij 1996).

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