A Recursive Approach to Argumentation: Motivation and Perspectives[*]

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Abstract

In the context of Dung's argumentation framework, we propose a general recursive schema for argumentation semantics, based on decomposition along the strongly connected components of the argumentation framework. We introduce the fundamental notion of SCC-recursiveness and show that all Dung's admissibility-based semantics are SCC-recursive, and therefore a special case of our schema. The space of SCC-recursive semantics provides a basis for the investigation of novel proposals: starting from the analysis of several examples where Dung's preferred semantics gives rise to counterintuitive results, we introduce four novel SCC-recursive semantics, able to overcome the limitations of preferred semantics, while differing in other respects. Among them, the $CF2$ semantics, which more radically departs from the traditional notion of admissibility, turns out to be the most satisfactory in all cases and, in particular, is able to correctly deal with the thorny problem of self-defeating arguments.

Keywords: Argumentation Semantics, Self-defeating arguments, Odd-length cycles.

Introduction

In (Dung 1995), an abstract framework for argumentation has been introduced where the origin and structure of the arguments are not specified, and the interaction between them is modeled by a binary relation indicating that an argument *attacks* another one. Thanks to its generality, this framework has been recognized as a unifying view of several existing approaches for argumentation and non-monotonic reasoning, and has also inspired subsequent proposals of argumentation systems, e.g. (Prakken & Sartor 1997; Vreeswijk 1997). The fundamental idea of Dung's approach is that of identifying a number of *extensions*, each representing a conflict-free set of arguments deemed to be collectively acceptable. Defining a specific *argumentation semantics* amounts to specifying the criteria for deriving a set of extensions from an argumentation framework: an argument is considered as justified if it belongs to all of the extensions.

The *preferred semantics* introduced in (Dung 1995) overcomes the limitations of the previously proposed *grounded*

semantics (concerning so-called floating arguments) and *stable semantics* (concerning existence of extensions). However, we have shown in (Baroni & Giacomin 2003) that some counterintuitive results arise in some significant examples concerning cyclic attack relationships, and we have proposed a solution based on a recursive approach in the definition of the extensions. In this paper, we generalize this result in two directions. First, we point out that also all the semantics introduced by Dung adhere to a general recursive schema. Then, exploiting this schema, we are able to introduce several novel semantics that overcome the problems described in (Baroni & Giacomin 2003) while differing in other aspects, including in particular the treatment of *self-defeating* arguments.

Dung's Approach: Counterexamples

The general theory proposed by Dung (Dung 1995) is based on the primitive notion of *argumentation framework*:

Definition 1 *An argumentation framework is a pair* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *where* $\mathcal{A}$ *is a set, and* $\rightarrow \subseteq (\mathcal{A} \times \mathcal{A})$ *is a binary relation on* $\mathcal{A}$.

The idea is that arguments are simply conceived as the elements of the set $\mathcal{A}$, whose origin is not specified, and the interaction between them is modeled by the binary relation of attack $\rightarrow$.

In the following, nodes that attack a given $\alpha \in \mathcal{A}$ are called *defeaters* of α, and form a set denoted as $\mathrm{parents}(\alpha) = \{\beta \in \mathcal{A} \mid \beta \rightarrow \alpha\}$. If $\mathrm{parents}(\alpha) = \emptyset$, then α is called an *initial* node. Since we will frequently consider properties of sets of arguments, we extend to them the notations defined for the nodes.

Definition 2 *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *a node* $\alpha \in \mathcal{A}$ *and two sets* $S, P \subseteq \mathcal{A}$, *we define:*

$$S \rightarrow \alpha \text{ iff } \exists \beta \in S : \beta \rightarrow \alpha$$
$$\alpha \rightarrow S \text{ iff } \exists \beta \in S : \alpha \rightarrow \beta$$
$$S \rightarrow P \text{ iff } \exists \alpha \in S, \beta \in P : \alpha \rightarrow \beta$$
$$\mathrm{parents}(S) = \{\alpha \in \mathcal{A} \mid \alpha \rightarrow S\}$$
$$\mathrm{outparents}(S) = \{\alpha \in \mathcal{A} \mid \alpha \notin S \wedge \alpha \rightarrow S\}$$

Dung's theory relies on the fundamental notions of acceptability and admissibility:
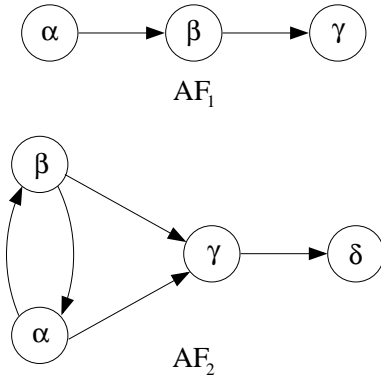
Figure 1: Two argumentation frameworks

Definition 3 *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$:

- *A set* $S \subseteq \mathcal{A}$ *is* conflict-free *if and only if* $\nexists \alpha, \beta \in S$ *such that* $\alpha \rightarrow \beta$.
- *An argument* $\alpha \in \mathcal{A}$ *is* acceptable *with respect to a set* $S \subseteq \mathcal{A}$ *iff* $\forall \beta \in \mathcal{A}$, *if* $\beta \rightarrow \alpha$ *then also* $S \rightarrow \beta$.
- *A set* $S \subseteq \mathcal{A}$ *is* admissible *iff* S *is conflict-free and each argument in S is acceptable with respect to S, i.e.* $\forall \beta \in \mathcal{A}$ *such that* $\beta \rightarrow S$ *we have that* $S \rightarrow \beta$.

On this basis, the notion of *complete extension*, introduced as a unifying concept underlying all of the proposed semantics, is defined as an admissible set $E \subseteq \mathcal{A}$ such that every argument $\alpha \in \mathcal{A}$ which is acceptable with respect to E belongs to E. Then, two main notions of extensions are introduced, corresponding to the *grounded* and *preferred* semantics, respectively.

The first follows the so-called *unique-status approach*, since for a given argumentation framework AF it always identifies a single extension $\mathrm{GE}_{\mathrm{AF}}$, called *grounded extension*, corresponding to the least (wrt. $\subseteq$) complete extension of AF: the set of arguments $\mathcal{A}$ can then be partitioned into *undefeated arguments*, that belong to $\mathrm{GE}_{\mathrm{AF}}$ and are considered as justified, *defeated argument*, that are attacked by $\mathrm{GE}_{\mathrm{AF}}$ and are rejected, and *provisionally defeated arguments*, that are neither included in $\mathrm{GE}_{\mathrm{AF}}$ nor attacked by it, reflecting, in a sense, a sort of undecided status. Following (Pollock 1992), the idea underlying the definition of grounded extension is to start by considering as undefeated the initial nodes, that have no defeaters, and then to proceed towards the inside of the graph by rejecting (i.e. considering as defeated) nodes that have undefeated defeaters and by accepting (i.e. considering as undefeated) nodes whose defeaters are all defeated. Finally, nodes that cannot be labeled in this way are all considered as provisionally defeated. For instance, for the argumentation framework AF_1 shown in Figure 1 we have that $\mathrm{GE}_{\mathrm{AF}_1} = \{\alpha, \gamma\}$.

The grounded semantics, in a sense, makes the most restrictive choice among complete extensions, as in fact it selects the least one. As a consequence, it is the most skeptical among semantics encompassed in Dung's framework and, as such, provides a reference since arguments undefeated

according to the grounded semantics should be justified in any other semantics. This extremely skeptical attitude prevents grounded semantics to properly deal with some cases related to the so-called *floating arguments*. Considering for instance the argumentation framework AF_2 of Figure 1, it is easy to see that all arguments are provisionally defeated, while node δ should be justified since its unique defeater γ is attacked independently of the way the conflict between α and β is solved. More generally, it has been shown in (Schlechta 1993) that the inability to discriminate floating arguments is not a specific disadvantage of grounded semantics, but affects any possible single-status approach.

Preferred semantics overcomes such limitations by following a multiple-status approach, where the set of *preferred extensions*, denoted as $\mathcal{PE}_{\mathrm{AF}}$, is defined as the set of all maximal admissible sets, or equivalently of all maximal complete extensions. Thus, it turns out that $\mathcal{PE}_{\mathrm{AF}_2} = \{\{\alpha, \delta\}, \{\beta, \delta\}\}$: δ is therefore included in all extensions and is justified.

Limitations of preferred semantics have been pointed out in (Baroni & Giacomin 2003): to have an idea, consider the argumentation frameworks shown in Figure 2, that differ only in the length of the leftmost cycle. It can be seen that AF_3 admits only one extension consisting of the node ϕ_2, while AF_4 admits three extensions whose intersection is empty. Therefore, ϕ_2 is justified in AF_3 while no argument is justified in AF_4. This turns out to be counterintuitive considering, for instance, that these argumentation frameworks can be regarded as simple variants of the example of conflicting witnesses proposed in (Pollock 1994). Here, the leftmost cycle represents a set of witnesses such that each of them questions the reliability of another one, and this *undercut* relation is arranged in a cycle. The two-length cycle on the right represents a couple of arguments with contradictory conclusions, known in the literature as *Nixon Diamond*, where ϕ_1 is based on an assertion of one of the witnesses. Intuitively, the length of the leftmost cycle, i.e. the number of witnesses, should be irrelevant. However, as described above, preferred semantics give a different treatment to ϕ_2 in the two cases.

To solve this problem, it is of course necessary to modify the definition of preferred extension. To this purpose, two directions can be envisaged: the requirement of admissibility can be replaced by a less restrictive one, or one should give up the maximality property. However, this kind of modifications alone can be easily seen to give rise to counterintuitive results even in the simplest cases of argumentation frameworks. For instance, replacing maximal admissible sets (i.e. preferred extensions) with maximal conflict-free sets yields an intuitively acceptable treatment of AF_3 and AF_4, but in general would produce a too large number of extensions, drastically reducing the number of justified arguments (e.g. none of the arguments in AF_1 would be justified). In order to properly constrain the set of extensions, in (Baroni & Giacomin 2003) a recursive definition has been devised where the recursion step is based on the decomposition of the argumentation framework into strongly connected components (SCCs). Though this definition was driven by the critical examples considered, a subsequent deeper analysis
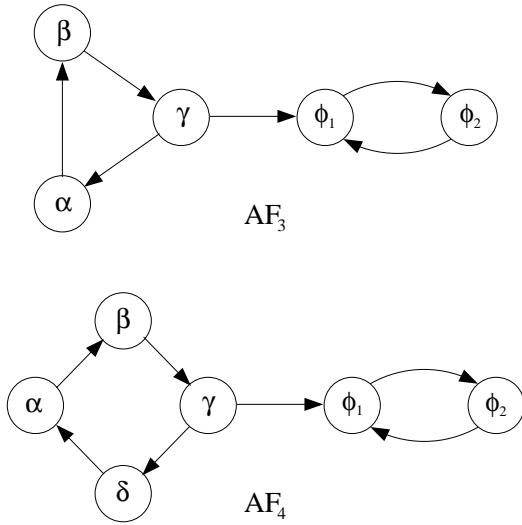
Figure 2: Different handling of cycles by preferred semantics

has shown that recursion along SCCs plays a more general role in the foundations of argumentation semantics. In the next section, we report the results of this analysis.

A General Recursive Schema

This section presents in a synthetic way our recent results concerning a general recursive schema for argumentation semantics, more details are given in (Baroni & Giacomin 2004). First, we need to introduce the notion of strongly connected components.

Definition 4 *Given an argumentation framework* AF $= \langle \mathcal{A}, \rightarrow \rangle$*, two nodes* $\alpha, \beta \in \mathcal{A}$ *are* path-equivalent *iff either* $\alpha = \beta$ *or there is a path from* α *to* β *and a path from* β *to* α*. The strongly connected components of* AF *are the equivalence classes of vertices under the relation of path-equivalence. The set of the strongly connected components of* AF *is denoted as* SCC(AF).

Given a node $\alpha \in \mathcal{A}$, we will indicate the strongly connected component α belongs to as SCC(α).

It is well-known that the graph made up by SCCs is acyclic. Taking into account this property, the key idea of our recursive schema consists in exploiting the order of SCCs to construct extensions. More specifically, each extension E can be regarded as the union of disjoint subsets each taken from a different SCC, namely:

$$E = \bigcup_{S \in \text{SCC(AF)}} (E \cap S)$$

Our idea is that, for a given SCC S, the choice of the subset $E \cap S$ is only affected by the choices made in the SCCs that precede S in the order mentioned above. In particular, such choices determine a partition of the nodes of S into three subsets, that are defined according to the following definition:

Definition 5 *Given an argumentation framework* AF $= \langle \mathcal{A}, \rightarrow \rangle$*, a set* $E \subseteq \mathcal{A}$ *and a strongly connected component* $S \in$ SCC(AF)*, we define:*

- $S^D(E) = \{\alpha \in S \mid (E \cap \text{outparents}(S)) \rightarrow \alpha\}$
- $S^P(E) = \{\alpha \in S \mid (E \cap \text{outparents}(S)) \not\rightarrow \alpha \wedge \exists \beta \in (\text{outparents}(S) \cap \text{parents}(\alpha)) : E \not\rightarrow \beta\}$
- $S^U(E) = S \setminus (S^D(E) \cup S^P(E))$

In words, if the set E is an extension, the set $S^D(E)$ consists of the nodes of S attacked by E from outside S, the set $S^U(E)$ consists of the nodes of S that are not attacked by E and are defended by E (i.e. their defeaters from outside S are all attacked by E), and $S^P(E)$ consists of the nodes of S that are not attacked nor defended by E. We remark that, thanks to the acyclicity of the SCC-graph, each of these sets is determined by the elements of E that belong to the SCCs antecedent to S in this graph.

To illustrate the above concepts, let us consider the argumentation framework AF$_2$ shown in Figure 1. It is easy to see that AF$_2$ admits three strongly connected components, i.e. SCC(AF$_2$) $= \{S_1, S_2, S_3\}$, where $S_1 = \{\alpha, \beta\}$, $S_2 = \{\gamma\}$ and $S_3 = \{\delta\}$. Assuming that $E \cap S_1 = \{\alpha\}$ (as in the case of one of the preferred extensions introduced in the previous section), we have that $S_2^D(E) = \{\gamma\}$, while $S_2^P(E) = S_2^U(E) = \emptyset$. In turn, assuming that $E \cap S_2 = \emptyset$, it turns out that $S_3^U(E) = \{\delta\}$, since δ is not attacked by E and its unique defeater γ is attacked by E. Of course, $S_3^D(E) = S_3^P(E) = \emptyset$.

To give an example where $S^P(E)$ is not empty, let us consider the argumentation framework AF$_3$ shown in Figure 2. It is easy to see that, in this case, we have two SCCs, i.e. SCC(AF$_3$) $= \{S_1', S_2'\}$, where $S_1' = \{\alpha, \beta, \gamma\}$ and $S_2' = \{\phi_1, \phi_2\}$. Assuming that $E \cap S_1' = \emptyset$ (as in the case of the unique preferred extension admitted), we have that $S_2'^D(E) = \emptyset$, $S_2'^P(E) = \{\phi_1\}$ and $S_2'^U(E) = \{\phi_2\}$. In fact, ϕ_1 is attacked by γ which is not included in E nor attacked by E, while ϕ_2 has no defeaters outside S_2'.

Now, we need to investigate how $E \cap S$ can be defined for each S on the basis of the above three sets. Intuitively, if S is initial in the order of SCCs, the choice of $E \cap S$ depends on S itself only. In other words, the possible choices for $E \cap S$ are exactly the same as the extensions of an argumentation framework consisting only in S. If S is not initial, the basic requirement that an extension is conflict-free leads to exclude $S^D(E)$ from E, i.e. $E \cap S \subseteq (S^P(E) \cup S^U(E))$. Roughly, the effect of previous SCCs is summarized by the exclusion of $S^D(E)$, while subsequent SCCs do not affect the choice in S, therefore one is lead to consider a sort of reduced argumentation framework, consisting in $(S^P(E) \cup S^U(E))$. The idea of our schema is then to define recursively $E \cap S$ referring to this restricted argumentation framework. The recursion stops when the restricted argumentation framework consists in a unique SCC.

To put the above concepts in formal terms, we need, first of all, the notion of *restriction* of an argumentation framework to a given subset of its nodes:

Definition 6 *Let* AF $= \langle \mathcal{A}, \rightarrow \rangle$ *be an argumentation framework, and let us consider a set of arguments* $S \subseteq \mathcal{A}$.

The restriction *of* AF *to S is the argumentation framework* $\mathrm{AF}{\downarrow}_S = \langle S, \to \cap(S \times S)\rangle$.

The proposed general schema is then based on the property of *SCC-recursiveness*: a semantics is *SCC-recursive* iff its extensions are identified by means of a recursive definition along SCCs. Such definition is parametric with respect to a *base function*: a specific SCC-recursive semantics is characterized by its own base function.

Definition 7 *A given argumentation semantics is* SCC-*recursive iff, with reference to a generic argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \to \rangle$, *a set* $E \subseteq \mathcal{A}$ *is an extension iff* $E \in \mathcal{FG}_{\mathrm{AF}}(\mathcal{A})$,
where, for all sets $C \subseteq \mathcal{A}$, $E \in \mathcal{FG}_{\mathrm{AF}}(C)$ *iff*

- $E \in \mathcal{FG}^*_{\mathrm{AF}}(C)$ *if* $|\mathrm{SCC}(\mathrm{AF})| = 1$ *or* $\mathcal{A} = \emptyset$
- $\forall S \in \mathrm{SCC}(\mathrm{AF})$
  $(E \cap S) \in \mathcal{FG}_{\mathrm{AF}{\downarrow}_{(S^P(E) \cup S^U(E))}}(S^U(E) \cap C)$ *otherwise*

where $\mathcal{FG}^*_{\mathrm{AF}}(C)$ *is a function that, given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \to \rangle$ *such that* $|\mathrm{SCC}(\mathrm{AF})| = 1$ *and a set* $C \subseteq \mathcal{A}$, *gives a subset of* $2^{\mathcal{A}}$.

Definition 7 states that extensions of an argumentation framework AF are identified by applying the recursive function $\mathcal{FG}$ to AF itself and to the set $\mathcal{A}$ of its nodes. The function $\mathcal{FG}$ has two arguments, namely a generic argumentation framework $\mathrm{AF}' = \langle \mathcal{A}', \to' \rangle$ and a subset C of $\mathcal{A}'$, and returns a set of subsets of $\mathcal{A}'$ (in particular, it returns the set of extensions of AF' when $C = \mathcal{A}'$). The definition of $\mathcal{FG}$ reflects the intuitive ideas illustrated above: it is based on the decomposition of AF' into its SCCs and admits two cases. In the case AF' is made up of a unique strongly connected component, $\mathcal{FG}$ directly coincides with the *base function* denoted as $\mathcal{FG}^*_{\mathrm{AF}'}(C)$. Therefore, when the original argumentation framework AF has a unique SCC, it holds that $\mathcal{FG}_{\mathrm{AF}}(\mathcal{A}) = \mathcal{FG}^*_{\mathrm{AF}}(\mathcal{A})$. Otherwise, the second recursive part of the definition applies to AF and $\mathcal{FG}^*$, applied on restricted argumentation frameworks, will provide the base of the recursion. Turning to the recursive part, it is stated that for any SCC S $\mathcal{FG}$ is recursively applied on $\mathrm{AF}{\downarrow}_{(S^P(E) \cup S^U(E))}$, i.e. on the restricted argumentation framework corresponding to the subset of S which survives the attacks from the preceding SCCs. The argument $S^U(E) \cap C$ identifies those nodes of this restricted argumentation framework whose defeaters from outside S are all attacked by E (according to choices in previous SCCs). In particular, when the recursive step is invoked on a SCC of the original AF, $C = \mathcal{A}$, and therefore $(S^U(E) \cap C) = S^U(E)$. In deeper levels of recursion, the function is applied on inner SCCs of restricted argumentation frameworks: in this case, both parts of the definition rely on the argument C in order to take into account the effect of choices in previous levels of recursion within the restricted argumentation framework. In fact, let $\hat{S}$ be such an inner SCC: $\hat{S}^U(E)$ identifies only the nodes which are not attacked by E and whose defeaters are attacked by the subset of E belonging to the restricted argumentation framework, while C identifies the same kind of nodes referred to the part of E outside the restricted argumentation framework chosen in previous levels of recursion.

Intersecting $\hat{S}^U(E)$ and C yields, therefore, the nodes of $\hat{S}$ that feature the above properties from both points of view.

Definition 7 provides a generic framework, where a particular SCC-recursive semantics is identified by its own base function: it is interesting to notice that, in this context, to define a semantics it is sufficient to specify its behavior only on single-SCC argumentation frameworks.

One may now wonder whether the property of SCC-recursiveness characterizes a suitable family of semantics. On the one hand, such family should be large enough to include previous significant approaches to argumentation semantics, on the other hand it should be constrained enough to support the definition of novel proposals based on reasonable definitions of the base function.

As far as the first requirement is concerned, a confirmation of the generality of SCC-recursiveness is given by Proposition 1, which states that Dung's complete, preferred and grounded semantics are SCC-recursive (proof is omitted and can be found in (Baroni & Giacomin 2004)). To obtain this result, it is necessary to generalize some definitions of Dung's theory by considering a specific subset C of $\mathcal{A}$ from which acceptable arguments (that compose the extensions) are selected (Dung's original definitions are recovered by letting $C = \mathcal{A}$).

Definition 8 *Given* $\mathrm{AF} = \langle \mathcal{A}, \to \rangle$ *and a set* $C \subseteq \mathcal{A}$:

- *A set* $E \subseteq \mathcal{A}$ *is* admissible in C *iff* E *is admissible and* $E \subseteq C$. *We denote as* $\mathcal{AS}_{\mathrm{AF}}(C) \equiv \{E \subseteq C \mid E \text{ is admissible}\}$ *the set of admissible sets in* C.
- *A set* $S \subseteq \mathcal{A}$ *is a* complete extension in C *iff* $S \in \mathcal{AS}_{\mathrm{AF}}(C)$, *and every argument* $\alpha \in C$ *which is acceptable with respect to* S *belongs to* S. *The set of complete extensions in* C *will be denoted as* $\mathcal{CE}_{\mathrm{AF}}(C)$.
- *A* preferred extension in C *is a maximal element (wrt. set inclusion) of* $\mathcal{AS}_{\mathrm{AF}}(C)$. *The set of preferred extensions in* C *will be denoted as* $\mathcal{PE}_{\mathrm{AF}}(C)$.

Proposition 1 *Dung's complete, preferred and grounded semantics are SCC-recursive with the following base functions:*

- *for complete semantics,* $\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{CE}_{\mathrm{AF}}(C)$;
- *for preferred semantics,* $\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{PE}_{\mathrm{AF}}(C)$;
- *for grounded semantics,* $\mathcal{FG}^*_{\mathrm{AF}}(C) = $
  $\begin{cases} \{\{\alpha\}\} & \text{if } C = \mathcal{A} = \{\alpha\} \wedge \alpha \not\to \alpha \\ \{\emptyset\} & \text{otherwise} \end{cases}$

To better illustrate the general recursive schema, we now provide some examples of its application to the argumentation frameworks introduced in the previous section, with reference to grounded and preferred semantics.

Example 1 *Deriving the grounded extension of* AF_2

Let us consider the argumentation framework AF_2 shown in Figure 1. As discussed above, we have that $\mathrm{SCC}(\mathrm{AF}_2) = \{S_1, S_2, S_3\}$, where $S_1 = \{\alpha, \beta\}$, $S_2 = \{\gamma\}$ and $S_3 = \{\delta\}$. Let us show first how the general recursive schema, instantiated with the base function introduced above for grounded semantics, admits as the unique extension the grounded extension $E = \mathrm{GE}_{\mathrm{AF}_2} = \emptyset$. According to Definition 7, E is

an extension iff $E \in \mathcal{FG}_{\mathrm{AF}_2}(\mathcal{A})$. Since AF_2 includes more than one SCC, the recursive part of the definition applies to AF_2 with $C = \mathcal{A}$, yielding $E \in \mathcal{FG}_{\mathrm{AF}_2}(\mathcal{A})$ iff

$$\forall S \in \mathrm{SCC}(\mathrm{AF}_2)\, (E \cap S) \in \mathcal{FG}_{\mathrm{AF}_2\downarrow_{(S^P(E)\cup S^U(E))}}(S^U(E)) \tag{1}$$

Accordingly, we now orderly examine the SCCs of AF_2. Considering S_1, we have that $\mathrm{outparents}(S_1) = \emptyset$, therefore, independently of E, it holds that $S_1^D(E) = S_1^P(E) = \emptyset$ and $S_1^U(E) = S_1$. As a consequence, (1) becomes $(E \cap S_1) \in \mathcal{FG}_{\mathrm{AF}_2\downarrow_{S_1}}(S_1)$. Since $\mathrm{AF}_2\downarrow_{S_1}$ is made up of the unique SCC S_1, the base case of Definition 7 applies, therefore $(E \cap S_1) \in \mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_1}}(S_1)$. Now, the base function for the grounded semantics returns $\{\emptyset\}$ in this case, since $|S_1| > 1$, therefore we necessarily have that

$$E \cap S_1 = \emptyset \tag{2}$$

Since S_1 is the only SCC preceding S_2, (2) entails that, for any E, $S_2^D(E) = S_2^U(E) = \emptyset$ and $S_2^P(E) = S_2$. In fact, $(E \cap \mathrm{outparents}(S_2)) \not\rightarrow \gamma$, while both α and β, which belong to $(\mathrm{outparents}(S_2) \cap \mathrm{parents}(\gamma))$, are not attacked by E: according to Definition 5, this entails that $S_2^P(E) = \{\gamma\} = S_2$. As a consequence, (1) becomes $(E \cap S_2) \in \mathcal{FG}_{\mathrm{AF}_2\downarrow_{S_2}}(\emptyset)$, which according to the base case of Definition 7 yields $(E \cap S_2) \in \mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_2}}(\emptyset)$. Applying the base function of the grounded semantics, we get $\mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_2}}(\emptyset) = \emptyset$, since $C = \emptyset \neq \mathcal{A} = S_2$. As a consequence, for all E

$$E \cap S_2 = \emptyset \tag{3}$$

Considering now S_3, it can be seen that (2) and (3) entail that $S_3^D(E) = S_3^U(E) = \emptyset$ and $S_3^P(E) = S_3$. In fact, the only defeater of δ from the outside of S_3, i.e. γ, is neither included in E nor attacked by E. Reasoning in an analogous way as for S_2 we get then

$$E \cap S_3 = \emptyset \tag{4}$$

Finally, conditions (2), (3) and (4) uniquely identify E as $\emptyset$, which corresponds to the grounded extension $\mathrm{GE}_{\mathrm{AF}_2}$.

Example 2 *Deriving the preferred extensions of* AF_2

Let us now examine how Definition 7 exactly identifies the preferred extensions $\{\alpha, \delta\}$ and $\{\beta, \delta\}$ of AF_2. Again, equation (1) holds, leading to consider orderly the three SCCs. As for S_1, the same considerations of the above example apply, yielding $(E \cap S_1) \in \mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_1}}(S_1)$. Now, $\mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_1}}(S_1) = \mathcal{PE}_{\mathrm{AF}_2\downarrow_{S_1}}(S_1)$: since $C = S_1 = \mathcal{A}$, $\mathcal{PE}_{\mathrm{AF}_2\downarrow_{S_1}}(S_1)$ simply returns the preferred extensions of the *Nixon Diamond* made up of S_1, i.e. $\mathcal{PE}_{\mathrm{AF}_2\downarrow_{S_1}}(S_1) = \{\{\alpha\}, \{\beta\}\}$. Therefore, there are two alternatives for $(E \cap S_1)$: we will follow just one of them, i.e. $(E \cap S_1) = \{\alpha\}$, since the reasoning for the other is symmetric. Turning to S_2, we have that $S_2^D(E) = \{\gamma\} = S_2$, since γ is attacked by $\alpha \in E$, and of course $S_2^U(E) = S_2^P(E) = \emptyset$. Therefore, (1) yields $(E \cap S_2) \in \mathcal{FG}_{\mathrm{AF}_2\downarrow_\emptyset}(\emptyset)$. Since $\mathrm{AF}_2\downarrow_\emptyset = \langle \emptyset, \emptyset \rangle$, the base case of Definition 7 applies, yielding $(E \cap S_2) \in \mathcal{FG}^*_{\langle\emptyset,\emptyset\rangle}(\emptyset)$, which directly yields $(E \cap S_2) = \emptyset$. Only S_3 remains to be considered, whose unique defeater from outside, namely γ, does not belong to E and is attacked by E,

entailing that $S_3^U(E) = S_3$ and $S_3^D(E) = S_3^P(E) = \emptyset$. Therefore, $(E \cap S_3) \in \mathcal{FG}_{\mathrm{AF}_2\downarrow_{S_3}}(S_3)$, which applying the base case of Definition 7 yields $(E \cap S_3) \in \mathcal{FG}^*_{\mathrm{AF}_2\downarrow_{S_3}}(S_3)$: in words, $(E \cap S_3)$ is a preferred extension of the simple argumentation framework made up of the node δ only, i.e. $(E \cap S_3) = \{\delta\}$. In conclusion, the alternative $\{\alpha\}$ for $(E \cap S_1)$ considered so far yields the extension $E = \{\alpha, \delta\}$. On the other hand, choosing the other alternative $\{\beta\}$ and reasoning in a symmetric way we get the extension $\{\beta, \delta\}$. Therefore, Definition 7 gives exactly the preferred extensions.

Example 3 *Deriving the unique preferred extension of* AF_3

The argumentation framework AF_3 shown in Figure 2 has two strongly connected components, i.e. $\mathrm{SCC}(\mathrm{AF}_3) = \{S_1, S_2\}$ with $S_1 = \{\alpha, \beta, \gamma\}$ and $S_2 = \{\phi_1, \phi_2\}$. As in previous examples, we orderly consider the SCCs of AF_2 to get the possible intersections with E, which must satisfy

$$\forall S \in \mathrm{SCC}(\mathrm{AF}_3)\, (E \cap S) \in \mathcal{FG}_{\mathrm{AF}_3\downarrow_{(S^P(E)\cup S^U(E))}}(S^U(E)) \tag{5}$$

As for S_1, which is an initial SCC, the base case of Definition 7 applies, yielding $(E \cap S_1) \in \mathcal{FG}^*_{\mathrm{AF}_3\downarrow_{S_1}}(S_1)$. Therefore, $(E \cap S_1)$ must coincide with a preferred extension of an argumentation framework made up of a three-length cycle: actually, such argumentation framework admits the empty set as the unique preferred extension, entailing that $(E \cap S_1) = \emptyset$. Turning to S_2, it can be seen that, as a consequence, $S_2^D(E) = \emptyset$, $S_2^P(E) = \{\phi_1\}$ and $S_2^U(E) = \{\phi_2\}$. In fact, there are no nodes in S_2 that are attacked by E from the outside, ϕ_1 is attacked by γ which is neither included in E nor attacked by E, while ϕ_2 does not receive external attacks since its unique defeater ϕ_1 belongs to S_2. Therefore, (5) becomes $(E \cap S_2) \in \mathcal{FG}_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$, where $\mathrm{AF}_3\downarrow_{S_2}$ coincides with its unique SCC S_2. Then, according to the base case of Definition 7, we get $(E \cap S_2) \in \mathcal{FG}^*_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$, i.e. $(E \cap S_2) \in \mathcal{PE}_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$. According to Definition 8, $\mathcal{PE}_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$ gives the maximal admissible sets in $\mathrm{AF}_3\downarrow_{S_2}$ contained in $\{\phi_2\}$. Now, the admissible sets of $\mathrm{AF}_3\downarrow_{S_2}$ are $\{\phi_1\}$, $\{\phi_2\}$ and $\emptyset$. Clearly, $\{\phi_2\}$ is the only maximal one contained in $C = \{\phi_2\}$, therefore $(E \cap S_2) = \{\phi_2\}$. Summing up, the only extension admitted by Definition 7 is $\{\phi_2\}$.

Example 4 *Deriving the grounded extension of* AF_3

As in the previous example, we apply (5) to the two SCCs S_1 and S_2. The base function of the grounded semantics applied to S_1 gives the empty set as the only possibility for $E \cap S_1$. Therefore, we are lead again to $(E \cap S_2) \in \mathcal{FG}^*_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$. As $C = \{\phi_2\} \neq \mathcal{A} = S_2$, the base function for grounded semantics gives $\{\emptyset\}$ as the result. Therefore, the only possible E is the empty set.

While in this section the recursive schema has been presented as a unifying concept underlying existing semantics, we show in the next section that it provides a natural basis for the definition and study of novel proposals of semantics. We explore four alternative semantics that uniformly exhibit

the desired behavior in the examples described above, which are critical for classical approaches, while differing in interesting ways in the treatment of odd-length cycles and, as a special case, of self-defeating arguments, two largely debated issues in argumentation literature.

Exploring the Space of SCC-Recursiveness

In the framework of SCC-recursive semantics the investigation of new semantics definitions turns out to be particularly easy: on one hand, introducing a new semantics simply amounts to defining a particular base function, on the other hand, the task of analyzing the relationships among different semantics is simplified by the general schema. To better appreciate this advantageous feature of our proposal, we briefly investigate in this section four novel semantics, directly derived by specializing Definition 7 in two main directions.

Beyond Preferred Semantics

Our first direction of investigation aims at devising proposals alternative to preferred semantics, while preserving the fundamental notions encompassed by Dung's framework. In this context, a key role is played by the intuitive concept of defense, formally represented by admissible sets, therefore complete admissible sets, i.e. complete extensions, are regarded as the most general family of conceivable extensions, and any semantics should select its extensions among them. The recursive schema turns out to completely include this framework, since, as shown in previous section, all complete extensions are recursively characterized. Therefore, considering the recursive schema does not introduce any limitation in this respect.

Preferred semantics is generally considered as the most satisfactory approach in this framework. However, as previously discussed, it fails to correctly deal with the argumentation framework AF_3 of Figure 2 since the argument ϕ_2 turns out to be improperly justified. Let us briefly examine how a solution to this problem can be devised.

First of all, note that in any framework based on defense (i.e. admissibility), none of the arguments α, β, or γ can be included in any extension, since any of them defeats its only possible defender. As a consequence, also ϕ_1 can not be included in any extension, since there can not be defence against its defeater γ. Therefore, the only complete extensions of AF_3 are $\{\phi_2\}$ and $\emptyset$. While preferred semantics prescribes $\{\phi_2\}$ as the only extension, we aim at finding a definition which admits the empty set as extension. Using the general recursive schema to identify such definition, we are lead to investigate $E \cap S$ for all $S \in \text{SCC}(\text{AF}_3) = \{S_1, S_2\}$, where $S_1 = \{\alpha, \beta, \gamma\}$ and $S_2 = \{\phi_1, \phi_2\}$. In particular, since for any complete extension E we have that $(E \cap S_1) = \emptyset$, it must be the case that $S_2^P(E) = \{\phi_1\}$ and $S_2^U(E) = \{\phi_2\}$ (see Example 3). Therefore, Definition 7 yields $(E \cap S_2) \in \mathcal{FG}^*_{\text{AF}_3\downarrow_{S_2}}(\{\phi_2\})$, and we look for a base function such that $\mathcal{FG}^*_{\text{AF}_3\downarrow_{S_2}}(\{\phi_2\}) = \emptyset$. A very simple way to obtain this property in general is to impose that if $C \subsetneq \mathcal{A}$ $\mathcal{FG}^*_{\text{AF}}(C) = \emptyset$. Taking this for granted, to complete the definition of the base function only the case where
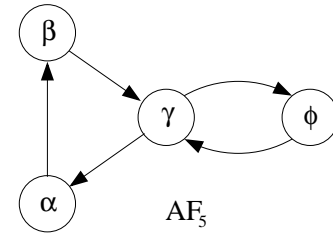

Figure 3: A problematic argumentation frameworks for $AD1$-semantics

$C = \mathcal{A}$ remains to be specified. In this respect, the more direct approach is to consider the set of preferred extensions, obtaining the following base function:

$$\mathcal{FG}^*_{\text{AF}}(C) = \begin{cases} \mathcal{PE}_{\text{AF}}(C) & \text{if } C = \mathcal{A} \\ \{\emptyset\} & \text{otherwise} \end{cases}$$

We denote the corresponding semantics as $AD1$.

While the $AD1$ semantics solves the problems related to AF_3, it fails in the argumentation framework AF_5, presented in Figure 3. In fact, since AF_5 consists of a single SCC, $AD1$ and preferred semantics identify a unique extension $\{\alpha, \phi\}$, yielding these two arguments justified: this behavior is counterintuitive since, as explained in (Baroni & Giacomin 2003), replacing the 3-length attack-cycle with a 4-length one (and, more generally, replacing an odd-length cycle with an even-length one) yields a radically different result, where no argument is justified (as it is intuitive to expect independently of the cycle length). To overcome this problem, we need to rule out $\{\alpha, \phi\}$ as an extension. Notice that the only defeater of both α and ϕ is γ, therefore in the desired definition the node γ should retain the capability of preventing α and ϕ to be accepted. To obtain this behavior, we note that one of the defeaters of γ, actually β, is not included in the extension. Exploiting this fact leads to require that an extension, in a sense, fully attacks its defeaters, i.e. it includes all the defeaters of its defeaters. In particular, $\{\alpha, \phi\}$ does not satisfy this condition since it does not include β. These considerations lead to define the following base function:

$$\mathcal{FG}^*_{\text{AF}}(C) = \begin{cases} \{E \mid E \text{ maximal in } \mathcal{AS}^*_{\text{AF}}\} & \text{if } C = \mathcal{A} \\ \{\emptyset\} & \text{otherwise} \end{cases}$$

where

$$\mathcal{AS}^*_{\text{AF}} = \{F \subseteq \mathcal{A} : F \text{ admissible and} \\ \forall \alpha \in \mathcal{A} : \alpha \to F, \text{ parents}(\alpha) \subseteq F\}$$

We denote the corresponding semantics as $AD2$.

It can be verified that $AD2$ correctly deals with AF_5, since $\mathcal{AS}^*_{\text{AF}_5} = \{\emptyset\}$, and preserves the desired behavior in all of the other problematic examples considered so far. In particular, in AF_1, AF_2, and AF_4 the base function $\mathcal{FG}^*_{\text{AF}}(C)$ is always invoked with $C = \mathcal{A}$, and it turns out that in all cases the preferred extensions of AF are exactly the elements of $\mathcal{AS}^*_{\text{AF}}$: therefore, $AD2$ prescribes the same extensions as preferred semantics. As far as AF_3 is concerned, it can be

seen that $AD2$ behaves as $AD1$, since in $\mathrm{AF}_3\!\downarrow_{S_1}$ the only admissible set is the empty set, which obviously belong to $\mathcal{AS}^*_{\mathrm{AF}_3\downarrow_{S_1}}$. Then, as for $AD1$, $(E \cap S_2)$ is determined by $\mathcal{FG}^*_{\mathrm{AF}_3\downarrow_{S_2}}(\{\phi_2\})$, which yields the empty set since $C \neq \mathcal{A}$.

It is interesting to note that the requirement of including all of the attackers of a node which attacks the extension, imposed in the base function of $AD2$, would be harmful rather than useful outside the recursive schema: examples can easily be found where even initial nodes would not be justified.

Finally, we show in Proposition 2 that both $AD1$ and $AD2$ fit in the Dung's framework, as all of their extensions are actually complete extensions. Proposition 2 relies on two lemmas, whose proof is omitted: Lemma 1 is a generalization of the well-known property that preferred extensions are complete extensions (Dung 1995), Lemma 2 is a direct consequence of Dung's Fundamental Lemma ((Dung 1995), page 327).

Lemma 1 *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ *and a set* $C \subseteq \mathcal{A}$, *any preferred extension in* C *is also a complete extension in* C, *i.e.* $\mathcal{PE}_{\mathrm{AF}}(C) \subseteq \mathcal{CE}_{\mathrm{AF}}(C)$.

Lemma 2 *Let* AF *be an argumentation framework* $\langle \mathcal{A}, \rightarrow \rangle$, *and let* E *be an admissible set of* AF. *If an argument* $\alpha \in \mathcal{A}$ *is acceptable with respect to* E, *then* $E \not\rightarrow \alpha$ *and* $\alpha \not\rightarrow E$.

Proposition 2 *For any argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *the extensions prescribed by* $AD1$ *and* $AD2$ *are complete extensions.*

Proof: According to the characterization of complete semantics as SCC-recursive given in Proposition 1, any set E which is decomposable according to Definition 7 with a base function whose elements belong to $\mathcal{CE}_{\mathrm{AF}}(C)$ is a complete extension. Therefore, to prove the claim it is sufficient to show that the base functions introduced for $AD1$ and $AD2$ give elements belonging to $\mathcal{CE}_{\mathrm{AF}}(C)$ for any AF (consisting of a unique SCC) and C.

Considering first $AD1$, we distinguish two cases for the base function. If $C = \mathcal{A}$, then $\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{PE}_{\mathrm{AF}}(C)$, and the conclusion directly follows from Lemma 1. In the other case, i.e. $C \subsetneq \mathcal{A}$, $\mathcal{FG}^*_{\mathrm{AF}}(C) = \{\emptyset\}$, therefore we have to prove that $\emptyset$ is a complete extension in C: since $\emptyset$ is obviously admissible in C, the only thing to show is that any $\alpha \in C$ is not acceptable with respect to $\emptyset$. This trivially holds if $C = \emptyset$. Otherwise, a generic $\alpha \in C$ could only be acceptable with respect to $\emptyset$ if $\mathrm{parents}(\alpha) = \emptyset$. However, this is impossible, since $\alpha \in C$ and $C \subsetneq \mathcal{A}$: taking into account that the argumentation framework admits a unique SCC, there must be an argument $\beta \in \mathcal{A}, \beta \neq \alpha$ such that $\beta \rightarrow \alpha$.

Let us turn now to $AD2$-semantics, and let us notice that, if $C \subsetneq \mathcal{A}$, the relevant base function coincides with that of $AD1$-semantics, therefore the proof proceeds as in the previous case. If instead $C = \mathcal{A}$, then by definition any $E \in \mathcal{FG}^*_{\mathrm{AF}}(C)$ is a maximal element of $\mathcal{AS}^*_{\mathrm{AF}}$, i.e. it is a maximal set such that it is admissible in AF and the following property holds:

$$\forall \beta \in \mathcal{A} : \beta \rightarrow E, \mathrm{parents}(\beta) \subseteq E \qquad (6)$$

Since $C = \mathcal{A}$ and E is admissible, we have only to prove that $\forall \alpha \in \mathcal{A}$ which is acceptable with respect to E, $\alpha \in E$.

We reason by contradiction, assuming that $\exists \hat{\alpha} \in \mathcal{A} : \hat{\alpha} \notin E$ and $\hat{\alpha}$ is acceptable with respect to E. In case $E = \emptyset$, the acceptability of $\hat{\alpha}$ would entail that $\mathrm{parents}(\hat{\alpha}) = \emptyset$, and therefore the set $\{\hat{\alpha}\}$ would be admissible. However, $\{\hat{\alpha}\}$ would clearly belong to $\mathcal{AS}^*_{\mathrm{AF}}$, contradicting the maximality of $E = \emptyset$. Therefore, let us now assume that $E \neq \emptyset$. Recalling that $\hat{\alpha} \notin E$ and that $|\mathrm{SCC(AF)}| = 1$, there must exist a path between $\hat{\alpha}$ and an element $\gamma \in E$. Since $\hat{\alpha}$ is acceptable with respect to E, Lemma 2 entails that $\hat{\alpha} \not\rightarrow E$, therefore the above mentioned path must have the following structure: $\hat{\alpha} \rightarrow \beta_1 \rightarrow \ldots \rightarrow \beta_n \rightarrow \gamma$, with $n \geq 1$, such that $\gamma \in E$ and $\beta_i \notin E$ for all $i : 1 \leq i \leq n$. In particular, this entails that $\beta_n \rightarrow E$ and $\mathrm{parents}(\beta_n) \not\subseteq E$, contradicting equation (6). $\qquad \square$

Beyond Admissibility

As pointed out in previous subsection, any admissibility-based semantics is bound to exclude from any extension the arguments included in an argumentation framework consisting only of an odd-length attack cycle. Therefore odd-length cycles admit only the empty extension, while instead even-length cycles admit multiple extensions. This asymmetry in the treatment of cycles has recently been indicated as puzzling by Pollock (Pollock 2001) and has been identified as the primary cause of the counterintuitive behavior of preferred semantics discussed in (Baroni & Giacomin 2003). In order to enforce a symmetric treatment of cycles, it is necessary to explore another direction of investigation not constrained by the admissibility requirement.

To start reasoning, consider that, to handle appropriately the case of floating arguments shown in Figure 1, it is necessary that two extensions exist for a two length-cycle, each including one node. Considering then a 3-length cycle, it turns out that, in an analogous case of floating defeat, three extensions must exist each including one node. We notice that this set of extensions coincides with the set of maximal conflict free sets of this simple argumentation framework, and the same holds in the two-length cycle case. Therefore, referring to the general recursive schema, in both cases we have that the whole set of extensions, i.e. $\mathcal{FG}_{\mathrm{AF}}(\mathcal{A})$, coincides with the set of maximal conflict-free sets of AF, which we will denote as $\mathcal{MI}_{\mathrm{AF}}$. Since in the considered cases $|\mathrm{SCC(AF)}| = 1$, it must be the case that $\mathcal{FG}^*_{\mathrm{AF}}(\mathcal{A}) = \mathcal{MI}_{\mathrm{AF}}$. Generalizing this idea, we obtain the specification of the base function for the case where $C = \mathcal{A}$, i.e. $\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{MI}_{\mathrm{AF}}$. Taking this for granted, to complete the definition of the base function only the case where $C \subsetneq \mathcal{A}$ remains to be specified. In this respect, it seems reasonable to exploit the notion of maximal conflict-free set to provide a uniform conceptual basis to the approach. Two ways of applying this notion can be envisaged. On the one hand, one may regard the inclusion in C as a requirement. This gives rise to the following base function:

$$\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{MI}_{\mathrm{AF}\downarrow_C}$$

We denote the corresponding semantics as $CF1$.

On the other hand, since the requirement of inclusion in C is not strictly necessary, one may consider the following base function where it is overlooked:

$$\mathcal{FG}^*_{\mathrm{AF}}(C) = \mathcal{MI}_{\mathrm{AF}}$$

The corresponding semantics, denoted as $CF2$, has been first proposed in (Baroni & Giacomin 2003), where it has been derived following an alternative conceptual analysis.

It can be seen that both CF1 and CF2-semantics are able to treat appropriately all of the argumentation frameworks introduced so far. We describe, as an example, the case of the argumentation framework AF_3.

Example 5 *Handling the argumentation framework* AF_3

Recall that $\mathrm{SCC}(\mathrm{AF}_3) = \{S_1, S_2\}$ with $S_1 = \{\alpha, \beta, \gamma\}$ and $S_2 = \{\phi_1, \phi_2\}$. As for S_1, which is an initial SCC, the base case of Definition 7 applies, yielding $(E \cap S_1) \in \mathcal{FG}^*_{\mathrm{AF}_3\downarrow_{S_1}}(S_1)$. Therefore, according to $CF1$-semantics $(E \cap S_1)$ must belong to $\mathcal{MI}_{\mathrm{AF}\downarrow_C}$ with $\mathrm{AF} = \mathrm{AF}_3\downarrow_{S_1}$ and $C = S_1$, which is of course equal to $\mathcal{MI}_{\mathrm{AF}_3\downarrow_{S_1}}$. This is the same set prescribed by $CF2$-semantics, therefore the two semantics agree in this case. Now, $\mathcal{MI}_{\mathrm{AF}_3\downarrow_{S_1}} = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$, leading to three alternative possibilities for $(E \cap S_1)$. Assuming that $(E \cap S_1) = \{\gamma\}$, we have that $S_2^D(E) = \{\phi_1\}$, $S_2^U(E) = \{\phi_2\}$ and $S_2^P(E) = \emptyset$. Therefore, the base function is applied to the argumentation framework $\mathrm{AF}_3\downarrow_{\{\phi_2\}}$ with $C = \{\phi_2\}$, yielding for both semantics $(E \cap S_2) = \{\phi_2\}$. Therefore, we have an extension $E_1 = \{\gamma, \phi_2\}$ for both semantics. Let us turn now to the alternative $(E \cap S_1) = \{\beta\}$, which entails that $S_2^U(E) = S_2$ (as γ is attacked by $\beta \in E$). Therefore, the base function is applied to the argumentation framework $\mathrm{AF}_3\downarrow_{S_2}$ with $C = S_2$: it is easy to see that both semantics give two alternatives for $(E \cap S_2)$, namely $\{\phi_1\}$ and $\{\phi_2\}$. Therefore, we have for both semantics the additional extensions $E_2 = \{\beta, \phi_1\}$ and $E_3 = \{\beta, \phi_2\}$. Finally, if $(E \cap S_1) = \{\alpha\}$, we have that $S_2^P(E) = \{\phi_1\}$, $S_2^U(E) = \{\phi_2\}$ and $S_2^D(E) = \emptyset$. Then, the base function is applied to the argumentation framework $\mathrm{AF}_3\downarrow_{S_2}$ with $C = \{\phi_2\}$. In this case, $CF1$ and $CF2$-semantics differ. In fact, $CF1$ gives $\mathcal{MI}_{\mathrm{AF}\downarrow_C}$ which is equal to $\mathcal{MI}_{\mathrm{AF}_3\downarrow_{\{\phi_2\}}}$, therefore $(E \cap S_2) = \{\phi_2\}$, giving $E_4 = \{\alpha, \phi_2\}$. On the other hand, $CF2$-semantics gives $\mathcal{MI}_{\mathrm{AF}_3\downarrow_{S_2}} = \{\{\phi_1\}, \{\phi_2\}\}$, yielding therefore the same extension E_4 as $CF1$ and an additional extension $E_5 = \{\alpha, \phi_1\}$.
Summing up, in both semantics no argument is justified, i.e. included in all extensions, which is the intuitively desirable handling.

Comparing SCC-Recursive Semantics
Agreement with Grounded Semantics
As emphasized in the first part of the paper, the grounded semantics represents a sort of lower bound among argumentation semantics, since the grounded extension is the least among all conceivable extensions, namely complete extensions. More generally, the agreement with grounded semantics can be regarded as a fundamental requirement for any argumentation semantics, as it appears evident considering the characterization of grounded semantics provided by Pollock (Pollock 1992) recalled above. As a confirmation of the

well-foundedness of the property of SCC-recursiveness, we have obtained a result showing that for any SCC-recursive semantics each of its extensions includes the grounded extension, as far as a very simple condition on the base function is satisfied (proof is omitted due to space limitations):

Proposition 3 *Let us consider a SCC-recursive semantics with base function* $\mathcal{FG}^*$. *Then, the fact that, for any argumentation framework* AF, $\mathrm{GE}_{\mathrm{AF}} \subseteq E$ *for all extensions* E *prescribed by the semantics holds iff*

$$\mathcal{FG}^*_{\langle\{\alpha\},\emptyset\rangle}(\{\alpha\}) = \{\{\alpha\}\}.$$

In words, agreement with grounded semantics is ensured if and only if the base function properly deals with the simplest case of argumentation framework, i.e. a single node which does not attack itself and therefore should be accepted. It is easy to see that all of the novel semantics introduced in previous section, namely $AD1$, $AD2$, $CF1$ and $CF2$, satisfy this basic condition. On the other hand, as it will be shown in the next section, they disagree among each other on some significant examples largely debated in the literature (Prakken & Vreeswijk 2001).

Odd-Length Cycles and Self-Defeating Arguments
As already explained, CF1 and CF2-semantics depart from admissibility-based approaches in the treatment of odd-length cycles. To make this difference concrete in a simple example, consider the argumentation framework AF_6 shown in Figure 4, corresponding to a floating defeat against argument δ by the nodes in the 3-length cycle. In this case, any admissibility-based semantics admits the empty set as the unique extension, and therefore this is the case for preferred, AD1 and AD2-semantics, which do not regard as justified any argument. On the contrary, both CF1 and CF2-semantics admit as extensions the sets $\{\alpha, \phi\}$, $\{\beta, \phi\}$ and $\{\gamma, \phi\}$, yielding δ defeated and ϕ justified. This behavior is coherent with the notions of *floating defeat* and *floating acceptance* introduced in (Makinson & Schlechta 1991). As a consequence, this kind of examples discriminates admissibility-based semantics from the proposed novel semantics based on maximal conflict-free sets.

Self-defeating arguments are the extreme case of odd-length cycles and have received a special attention in the literature (Pollock 1994; Prakken & Vreeswijk 2001). The argumentation framework AF_7 shown in Figure 4 has been considered as an example of problematic behavior of preferred semantics by Dung himself (Dung 1995). In fact, the only preferred extension here is empty, though one can argue that since α attacks itself, β should be justified. This problem is inherited by any admissibility-based semantics, and therefore also affects AD1 and AD2-semantics.

Let us now consider how this case is handled by CF1 and CF2-semantics. First, note that AF_7 is composed of two SCCs, namely $S_1 = \{\alpha\}$ and $S_2 = \{\beta\}$. Starting from the initial SCC S_1, the base function applies to $\mathrm{AF}_7\downarrow_{\{\alpha\}}$ with $C = \{\alpha\}$: both CF1 and CF2-semantics prescribe that $(E \cap S_1)$ is a maximal conflict-free set of S_1. Since S_1 consists just of a self-defeating argument, the only conflict-free set is the empty set, therefore both CF1
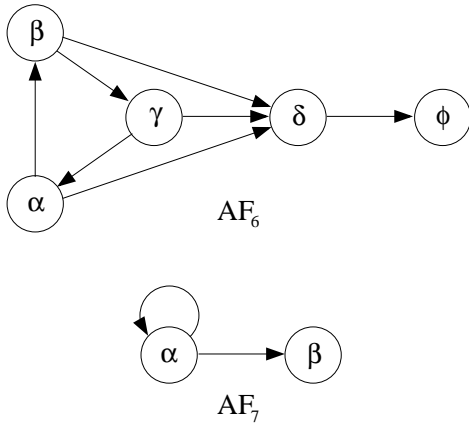
Figure 4: Handling odd-length cycles

and CF2-semantics exclude α from any extension. This in turn entails that, for any extension E, $S_2^P(E) = \{\beta\}$ while $S_2^U(E) = S_2^D(E) = \emptyset$. Then, $E \cap S_2 \in \mathcal{FG}_{\mathrm{AF}\downarrow_{\{\beta\}}}(\emptyset)$, and, since $\mathrm{AF}\downarrow_{\{\beta\}}$ has obviously only one SCC, this entails that $E \cap S_2 \in \mathcal{FG}^*_{\mathrm{AF}\downarrow_{\{\beta\}}}(\emptyset)$. CF1 and CF2-semantics behave differently in this case, due to the role of the argument C in the definition of the relevant base functions. In particular, according to CF1-semantics $(E \cap S_2) \in \mathcal{MI}_{\mathrm{AF}\downarrow_{\{\beta\}}\downarrow_\emptyset}$, i.e. $(E \cap S_2)$ must be equal to the empty set. On the other hand, C plays no role in the base function of CF2-semantics, yielding $(E \cap S_2) \in \mathcal{MI}_{\mathrm{AF}\downarrow_{\{\beta\}}}$, i.e. $(E \cap S_2) = \{\beta\}$. As a consequence, CF1-semantics admits the empty set as the unique extension, thus sharing with admissibility-based semantics the inability to properly handle self-defeating arguments. On the contrary, CF2-semantics is able to select the only desired extension $\{\beta\}$, thus ruling out the role of the self-defeating argument α.

Conclusions

In this paper, we have proposed a general recursive schema that, while including Dung's framework as a special case, can be regarded as an alternative foundation of argumentation theory. In particular, in this more general framework, the property of SCC-recursiveness appears to have the role of a unifying notion, in a similar way as admissibility in Dung's framework. As a matter of fact, this notion turns out to be effective as a basis supporting the development of alternative semantics. In fact, any SCC-recursive semantics is characterized by a base function defined over argumentation frameworks consisting of a single SCC, and the general requirement of agreement with the grounded semantics comes almost for free, since it is sufficient that the base function correctly treats single nodes without defeaters.

Exploiting the recursive schema, we have been able to introduce four novel semantics all solving counterintuitive examples affecting preferred semantics. The most satisfactory behavior is achieved by CF2-semantics, originally introduced in (Baroni & Giacomin 2003), which, in particular, is the only one able to deal with self-defeating arguments. It is worth noting that such semantics fully departs from the

admissibility notion in two respects: on one hand, it relies on the less demanding notion of maximal conflict-free set, on the other hand, as C plays no role in the base function, it prevents any argument outside the extension to defeat other arguments, while in case of admissibility-based semantics (and CF1 as well) this does not hold for those of them not attacked by the extension.

As for future work, we plan to further study the general properties of the SCC-recursive schema, as well as the relationships between different SCC-recursive semantics. In particular, it would be interesting to characterize such relationships in terms of the notion of *skepticism*, i.e. on the basis of the level of commitment concerning the choices about the justification status assigned to the arguments. Moreover, the meaning and use of the novel semantics proposed in this paper in different applications will be investigated.

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