Logical representation of preference & nonmonotonic reasoning

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- About the meaning of preference
- The need for compact representations and the role of logic
- Some logical languages for compact preference representation (a brief survey with examples)
- Preference representation and NMR
- Other issues

- **About the meaning of preference**
- The need for compact representations and the role of logic
- Some logical languages for preference representation
- Preference representation and NMR
- Other issues

preference

has different meanings in different communities

- in economics / decision theory:

  preference = relative or absolute satisfaction
  of an individual when facing
  various situations

preference

has different meanings in different communities

- in economics / decision theory:

  preference = relative or absolute satisfaction
  of an individual when facing
  various situations

- in KR / NMR

  preference = [weak] [strict] order
  with various meanings

  - A is more plausible / believed than B

    ↳ preferential models, preferential entailment etc.

  - rule A has priority over rule B

« *preference* »

has different meanings in different communities

- in economics / decision theory:

  preference = relative or absolute satisfaction
  of an individual when facing
  various situations

- in KR / NMR

  preference = [weak] [strict] order
  with various meanings

  - A is more plausible / believed than B

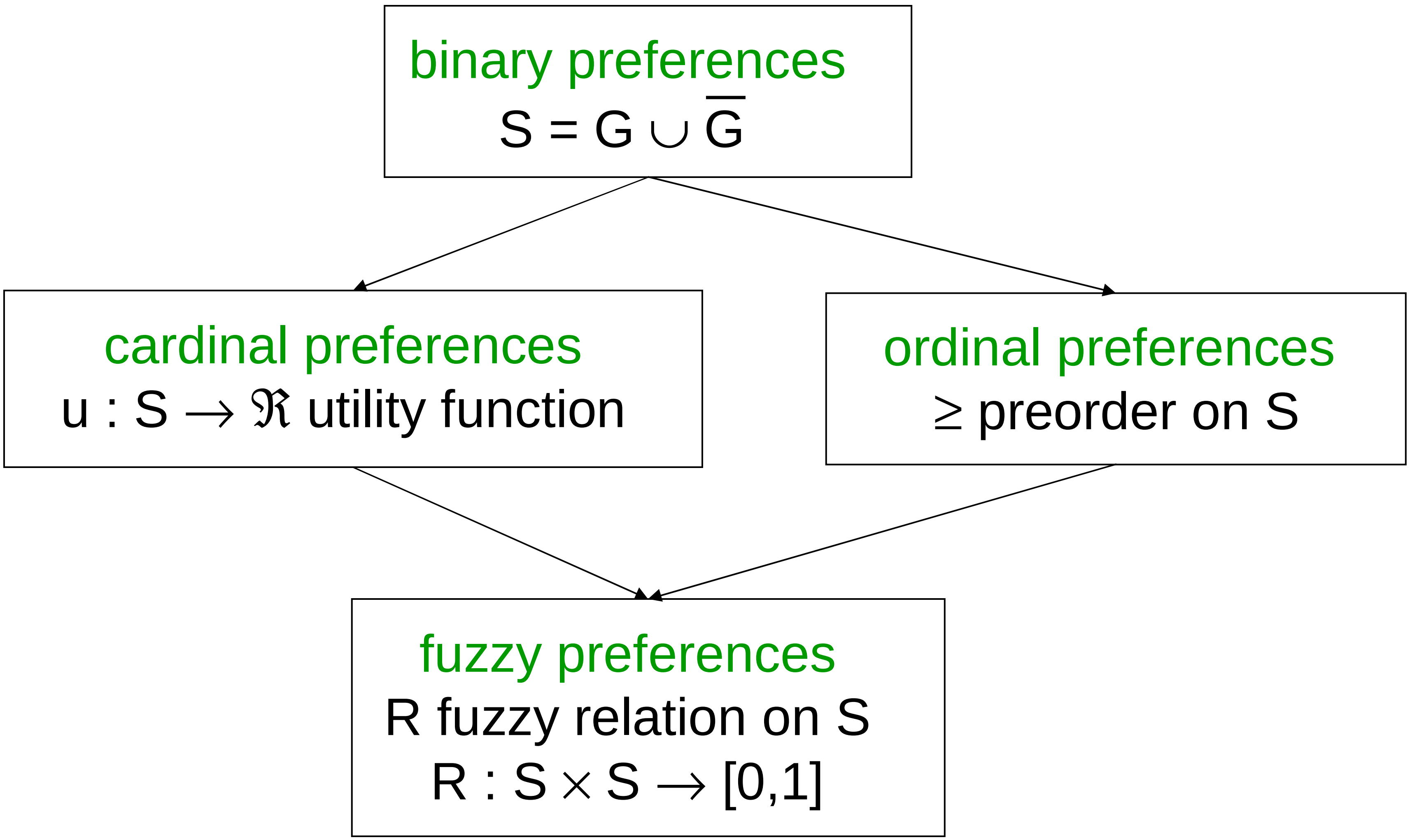
    preferential models, preferential entailment etc.

  - rule A has priority over rule B

relative (ordinal) uncertainty

control

Preference structure: represents the preferences of an agent over a set S of possible alternatives



- About the meaning of preference
- **The need for compact representations + the role of logic**
- Some logical languages for preference representation
- Preference representation and NMR
- Other issues

Complex domains: a state is defined by a tuple of values for a given set of variables

Example : preferences on airplane tikcets

option = (destination, price, dates, number-changes)

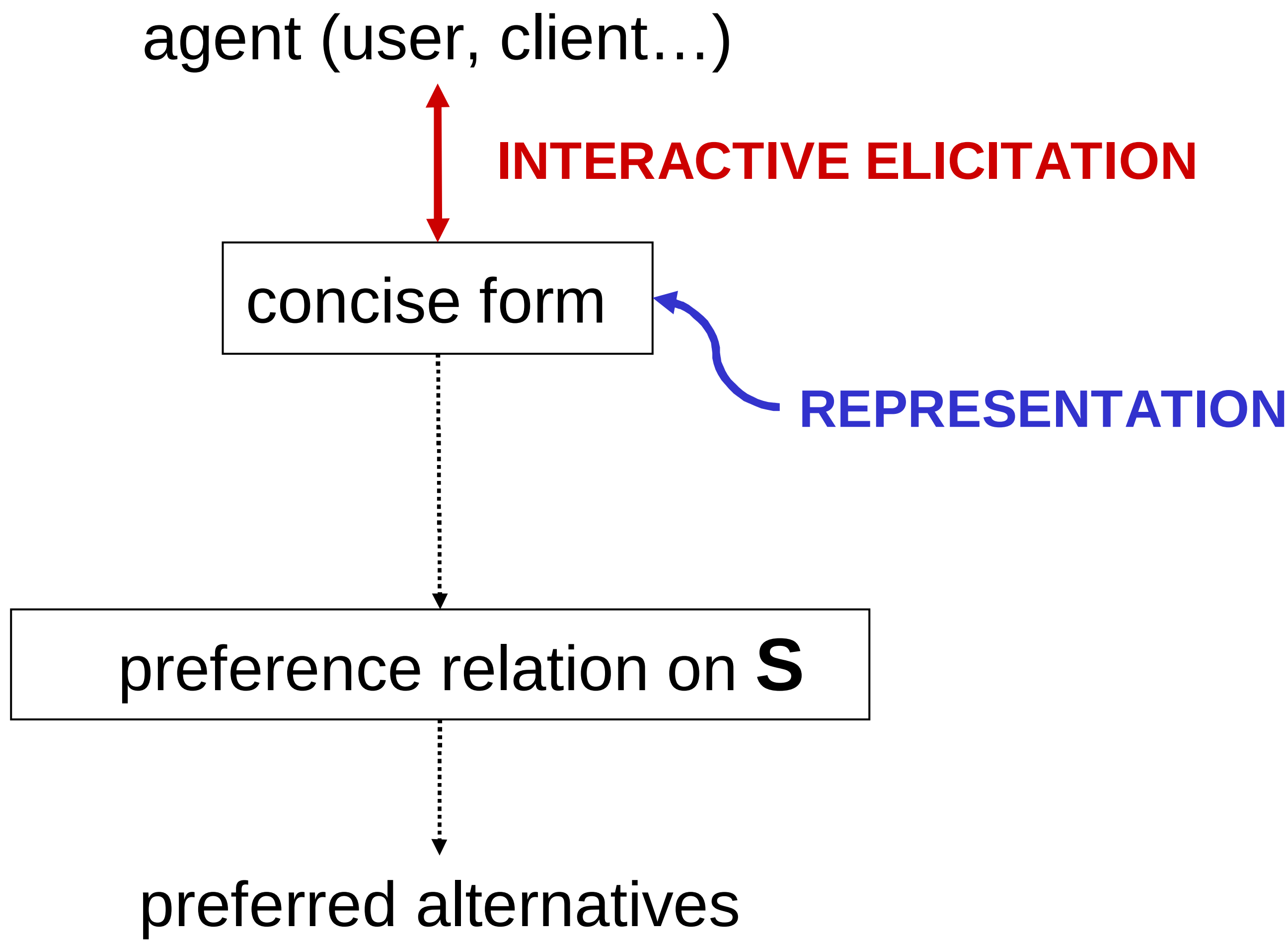
preferentially interdependent variables

Combinatorial explosion: prohibitive number of alternative

50 destinations, 10 price ranges, 10 departure dates and 10 return dates, 0/1/2 changes → 150 000 alternatives

→ Need for concise representations for preferences

Representation and elicitation of preferences



Why (propositional) logic?

- prototypical compact & structured language

  → good starting point

- expressive power
  + closeness to human intuition

  → elicitation issues

- efficient and well-studied algorithms
  (+ tractable fragments etc.)

  → optimization issues (find optimal alternatives)

- About the meaning of preference
- The need for compact representations + the role of logic
- **A brief survey on propositional logical languages for preference representation**
- Preference representation and NMR
- Other issues

Some logical languages for preference representation

1a. "Basic" propositional representation

K propositional formula

$S = \{\omega \mid \omega \models K\}$ set of possible alternatives

2 positions maximum to be filled
4 candidates A,B,C,D

$$K = (\neg A \wedge \neg B) \vee (\neg A \wedge \neg C) \vee (\neg A \wedge \neg D) \vee (\neg B \wedge \neg C) \vee (\neg B \wedge \neg D) \vee (\neg C \wedge \neg D)$$

$$K = [\leq 2 : A, B, C, D]$$

Some logical languages for preference representation

1a. "Basic" propositional representation

K

B = $\{\varphi_1, \ldots, \varphi_n\}$ set of goals

ω such that $\omega \models K \wedge \varphi_1 \wedge \ldots \wedge \varphi_n$

« good » states

$>$

ω such that $\omega \models K \wedge \neg (\varphi_1 \wedge \ldots \wedge \varphi_n)$

« bad » states

$\omega \models \neg K$

impossible states

Some logical languages for preference representation

1a. "Basic" propositional representation

$K = [\leq 2 : A, B, C, D]$

$G = \{ (A \vee B), (B \rightarrow \neg C), \neg D \}$

I would like to hire A or to hire B;
if B is hired then I would prefer not to hire C;
I would like not to hire D

« good » states

$(A, B, \neg C, \neg D)$	hire A and B
$(A, \neg B, C, \neg D)$	hire A and C
$(A, \neg B, \neg C, \neg D)$	hire A only
$(\neg A, B, \neg C, \neg D)$	hire B only

Some logical languages for preference representation

1b. "Basic" propositional representation + cardinality

K

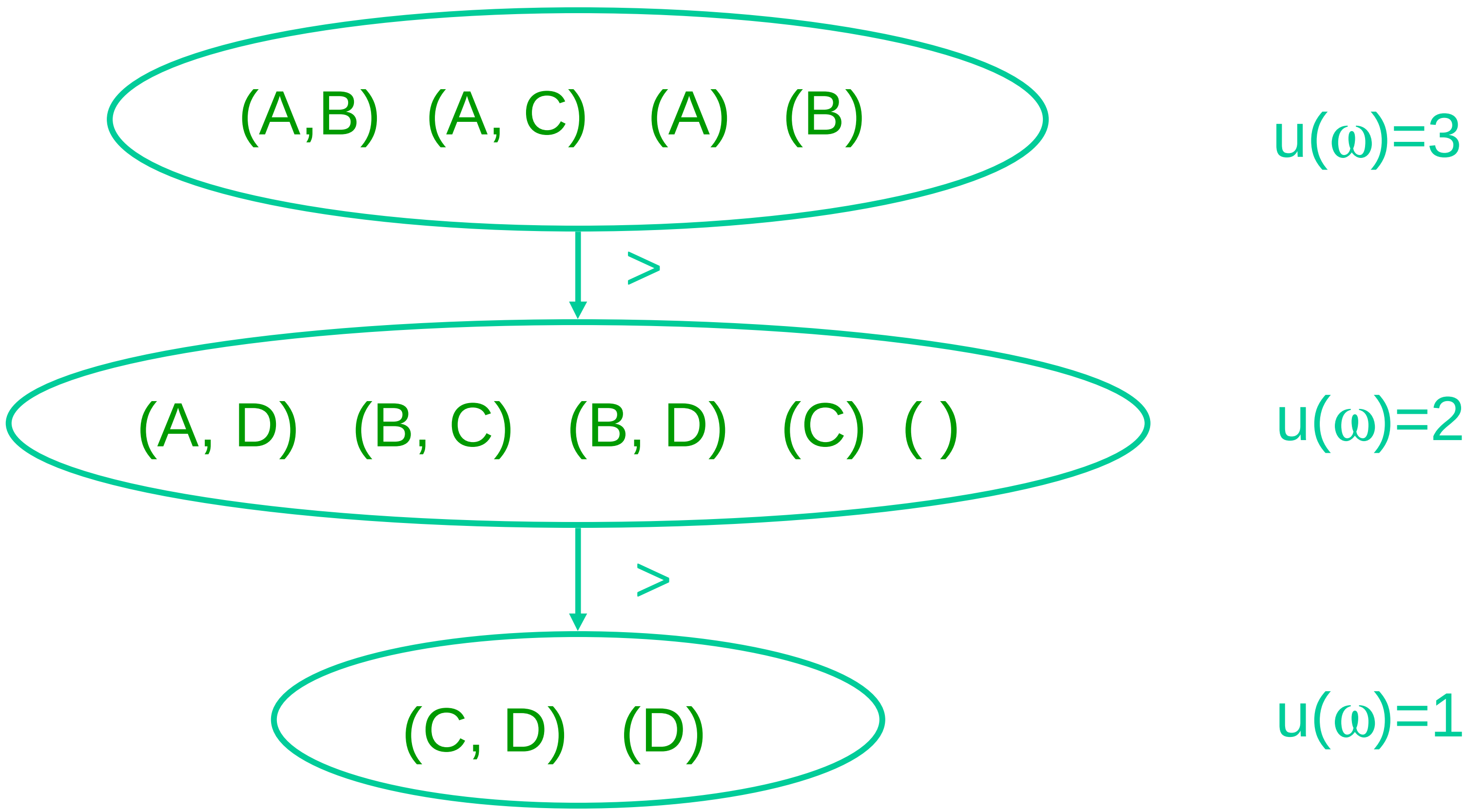
$B = \{\varphi_1, \ldots, \varphi_n\}$ set of goals

For all $\omega \in Mod(K)$, $u_B(\omega) = |\{i, \omega \models \varphi_i\}|$

Some logical languages for preference representation

1b. "Basic" propositional representation + cardinality

$K = [\leq 2 : A, B, C, D]$; $G = \{ (A \vee B), (B \rightarrow \neg C), \neg D \}$



Some logical languages for preference representation

1c. "Basic" propositional representation + inclusion

K

$B = \{\varphi_1, \ldots, \varphi_n\}$ set of goals

For all $\omega, \omega' \in Mod(K)$

$$\omega \geq \omega'$$

if and only if

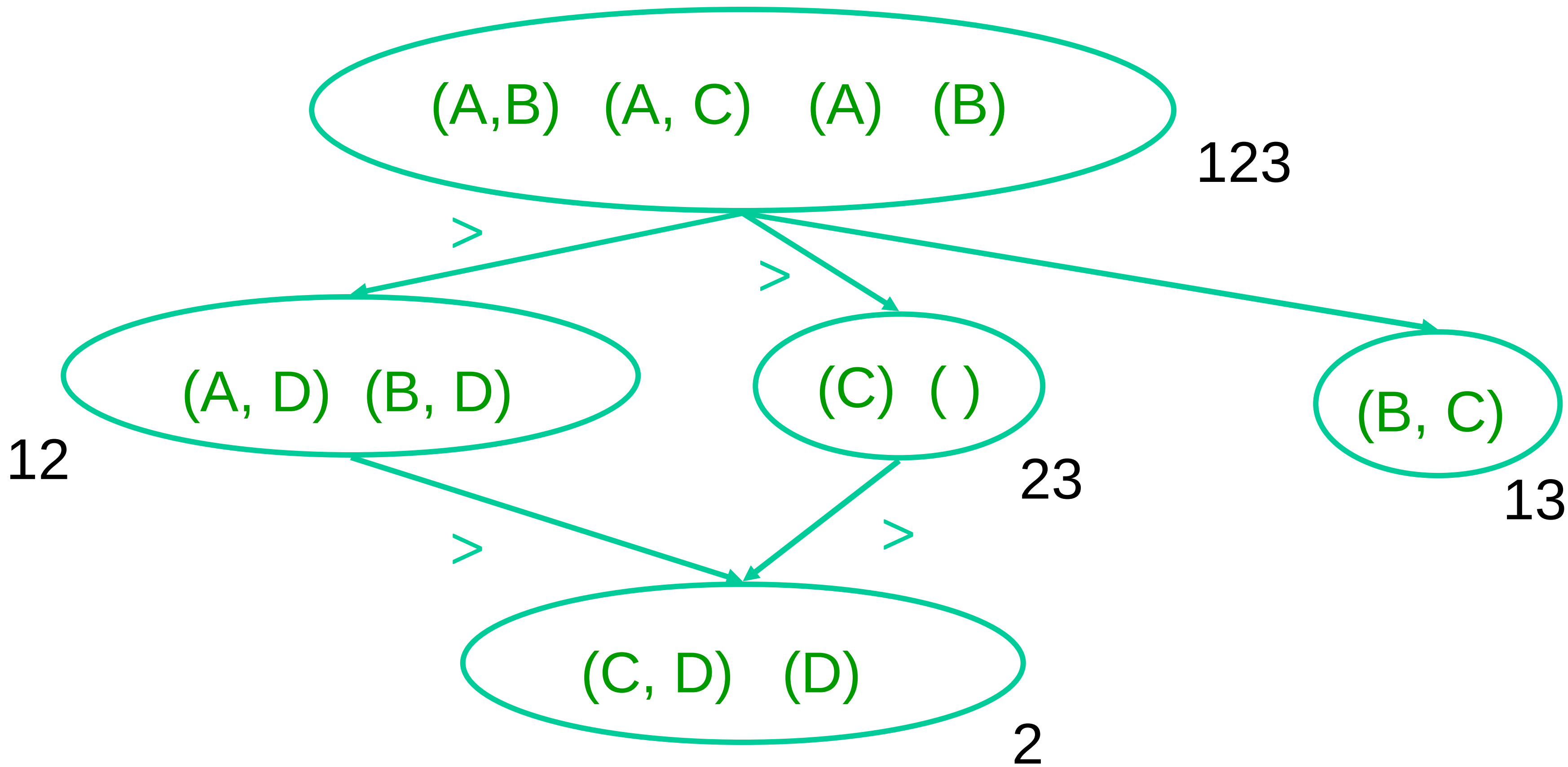
$$\{ i , \omega \models \varphi_i \} \supseteq \{ j , \omega' \models \varphi_j \}$$

Some logical languages for preference representation

1c. "Basic" propositional representation + inclusion

K = [≤ 2 : A, B, C, D] ; G = { (A ∨ B), (B → ¬ C), ¬ D }

1 2 3



Some logical languages for preference representation

2. Propositional logic + weights

K

$B = \{ (\varphi_1, x_1), \ldots, (\varphi_n, x_n) \}$

φ_l propositional formula

$x_i \in \Re^*$ → $x_i > 0$ reward; $x_i < 0$ penalty

Example: additive weights

For all $\omega \in \mathrm{Mod}(K)$,

$$u_B(\omega) = \sum_{\substack{i \in 1..N \\ \omega \models \varphi_i}} x_i$$

Some logical languages for preference representation

2. Propositional logic + weights

K

$B = \{ (\varphi_1, x_1), \ldots, (\varphi_n, x_n) \}$

φ_l propositional formula

$x_i \in \Re^*$ → $x_i > 0$ reward; $x_i < 0$ penalty

Example: additive weights

For all $\omega \in \mathrm{Mod}(K)$,

$$u_B(\omega) = \cancel{\sum_{\substack{i \in 1..N \\ \omega \models \varphi_i}}} x_i$$

other aggregation functions

Some logical languages for preference representation

2. Propositional logic + weights

K

$B = \{ (\varphi_1, x_1), \ldots, (\varphi_n, x_n) \}$

φ_I propositional formula

$x_i \in \Re^*$ → $x_i > 0$ reward; $x_i < 0$ penalty

For all $\omega \in$ Mod(K),

$$u_B(\omega) = F\left(\{x_i \mid \omega \models \varphi_i, i \in 1 .. N\} \right)$$

Some logical languages for preference representation

2. Propositional logic + weights

K

$B = \{ (\varphi_1, x_1), \ldots, (\varphi_n, x_n) \}$

φ_l propositional formula

$x_i \in \Re^*$
- $x_i > 0$ reward
- $x_i < 0$ penalty

For all $\omega \in \mathrm{Mod}(K)$,

$$u_B(\omega) = F\Big(G\big(\{x_i \mid \omega \models \varphi_i, i \in 1..N, x_i > 0\}\big), H\big(\{x_j \mid \omega \models \varphi_j, i \in 1..N, x_i < 0\}\big)\Big)$$

bipolarity

Some logical languages for preference representation

2. Propositional logic + (additive) weights

$K = [\leq 3 : A, B, C, D, E]$;

$G = \{ (B \vee C, +5)$;	only B and C can teach logic
$(A \vee C, +6)$;	only A and C can teach databases
$(A \wedge B, -3)$;	A and B would be in the same group (to be avoided)
$(D \wedge E, -3)$;	idem for D and E
$(D, +10)$;	D is the best candidate
$(E, +8)$;	E is the second best
$(A, +6)$;	etc.
$(B, +4)$; $(C, +2) \}$	

Some logical languages for preference representation

2. Propositional logic + weights

$K = [\leq 3 : A, B, C, D, E]$; $\omega = (A, D, E, \neg B, \neg C)$

$G = \{$ $(B \vee C, +5)$;

$(A \vee C, +6)$; +6

$(A \wedge B, -3)$;

$(D \wedge E, -3)$; -3

$(D, +10)$; +10

$(E, +8)$; +8

$(A, +6)$; +6

$(B, +4)$;

$(C, +2)$ $\}$

$u(\omega) = +27$

Some logical languages for preference representation

2. Propositional logic + weights

$K = [\leq 3 : A, B, C, D, E]$; $\omega' = (C, D, E, \neg A, \neg D)$

$G = \{$ $(B \vee C, +5)$; +5

$(A \vee C, +6)$; +6

$(A \wedge B, -3)$;

$(D \wedge E, -3)$;

$(D, +10)$; +10

$(E, +8)$; +8

$(A, +6)$;

$(B, +4)$;

$(C, +2)$ $\}$ +2

$u(\omega') = +31$

Some logical languages for preference representation

2. Propositional logic + weights

$K = [\leq 3 : A, B, C, D, E]$;

$G = \{ \; (B \vee C, +5)$;

$(A \vee C, +6)$;

$(A \wedge B, -3)$;
$(D \wedge E, -3)$;

$(D, +10)$;
$(E, +8)$;
$(A, +6)$;
$(B, +4)$;
$(C, +2) \; \}$

ω	$u(\omega)$
(C,D,E)	31
(A,C,D)	29
(A,B,D)	28
(A,D,E) (B,C,D) (A,C,E)	27
(B,D,E)	24
(C,D)	23
…	

Some logical languages for preference representation

3a. Propositional logic + priorities

K

$B = \langle B_1, \ldots, B_p \rangle$ stratification of B

B_1
…
B_p

increasing priority

$K = [\leq 2 : A, B, C, D, E]$;

$B_1 = \{B \vee C, A \vee C, \neg (D \wedge E), \neg (D \wedge E)\}$

1 2 3 4

$B_2 = \{D, A\}$ 5 6

$B_3 = \{E\}$ 7

$B_4 = \{B, C\}$ 8 9

3a. Propositional logic + priorities

$K = [\leq 3 : A, B, C, D, E]$;

$B1 = \{B \vee C, A \vee C, \neg (A \wedge B), \neg (D \wedge E)\}$

1 2 3 4

$B2 = \{D,A\}$ $B3 = \{E\}$ $B4 = \{B, C\}$

5 6 7 8 9

« Best-out » ordering

$$u(\omega) = \min \left\{ i,\ \omega \text{ violates at least a formula of } B_i \right\}$$

(= + ∞ if there is no such i)

3a. Propositional logic + priorities

$K = [\leq 3 : A, B, C, D, E]$;

$\omega = (A, B, C, \neg D, \neg E)$

$B1 = \{B \vee C, A \vee C, \neg (A \wedge B), \neg (D \wedge E)\}$

1 2 3 4

$u(\omega) = 1$

$B2 = \{D, A\}$ $B3 = \{E\}$ $B4 = \{B, C\}$

5 6 7 8 9

« Best-out » ordering

$u(\omega) = \min \left\{ i, \ \omega \text{ violates at least a formula of } B_i \right\}$

3a. Propositional logic + priorities

$K = [\leq 3 : A, B, C, D, E]$; $\omega = (A, C, D, \neg B, \neg E)$

$B1 = \{B \vee C, A \vee C, \neg (A \wedge B), \neg (D \wedge E)\}$

1 2 3 4

$u(\omega) = 3$

$B2 = \{D, A\}$ $B3 = \{E\}$ $B4 = \{B, C\}$

5 6 7 8 9

« Best-out » ordering

$u(\omega) = \min \{ i, \ \omega$ violates at least a formula of $B_i \}$

3a. Propositional logic + priorities

K = [$\leq$ 2 : A, B, C, D, E] ;

B1 = {B $\vee$ C, A $\vee$ C, A $\vee$ B, D $\vee$ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}

5 6 7 8 9

« leximin » ordering [Benferhat et al. 93]

$\omega > \omega'$

iff (ω satisfies more formulas of B1 than ω')

or (ω and ω' satisfy the same number of formulas of B1, and ω satisfies more formulas of B2 than ω')

or (ω et ω' satisfy the same number of formulas of B1 and of B2, and ω satisfies more formulas of B3 than ω')

etc.

3a. Propositional logic + priorities: leximin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}
1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}
5 6 7 8 9

	B1	B2	B3	B4
(A,C)	3	0	1	1
(A,D)	3	1	1	0
(B,C)	3	0	0	2
(C,D)	3	1	0	1

3a. Propositional logic + priorities: leximin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}

5 6 7 8 9

	B1	B2	B3	B4
(A,C)	3	0	1	1
(A,D)	3	1	1	0
(B,C)	3	0	0	2
(C,D)	3	1	0	1
~~(D,E)~~	~~1~~	~~1~~	~~1~~	~~0~~

3a. Propositional logic + priorities: leximin ordering

K = [$\leq$ 2 : A, B, C, D, E] ;

B1 = {B $\vee$ C, A $\vee$ C, A $\vee$ B, D $\vee$ E}
1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}
5 6 7 8 9

	B1	B2	B3	B4
~~(A,C)~~	~~3~~	~~0~~	~~1~~	~~1~~
(A,D)	3	1	1	0
~~(B,C)~~	~~3~~	~~0~~	~~0~~	~~2~~
(C,D)	3	1	0	1
~~(D,E)~~	~~1~~	~~1~~	~~1~~	~~0~~

3a. Propositional logic + priorities: leximin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}
1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}
5 6 7 8 9

	B1	B2	B3	B4
~~(A,C)~~	~~3~~	~~0~~	~~1~~	~~1~~
✦ (A,D)	3	1	1	0
~~(B,C)~~	~~3~~	~~0~~	~~0~~	~~2~~
~~(C,D)~~	~~3~~	~~1~~	~~0~~	~~1~~
~~(D,E)~~	~~1~~	~~1~~	~~1~~	~~0~~

3b. Propositional logic + ordered disjunction

K

[Brewka, Benferhat & Le Berre 02]

$\Psi = (\varphi_1 \times \varphi_2 \ \ldots \times \varphi_p)$

ideally φ_1;

otherwise (sub-ideally) φ_2

otherwise φ_3

etc.

$B = \langle \Psi_1, \ \ldots, \Psi_p \rangle$

Some logical languages for preference representation

3b. Propositional logic + ordered disjunction

K

$\Psi = (\varphi_1 \times \varphi_2 \; \ldots \times \varphi_p)$

ideally φ_1;

otherwise (sub-ideally) φ_2

otherwise φ_3

etc.

For all $\omega \in$ Mod(K),

$$\begin{aligned} disu(\omega, \Psi) &= 0 && \text{if } \omega \models \varphi_1 \\ &= i && \text{if } \omega \models \neg \varphi_1 \wedge \ldots \wedge \neg \varphi_{i-1} \wedge \varphi_i \\ &= p+1 && \text{if } \omega \models \neg \varphi_1 \wedge \ldots \wedge \neg \varphi_n \end{aligned}$$

Some logical languages for preference representation

3b. Propositional logic + ordered disjunction

K

$B = \langle \Psi_1, \ldots, \Psi_p \rangle$

$\overrightarrow{disu}(\omega, B) = \langle disu(\omega, \Psi_1), \ldots, disu(\omega, \Psi_p) \rangle$

For all $\omega, \omega' \in Mod(K)$,

$\omega >_B \omega'$ iff $\overrightarrow{disu}(\omega, B) <_{leximin} \overrightarrow{disu}(\omega', B)$

Some logical languages for preference representation

3b. Propositional logic + ordered disjunction

K

$B = \langle \Phi_1, \ \ldots, \Phi_p \rangle$

$K = [\leq 2 : A, B, C, D, E]$;

	$\omega = (A,E)$	$\omega' = (A,B)$
$\Phi_1 : (B \wedge C) \times (B \vee C)$	3	2
$\Phi_2 : (A \wedge C) \times (A \vee C)$	2	2
$\Phi_3 : \neg (D \wedge E)$	1	1
$\Phi_4 : \neg (A \wedge B)$	1	2
$\Phi_5 : D \times A \times E \times B \times C$	2	2
$\Phi_5 : (= 2 : A,B,C,D,E) \times (= 2 : A,B,C,D,E)$	1	1

Some logical languages for preference representation

3b. Propositional logic + ordered disjunction

K

$B = \langle \Phi_1, \ \ldots, \Phi_p \rangle$

$K = [\leq 2 : A, B, C, D, E] \quad ;$

$>_B \longrightarrow$

	ω = (A,E)	ω' = (A,B)
$\Phi_1 : (B \wedge C) \times (B \vee C)$	3	2
$\Phi_2 : (A \wedge C) \times (A \vee C)$	2	2
$\Phi_3 : \neg (D \wedge E)$	1	1
$\Phi_4 : \neg (A \wedge B)$	1	2
$\Phi_5 : D \times A \times E \times B \times C$	2	2
$\Phi_5 : (= 2 : A,B,C,D,E) \times (= 2 : A,B,C,D,E)$	1	1

3. Propositional logic + priorities

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B, C}

5 6 7 8 9

« discrimin » ordering [Brewka 89]

strict inclusion ↓

ω > ω'

iff { φ ∈ B1, ω satisfies φ } ⊃ { φ ∈ B1, ω' satisfies φ }

or ({ φ ∈ B1, ω satisfies φ } = { φ ∈ B1, ω' satisfies φ }

and { φ ∈ B2, ω satisfies φ } ⊃ { φ ∈ B2, ω' satisfies φ })

etc.

3. Propositional logic + priorities : discrimin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}

5 6 7 8 9

	B1	B2	B3	B4
(A,C)	123-	-	6-	9
(B,C)	123-	-	--	89
(A,D)	-234	5	--	9
(C,D)	12-4	5	--	9

3. Propositional logic + priorities: discrimin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {$B \vee C$, $A \vee C$, $A \vee B$, $D \vee E$}
1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}
5 6 7 8 9

	B1	B2	B3	B4
(A,C)	123-	-	6-	9
(B,C)	123-	-	--	89
(A,D)	2--4	5	--	9
(C,D)	12-4	5	--	9

3. Propositional logic + priorities: discrimin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}
1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}
5 6 7 8 9

	B1	B2	B3	B4
(A,C)	123-	-	6-	-9
~~(B,C)~~	~~123-~~	~~-~~	~~--~~	~~89~~
(A,D)	-234	5	6-	--
(C,D)	12-4	5	--	-9

3. Propositional logic + priorities: discrimin ordering

K = [$\leq$ 2 : A, B, C, D, E] ;

B1 = {B $\vee$ C, A $\vee$ C, A $\vee$ B, D $\vee$ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}

5 6 7 8 9

incomparable

	B1	B2	B3	B4
(A,C)	123-	-	6-	-9
~~(B,C)~~	~~123-~~	~~-~~	~~--~~	~~89~~
(A,D)	-234	5	6-	--
(C,D)	12-4	5	--	-9

3. Propositional logic + priorities: discrimin ordering

K = [≤ 2 : A, B, C, D, E] ;

B1 = {B ∨ C, A ∨ C, A ∨ B, D ∨ E}

1 2 3 4

B2 = {D} B3 = {A,E} B4 = {B,C}

5 6 7 8 9

	B1	B2	B3	B4	leximin	discrimin	best-out
(A,C)	123-	-	6-	-9	no	yes	yes
(B,C)	123-	-	--	89	no	no	yes
(A,D)	-234	5	6-	--	yes	yes	yes
(C,D)	12-4	5	--	-9	no	yes	yes

Some logical languages for preference representation

4. Propositional logic + distances

- K
- B = $\langle \varphi_1, \ldots, \varphi_p \rangle$
- d: $S \times S \rightarrow \Re$

  $d(\omega, \omega') = d(\omega', \omega)$

  $d(\omega, \omega') = 0$ iff $\omega = \omega'$

  (example: d = Hamming distance)

$d(\omega, \varphi_i) = \min \{(\omega, \omega') \mid \omega'$ satisfies $K \wedge \varphi_i\}$

$d(\omega, B) = F(d(\omega, \varphi_1), d(\omega, \varphi_2), \ldots, d(\omega, \varphi_n))$

$\omega \geq \omega'$ iff $d(\omega, B) \leq d(\omega', B)$

⇒ distance-based merging

Some logical languages for preference representation

5. « ceteris paribus » preferences

[von Wright 63; Hansson 66; Doyle & Wellman 91]

$$\gamma : \varphi > \psi$$

For any two states ω, ω' such that

- ω satisfies $\gamma \wedge \varphi \wedge \neg\psi$
- ω' satisfies $\gamma \wedge \neg \varphi \wedge \psi$
- ω and ω' coincide on « irrelevant » variables

then $\omega >_B \omega'$ (+ transitive closure)

Some logical languages for preference representation

5. « ceteris paribus » preferences

$$\gamma : \varphi > \psi$$

For any two states ω, ω' such that

- ω satisfies $\gamma \wedge \varphi \wedge \neg\psi$
- ω' satisfies $\gamma \wedge \neg \varphi \wedge \psi$
- ω and ω' coincide on « irrelevant » variables

then $\omega >_B \omega'$ (+ transitive closure)

↓

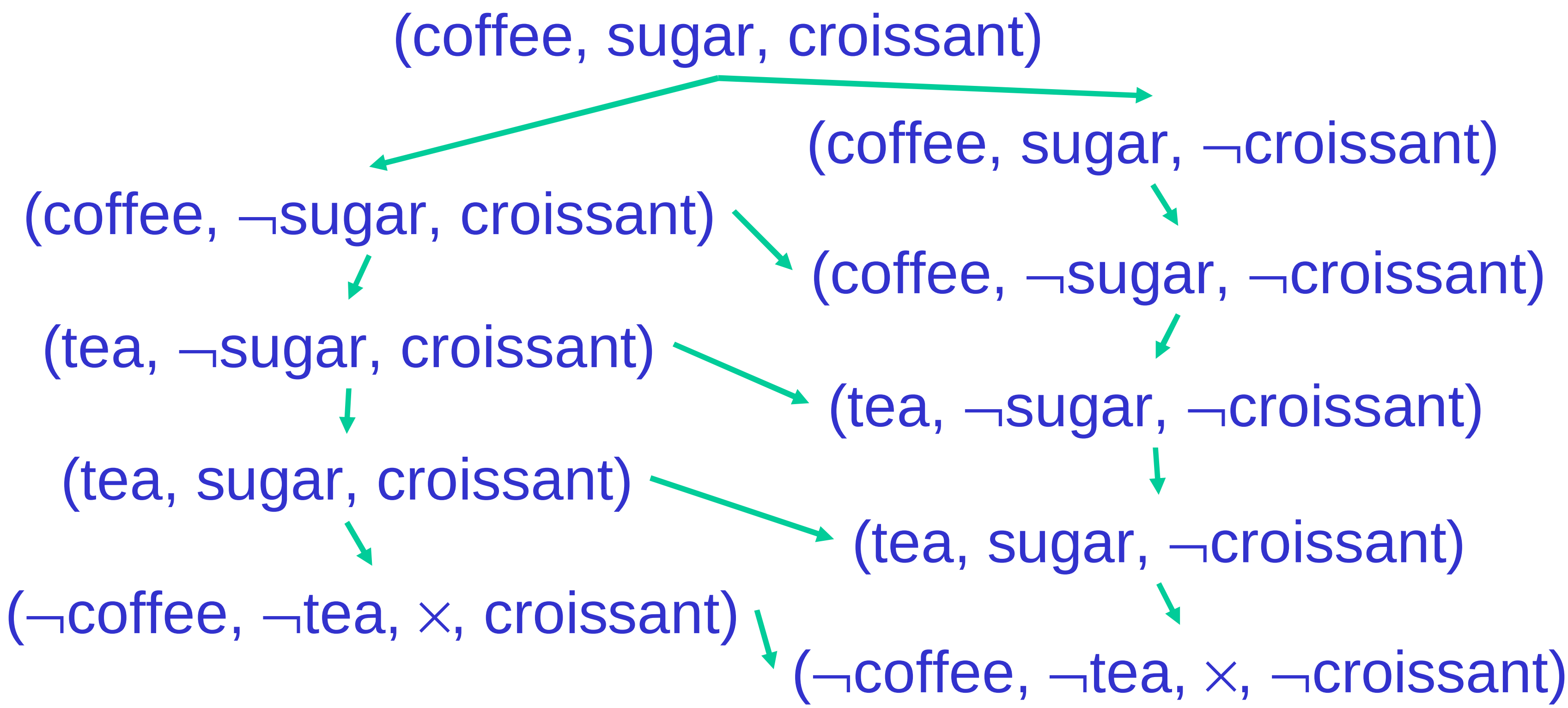
e.g. variables outside Var (γ) $\cup$ Var (φ) $\cup$ Var (ψ)

(more sophisticated definitions are possible)

5. « ceteris paribus » preferences

K = {¬ (coffee ∧ tea) }

B = {coffee: sugar > ¬ sugar ; tea: ¬ sugar > sugar ;
T : coffee > tea > ¬ coffee ∧ ¬ tea ;
T : croissant > ¬ croissant }



Some logical languages for preference representation

5. « ceteris paribus » preferences: CP-nets

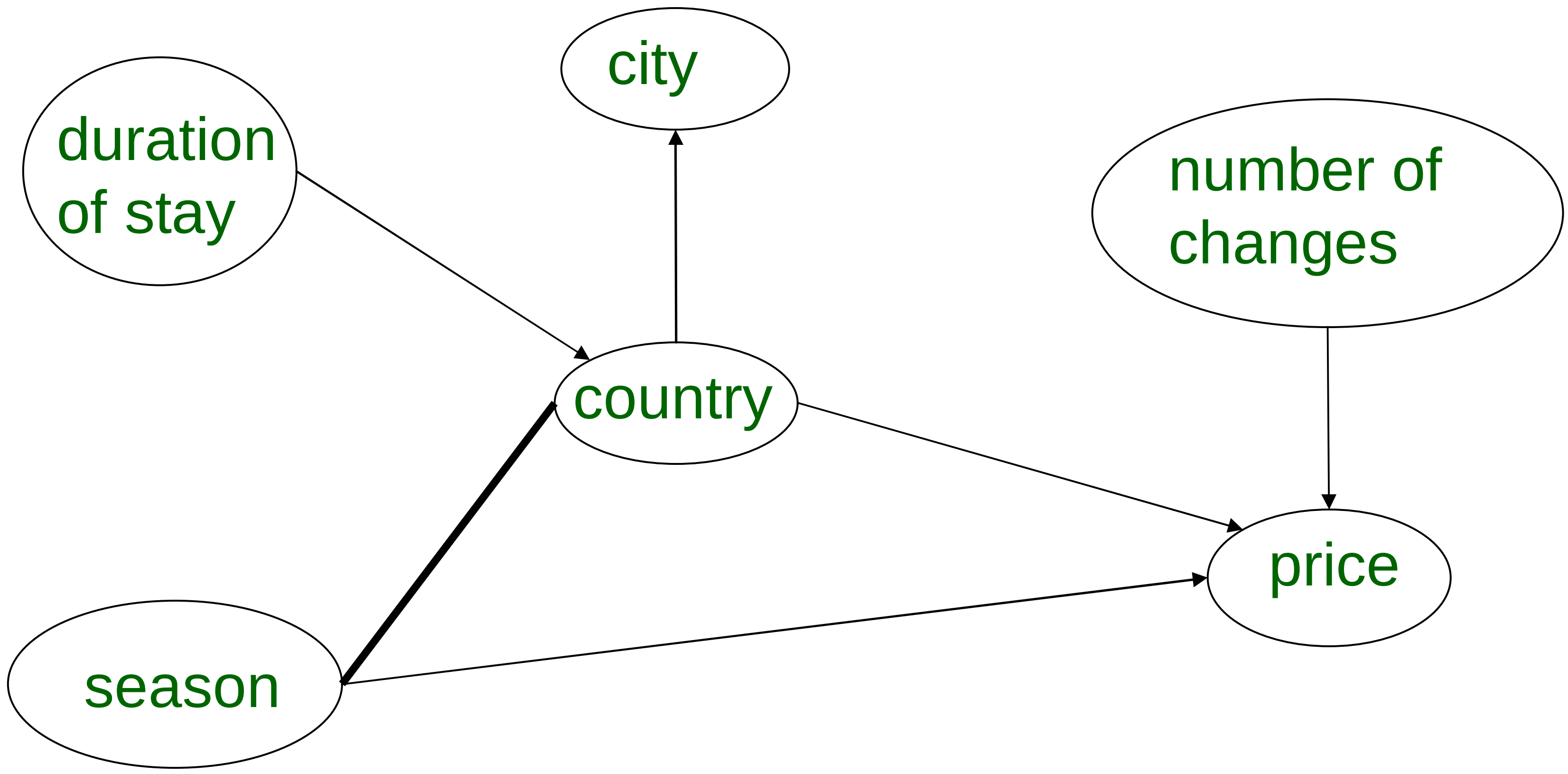
[Boutilier et al. 99; Brafman & Domshlak 02; ...]

- variables stuctured in a network
- restriction on syntax

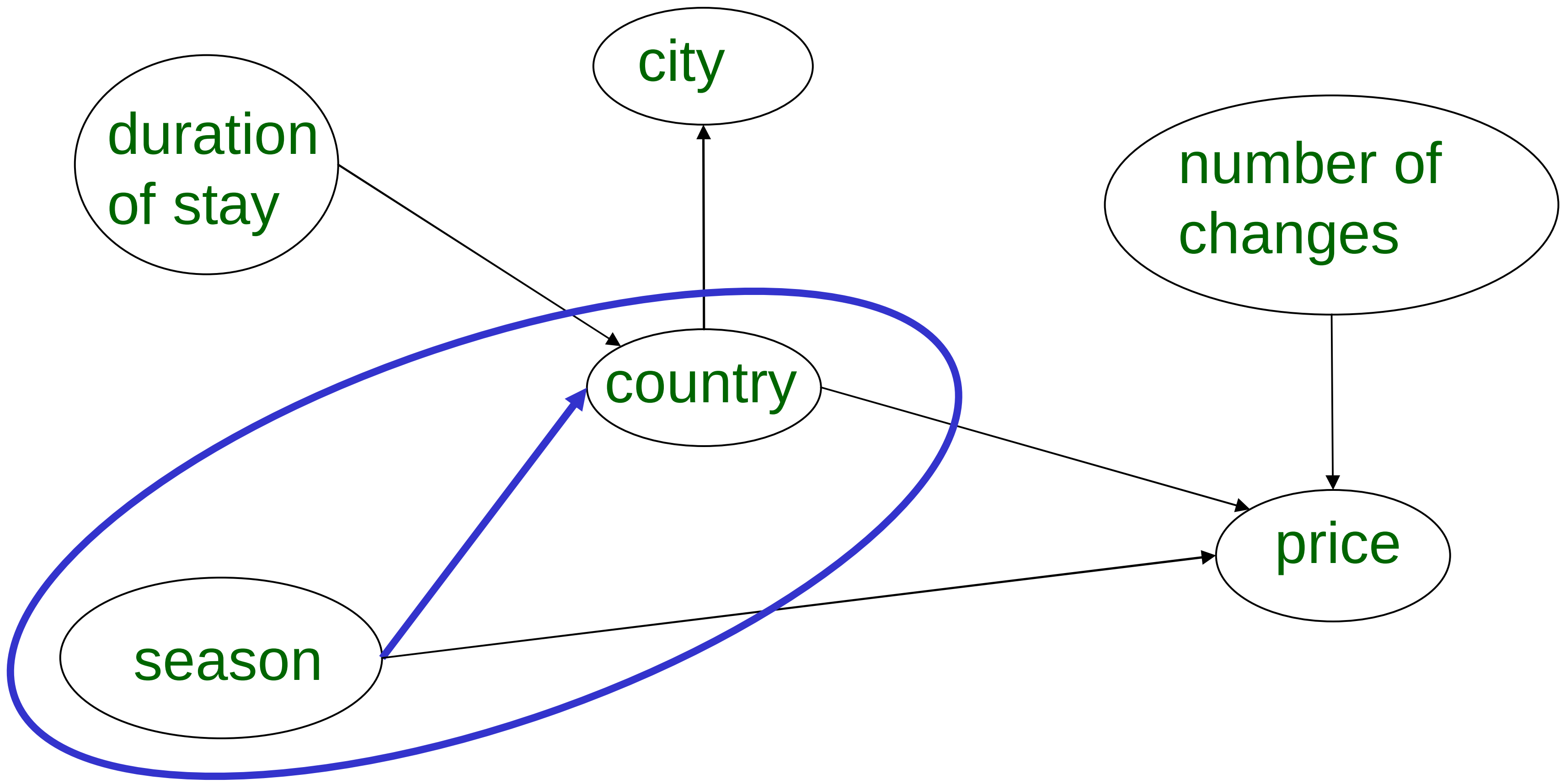
$$\gamma : (x=a) > (x=a)$$

where the variables appearing in γ are parents of x in the network

5. « ceteris paribus » preferences: CP-nets



5. « ceteris paribus » preferences: CP-nets



JANUARY : INDIA > BRAZIL > TURKEY > RUSSIA

5. « ceteris paribus » preferences: CP-nets

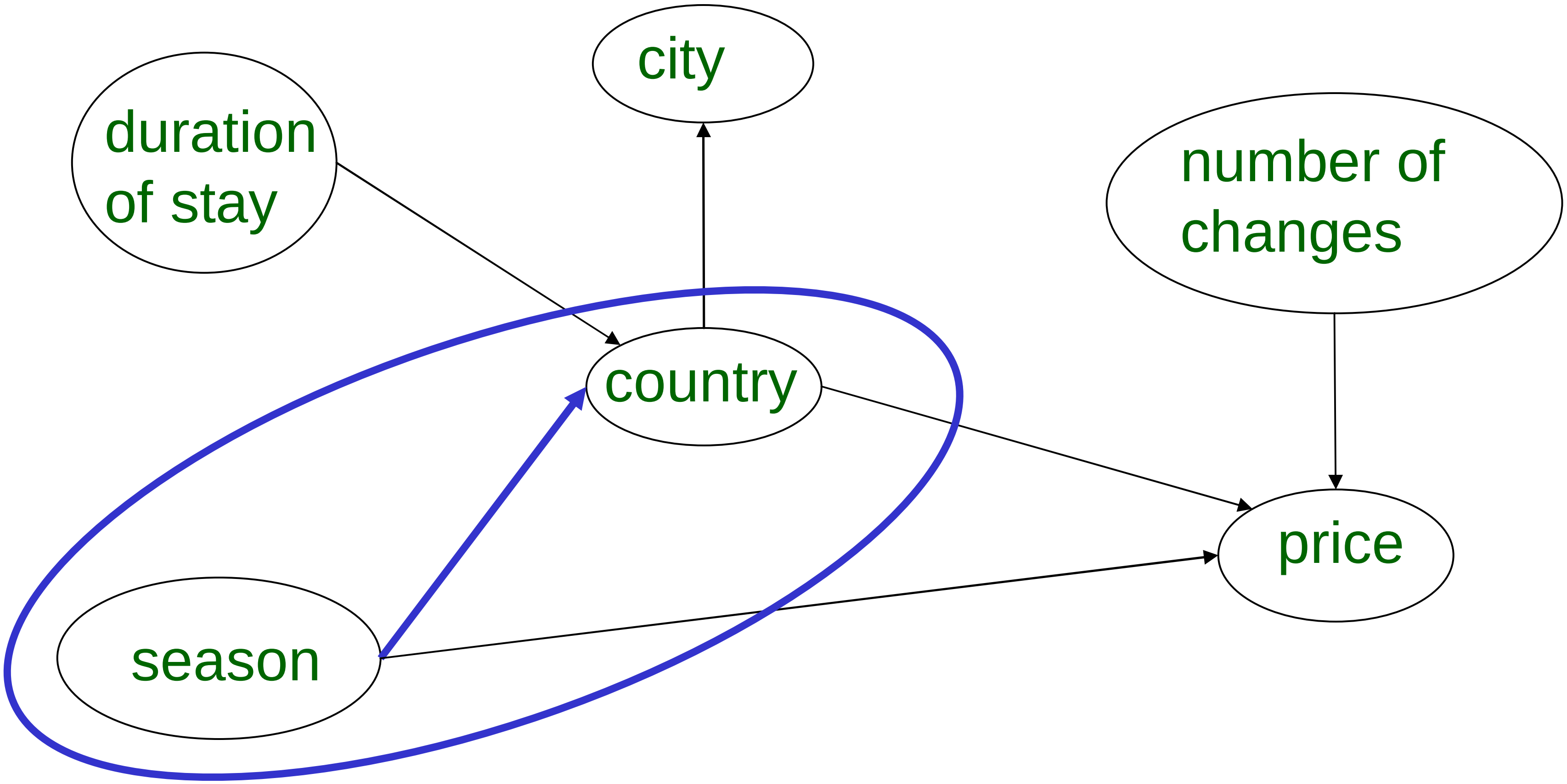
JANUARY : INDIA > BRAZIL > TURKEY > RUSSIA

Given two states o,o’ such that

- departure in January for o and o’
- destination(o) = **INDIA**, destination(o’) = **BRAZIL**
- o and o’ coincide on all other variables

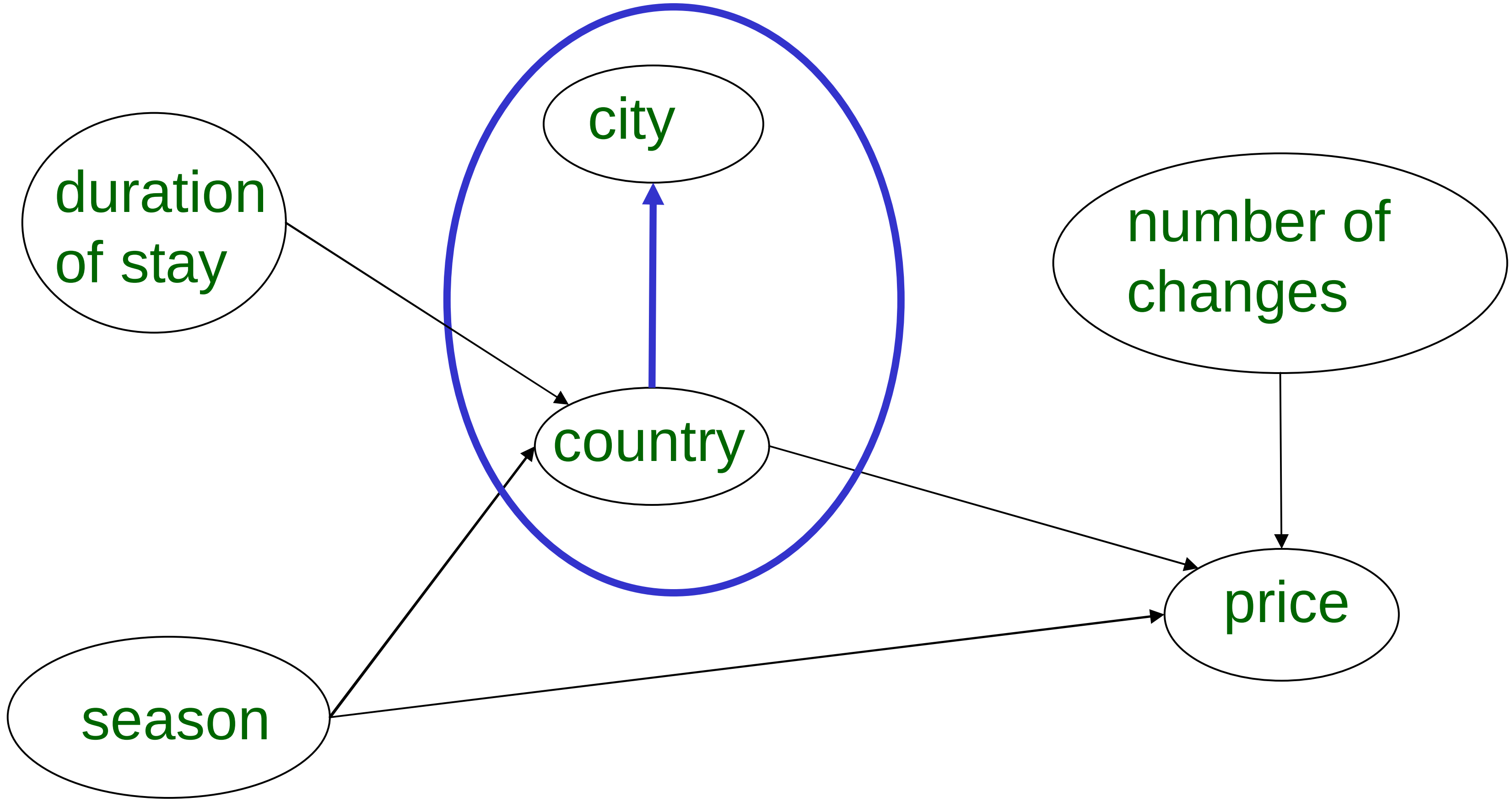
«*Ceteris paribus*, in January I prefer to go to India than to Brazil, to Brazil than to Turkey etc. »

5. « ceteris paribus » preferences: CP-nets



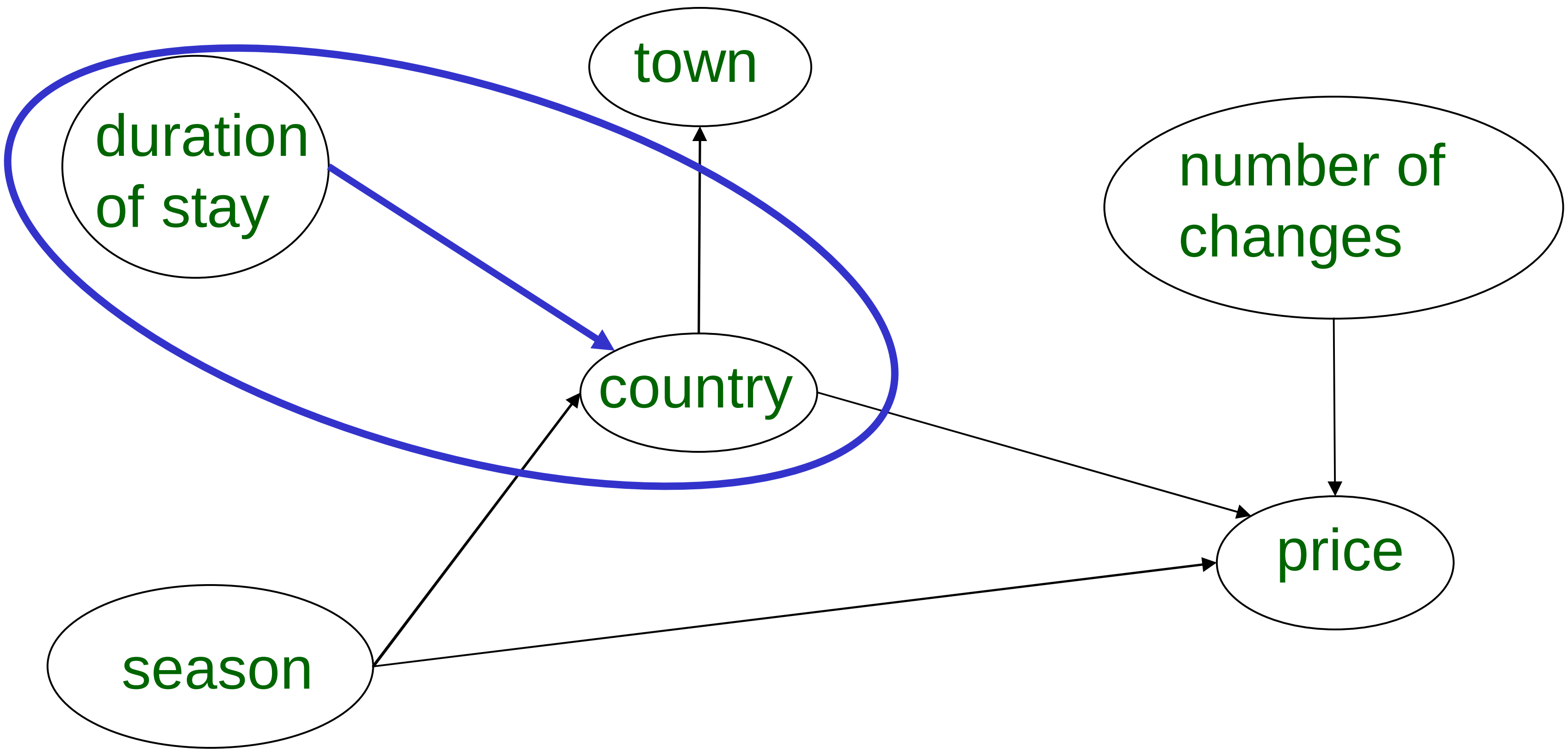
JANUARY ∨ FEBRUARY : INDIA > BRAZIL > TURKEY > RUSSIA
APRIL ∨ MAY : TURKEY > RUSSIA > BRAZIL > INDIA
JUNE ∨ JULY : RUSSIA > TURKEY > INDIA > BRAZIL
...

5. « ceteris paribus » preferences: CP-nets



RUSSIA : ST-PETERSBURG > MOSCOW
INDIA : NEW-DELHI > MADRAS > CALCUTTA
...

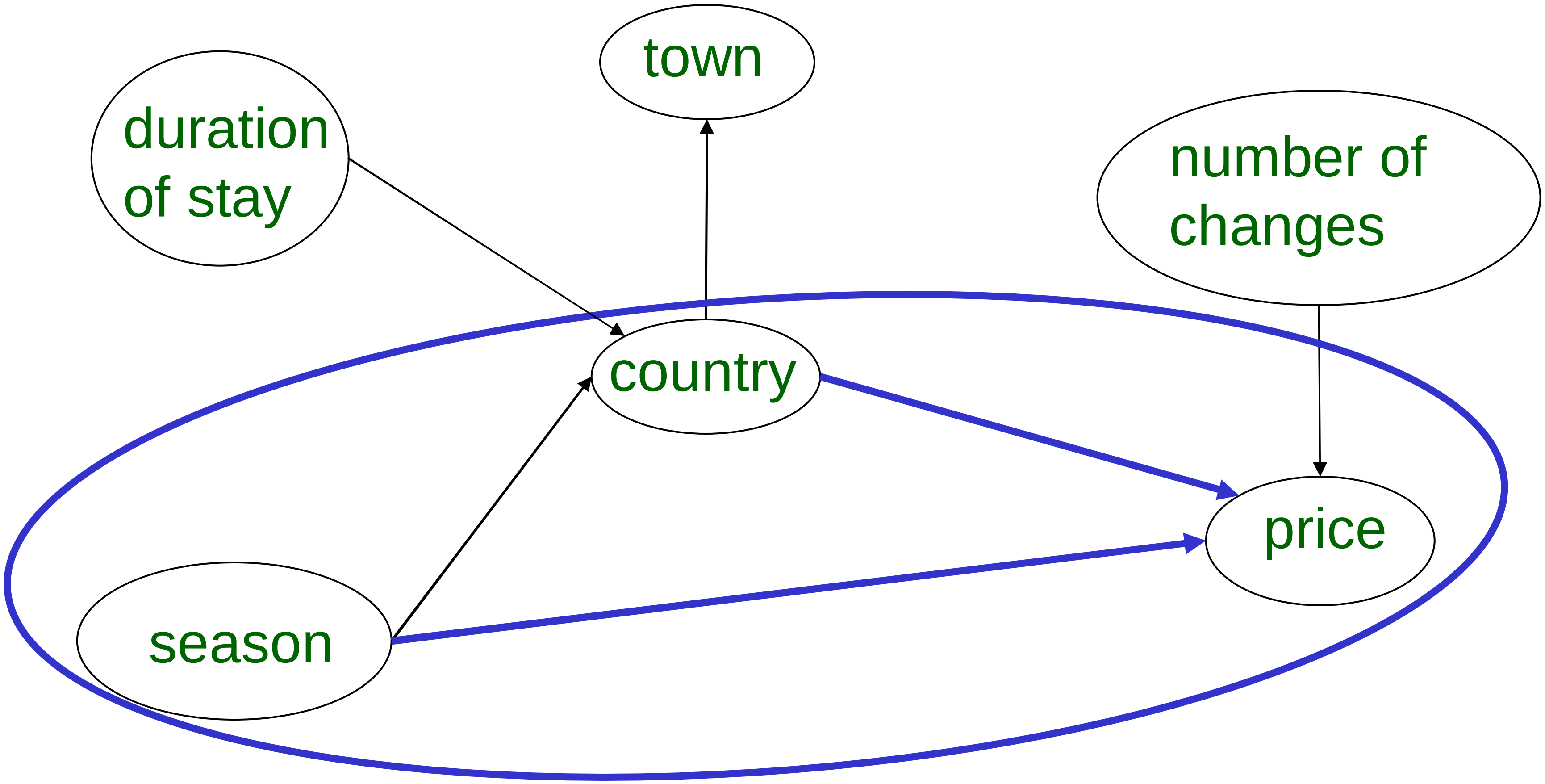
5. « ceteris paribus » preferences: CP-nets



DURATION < 10 DAYS : TURKEY > RUSSIA > INDIA ∨ BRAZIL

...

5. « ceteris paribus » preferences: CP-nets



TURKEY : (PRICE < 350 EUROS)
BRAZIL ∧ DECEMBER : (PRICE < 750 EUROS)
BRAZIL ∧ JUNE : (PRICE < 500 EUROS)
...

Some logical languages for preference representation

6. Conditional desires

$D(\psi \mid \varphi)$: in context ψ, ideally φ is true [Boutilier 94]

R preference relation (complete preorder)

R satisfies $D(\psi \mid \varphi)$ iff $Max(Mod(\varphi), R) \subseteq Mod(\psi)$

Intuitively :

the best states satisfying φ satisfy ψ too

or equivalently

the best states satisfying $\varphi \wedge \psi$
are better than the best states satisfying $\varphi \wedge \neg \psi$

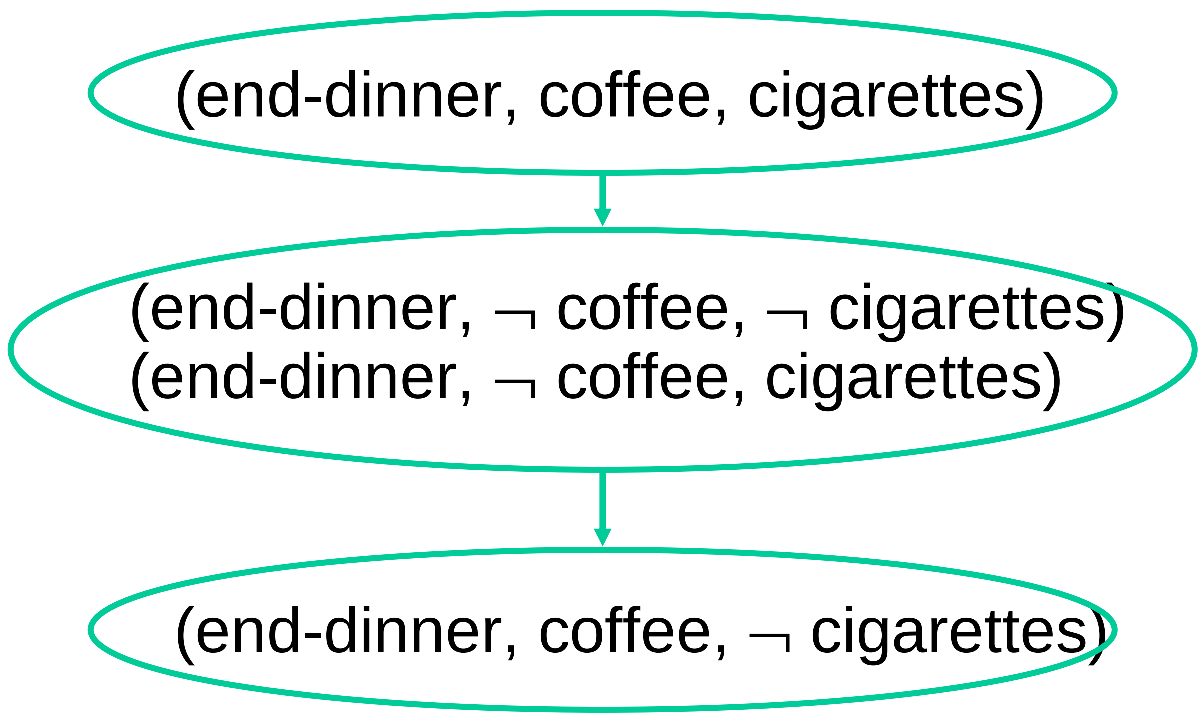
6. Conditional desires

R satisfies D (ψ | φ) iff Max (Mod (φ), R) $\subseteq$ Mod(ψ)

D(coffee | end-dinner)

D($\neg$ coffee | end-dinner $\wedge$ $\neg$ cigarettes)

For instance:



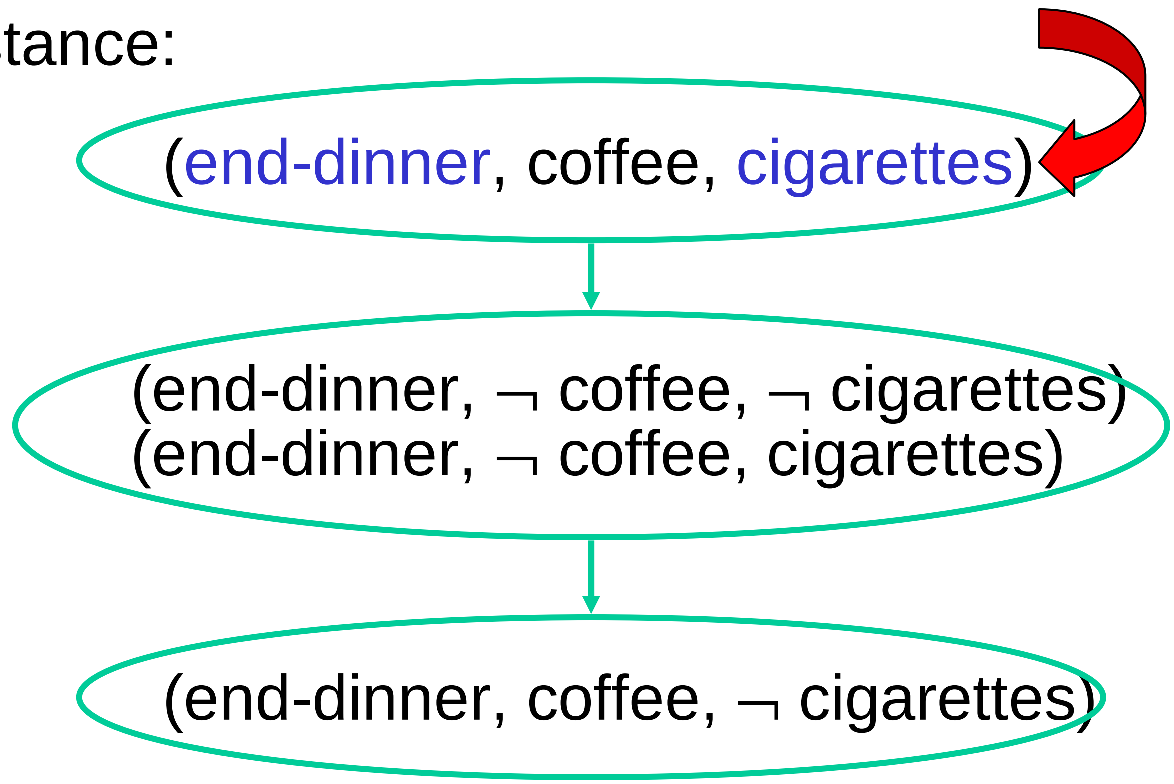
6. Conditional desires

R satisfies D $(\psi \mid \varphi)$ iff Max (Mod (φ), R) $\subseteq$ Mod(ψ)

D(coffee | end-dinner)

D(¬ coffee | end-dinner ∧ ¬ cigarettes)

For instance:



6. Conditional desires

R satisfies $D(\psi \mid \varphi)$ iff $Max(Mod(\varphi), R) \subseteq Mod(\psi)$

D(coffee | end-dinner)

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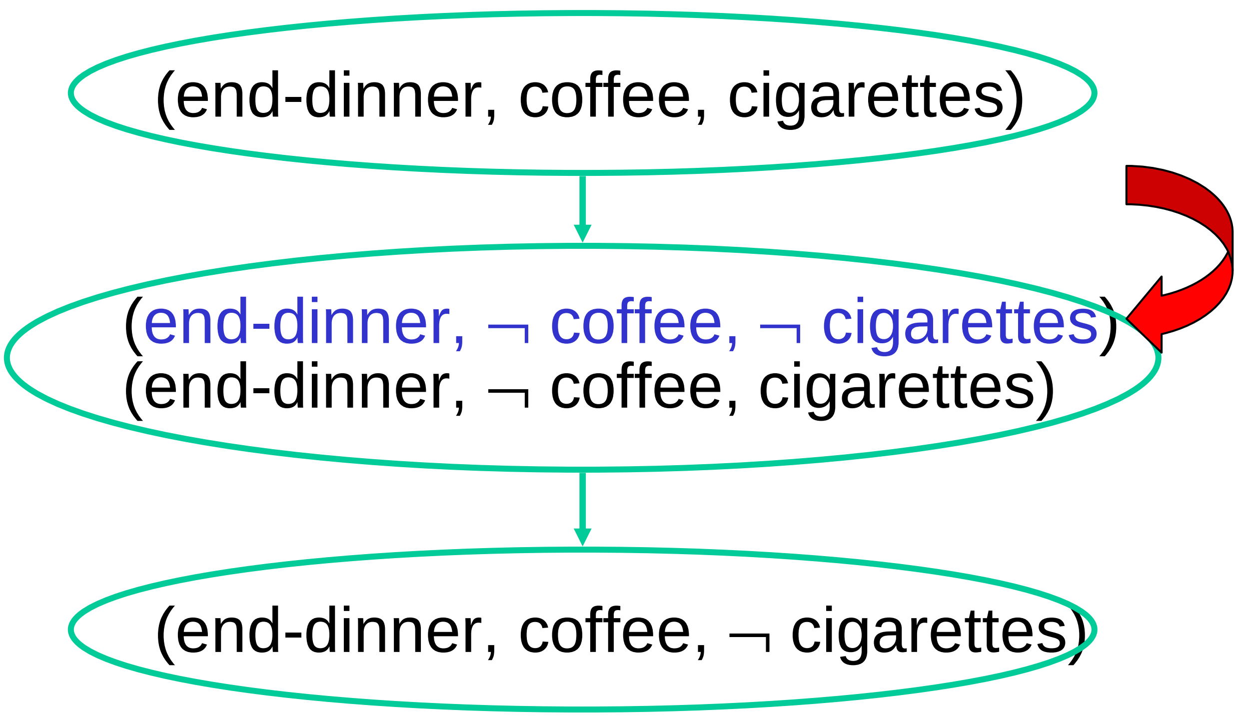
For instance:



6. Conditional desires

« Drowning effect»

D(coffee | end-dinner) ; D($\neg$ coffee | end-dinner $\wedge$ $\neg$ cigarettes)
D(dessert | end-dinner)

(end-dinner, coffee, cigarettes, dessert)

↓

(end-dinner, $\neg$ coffee, $\neg$ cigarettes, dessert)
(end-dinner, $\neg$ coffee, $\neg$ cigarettes, $\neg$ dessert)
(end-dinner, $\neg$ coffee, cigarettes, dessert)

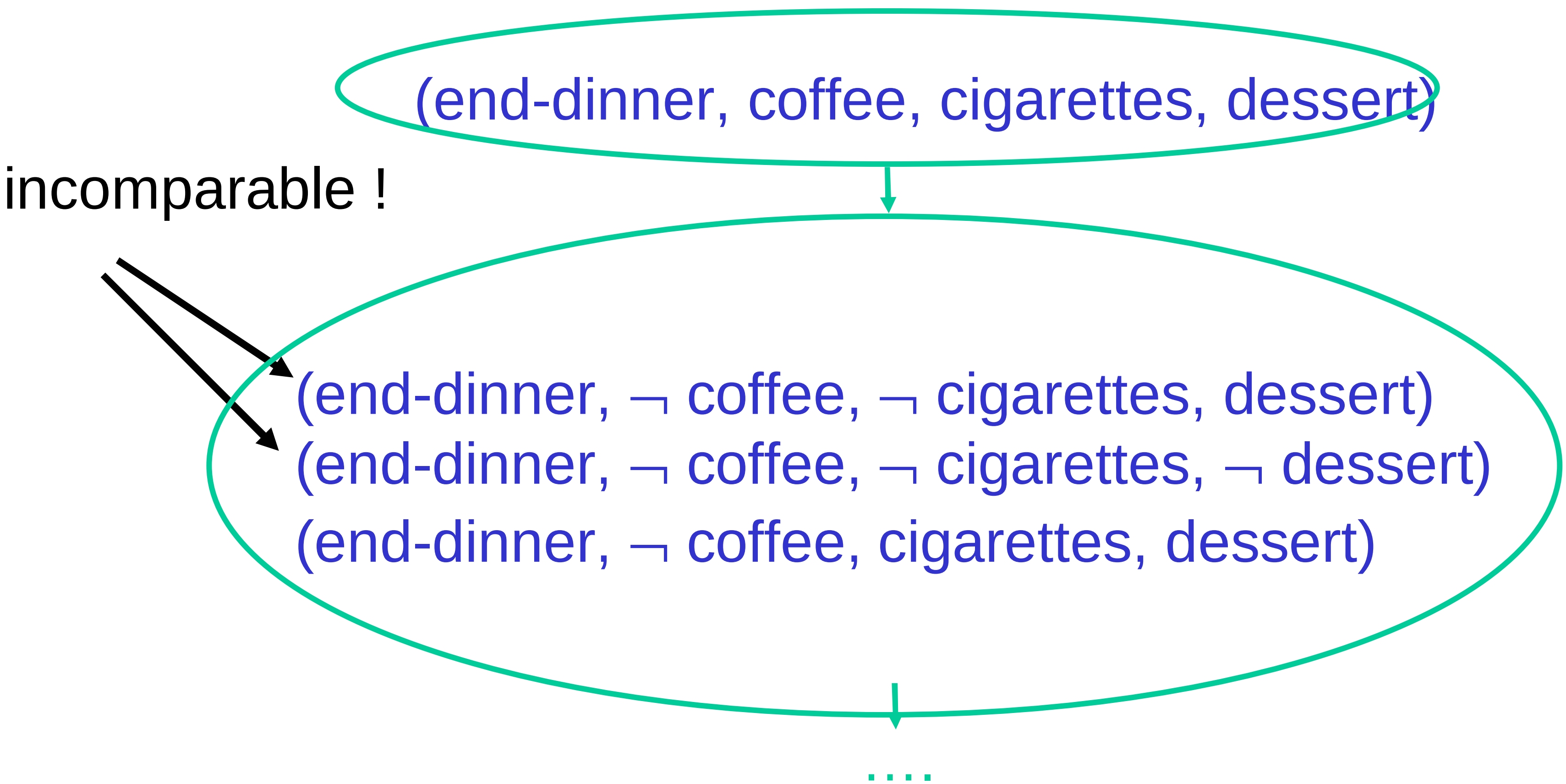
↓

....

6. Conditional desires

« Drowning effect»

D(coffee | end-dinner) ; D($\neg$ coffee | end-dinner $\wedge$ $\neg$ cigarettes)
D(dessert | end-dinner)



6. Conditional desires

« Drowning effect»

D(coffee | end-dinner)
D(¬ coffee | end-dinner ∧ ¬ cigarettes)
D(dessert | end-dinner)

The lack of cigarettes « inhibits » the desire for coffee
but the desire for dessert as well (« inheritance blocking »)

→ need to be improved

[Lang 96; Lang, van der Torre & Weydert 02]

More references about logical preference representation can be found in the paper

Coste-Marquis, Lang, Liberatore & Marquis, KR04

Expressive power and succinctness of propositional languages for preference representation

- About the meaning of preference
- The need for compact representations + the role of logic
- A brief survey on propositional logical languages for preference representation
- **Preference representation and NMR**
- Other issues

Preference representation and NMR

1. Preference representation makes use of *default preferential independence between variables*

As long as no preferential dependence between variables a and b was not explicitely stated, they are considered as preferentially independent

I prefer coffee to tea

(coffee, $\neg$ sugar) > (tea, $\neg$ sugar)

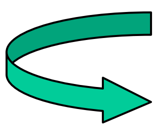
as long as no interaction between drinks and sugar is specified

Preference representation and NMR

1. Preference representation makes use of
default preferential independence between variables

As long as no preferential dependence between variables a and b was not explicitely stated, they are considered as preferentially independent

birds fly



red birds by

as long as no interaction between flying and colour is specified

Preference representation and NMR

2. Are the preference representation languages given in this overview monotonic or nonmonotonic ?

- the preference relation induced by B satisfies $\omega > \omega'$
- $B \subset B'$

$\Rightarrow$ does the preference relation induced by B' satisfy $\omega > \omega'$?

Preference representation and NMR

2. Are the preference representation languages given in this overview monotonic or nonmonotonic ?

- the preference relation induced by B satisfy $\omega > \omega'$
- $B \subset B'$

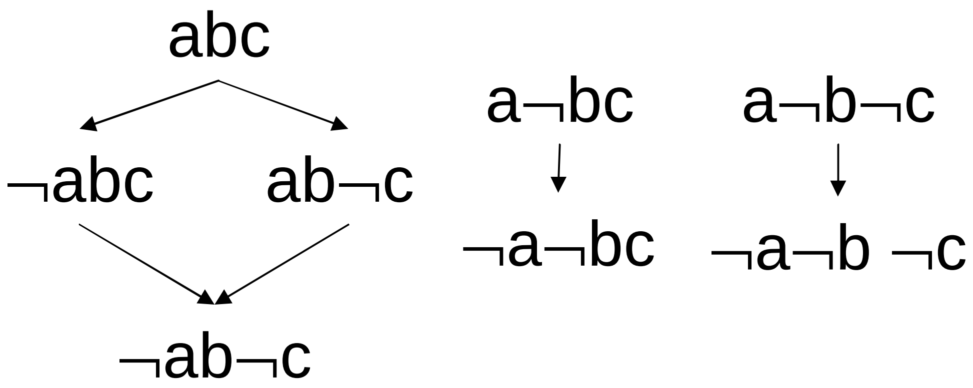
$\Rightarrow$ does the preference relation induced by B' satisfy $\omega > \omega'$?

YES for ceteris paribus statements (and CP-nets)
NO for almost all other languages

Preference representation and NMR

2. Are the preference representation languages given in this overview monotonic or nonmonotonic ?

I prefer a to be true
if b then I prefer c to be true

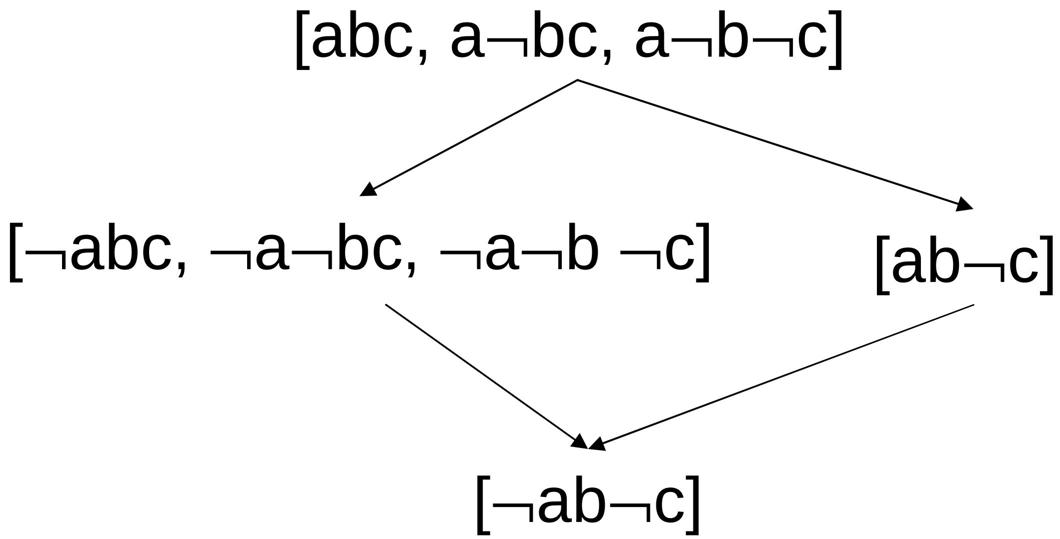


ceteris paribus preferences: monotonic and cautious

Preference representation and NMR

2. Are the preference representation languages given in this overview monotonic or nonmonotonic ?

I prefer a to be true
if b then I prefer c to be true



Preference representation and NMR

2. Are the preference representation languages given in this overview monotonic or nonmonotonic ?

I prefer a to be true
if b then I prefer c to be true

[abc, a¬bc, a¬b¬c]

↓

[¬abc, ¬a¬bc, ¬a¬b ¬c, ab¬c]

↓

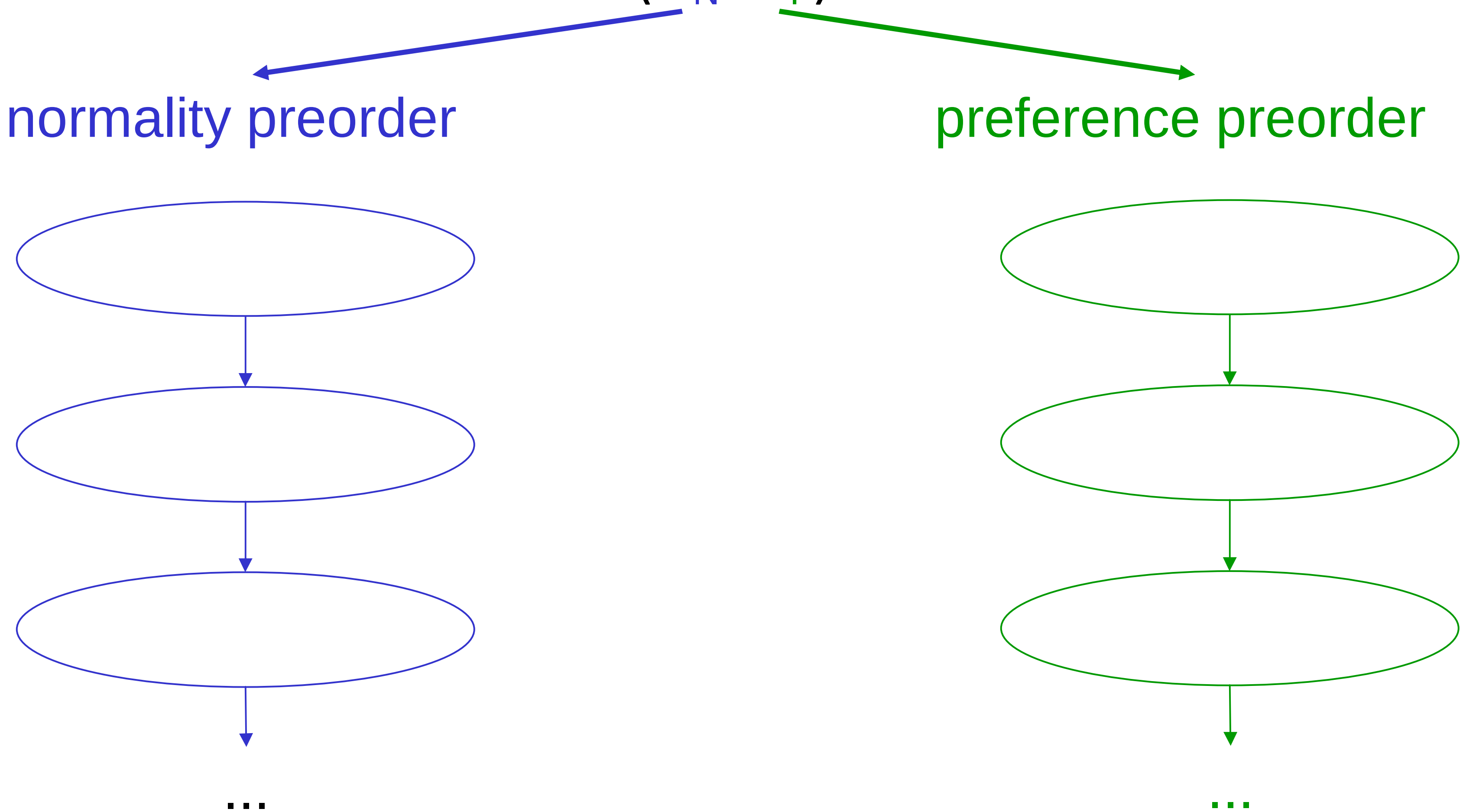
[¬ab¬c]

Preference representation and NMR

3. Hidden uncertainty in the expression of preference (normality and preference)

[Lang, van der Torre & Weydert 03]

$$M = \langle R_N, R_P \rangle$$



Hidden uncertainty in the expression of preference (normality and preference)

$N(\psi \mid \varphi)$: « normally ψ if φ »

$M = \langle R_N, R_P \rangle$ satisfies $N(\psi \mid \varphi)$ ssi $\mathrm{Max}\,(\mathrm{Mod}\,(\varphi), R_N) \subseteq \mathrm{Mod}(\psi)$

in the most normal (« typical ») states
among those where φ is true, ψ is true as well.

...

Hidden uncertainty in the expression of preference (normality and preference)

$P(\psi \mid \varphi)$: « I prefer ψ if φ »

$$M = \langle R_N, R_P \rangle \text{ satisfies } D(\psi \mid \varphi)$$

iff

$$\text{Max}(\text{Max}(\text{Mod}(\varphi), R_N), R_P) \subseteq \text{Mod}(\psi)$$

the preferred states among those where φ is true satisfy ψ

the most normal states where $\varphi \wedge \psi$ is true are preferred to the most normal states where $\varphi \wedge \neg \psi$ is true

...

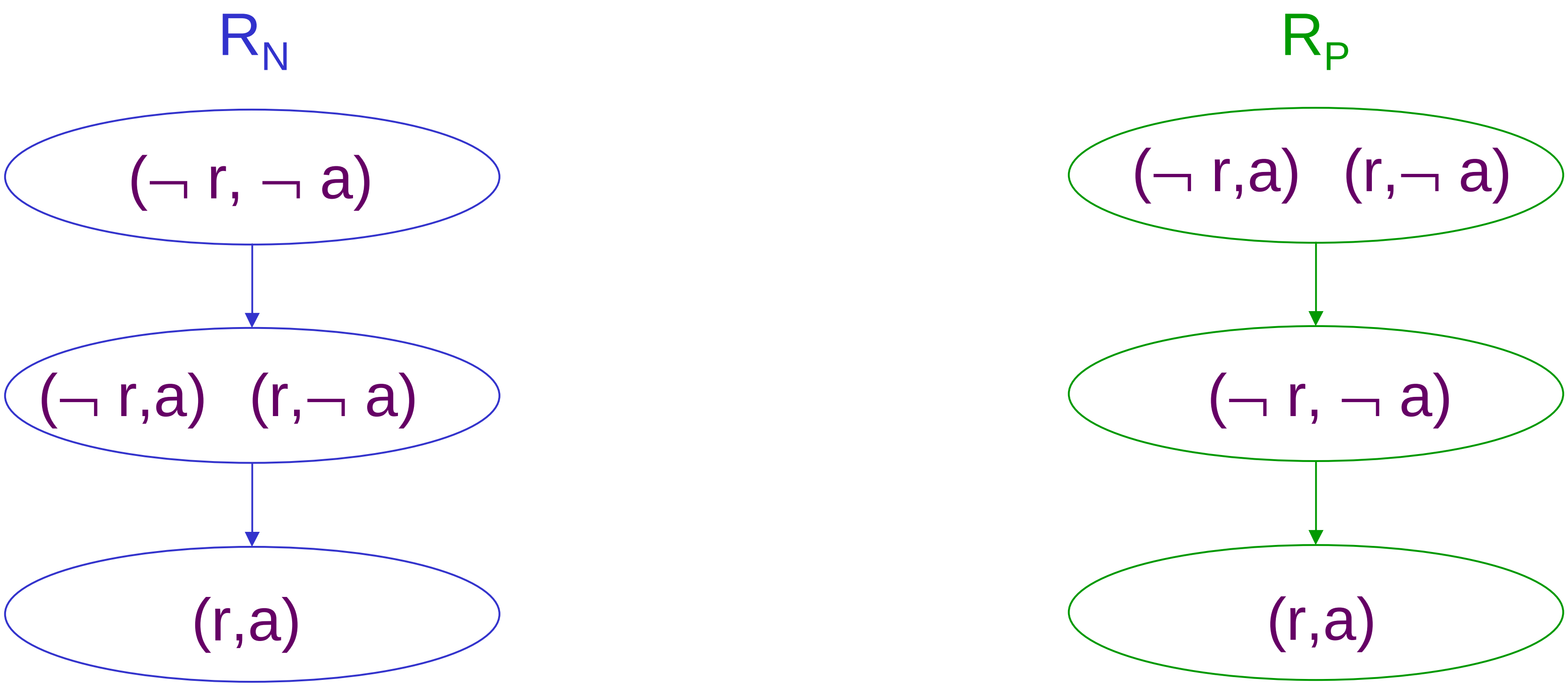
Hidden uncertainty in the expression of preference (normality and preference)

1. I would like an ticket to Rome $D(r)$
2. I would like a ticket to Amsterdam $D(a)$
3. I would not like having both a ticket to Rome and a ticket to Amsterdam $\neg D(r \wedge a)$
4. In the actual situation, I do not have any ticket to Rome nor to Amsterdam. $N(\neg r)$ $N(\neg a)$

...

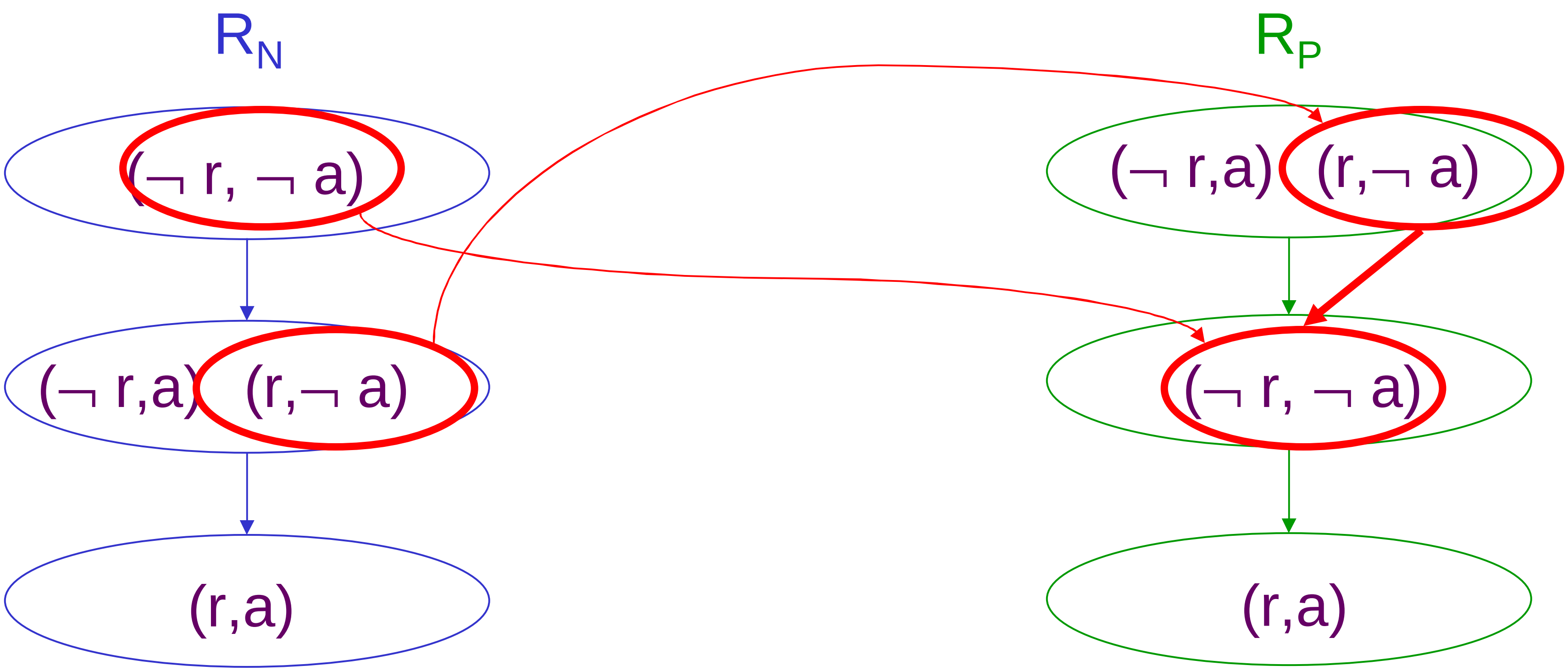
Hidden uncertainty in the expression of preference (normality and preference)

$P(r)$ $P(a)$ $\neg P(r \wedge a)$ $N(\neg r)$ $N(\neg a)$



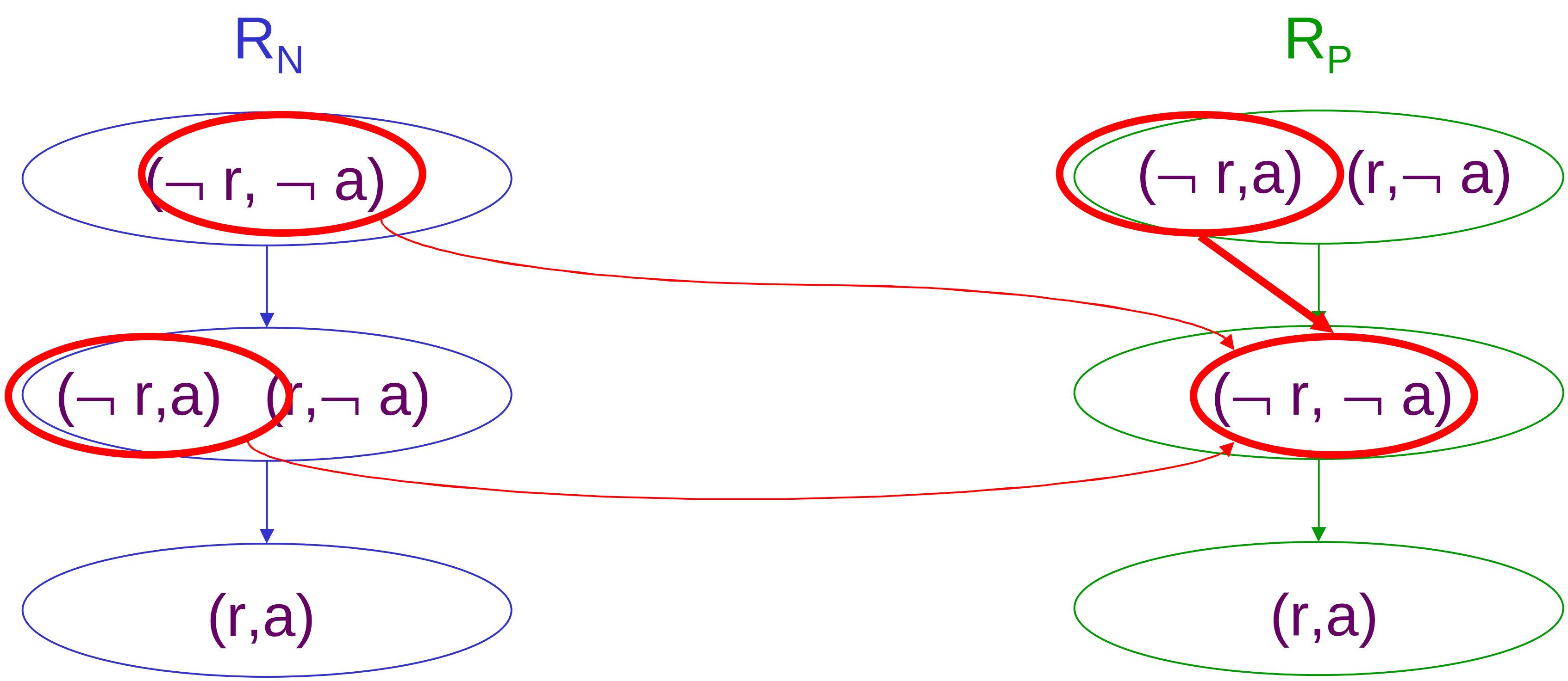
Hidden uncertainty in the expression of preference (normality and preference)

P(r)   P(a) $\neg$ P(r $\wedge$ a)   N($\neg$r) N($\neg$a)

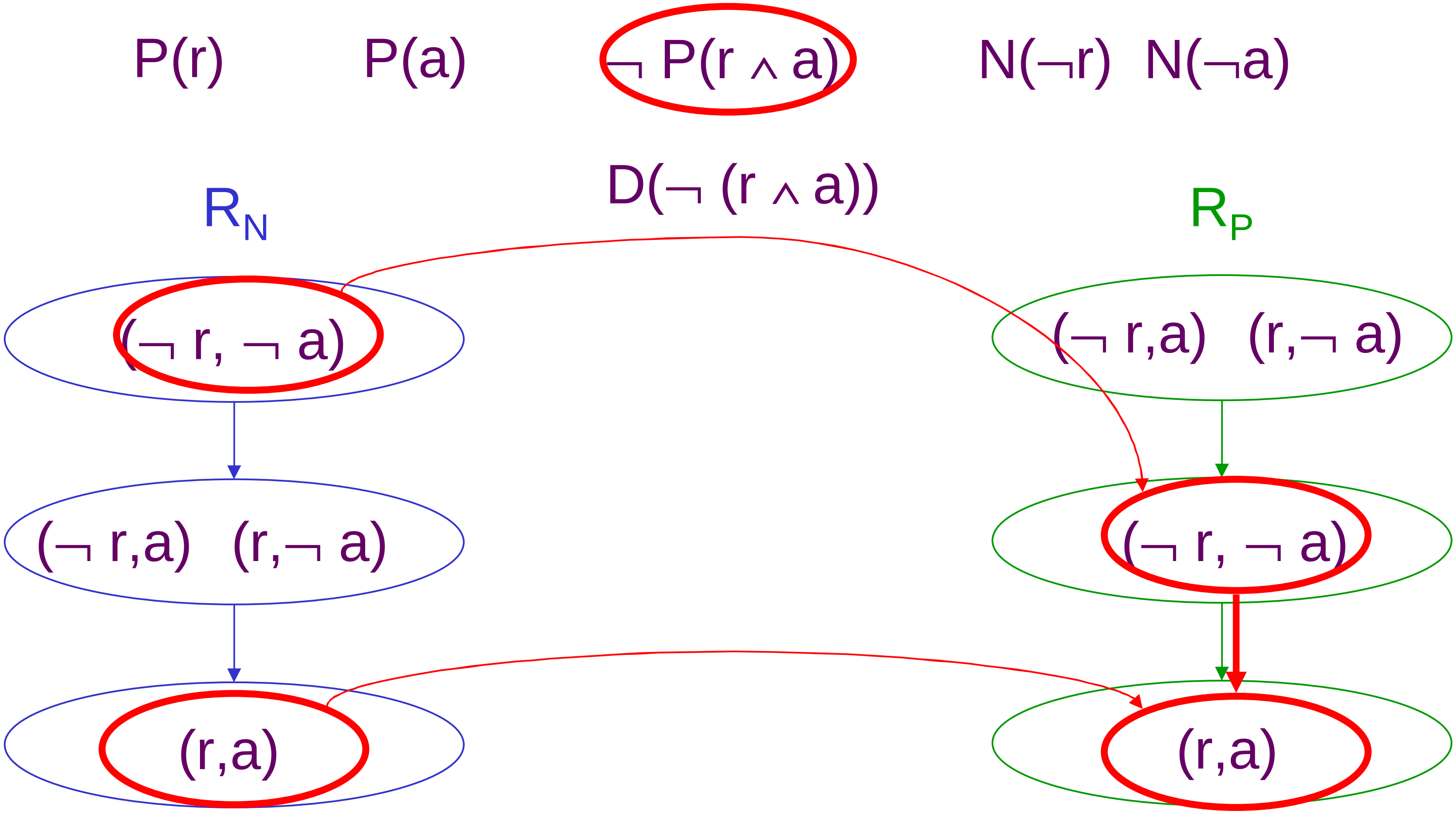


Hidden uncertainty in the expression of preference (normality and preference)

P(r) P(a) ¬ P(r ∧ a) N(¬r) N(¬a)

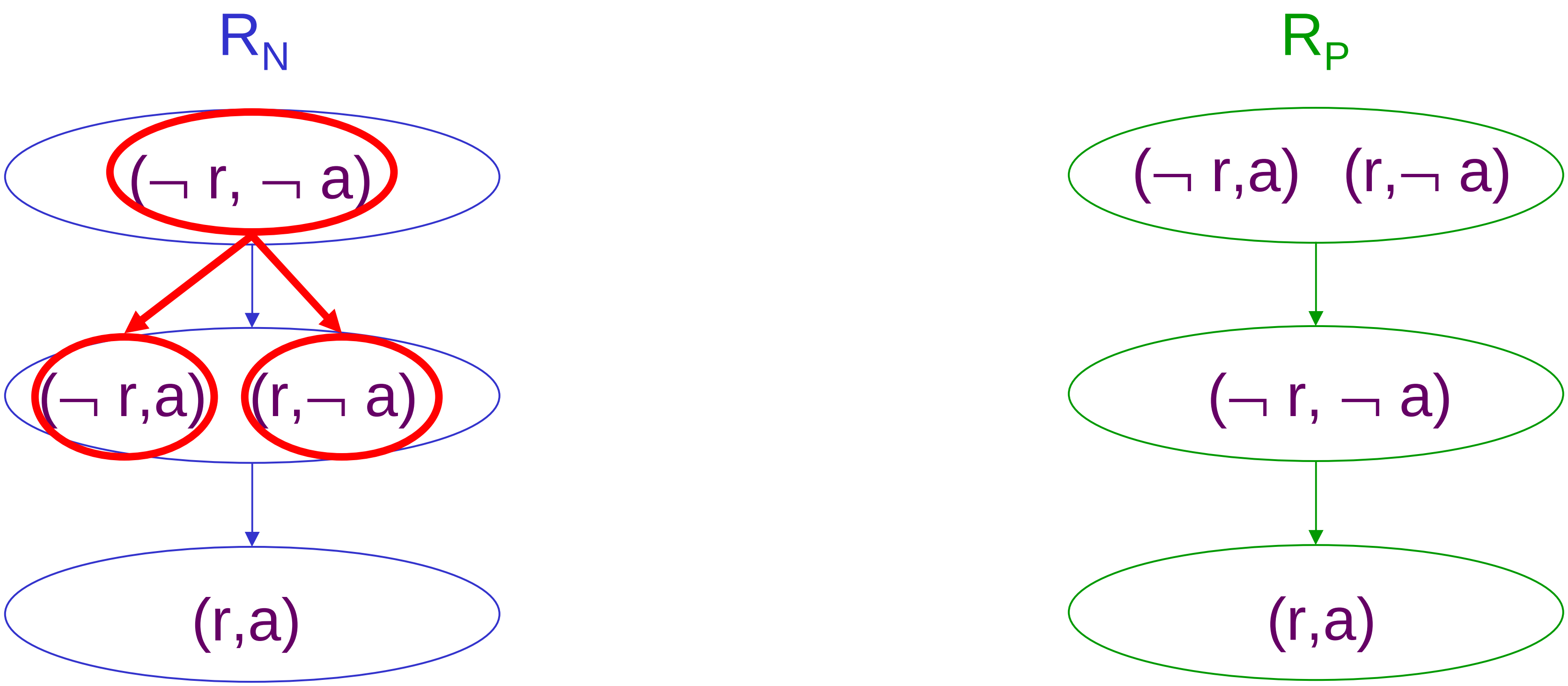


Hidden uncertainty in the expression of preference (normality and preference)



Hidden uncertainty in the expression of preference (normality and preference)

$P(r)$ $P(a)$ $\neg P(r \wedge a)$ $N(\neg r)$ $N(\neg a)$



Preference representation and NMR

4. From belief change to preference change

Does it make sense to revise / update preferences ?

Preference representation and NMR

4. From belief change to preference change

a. revision of beliefs about preferences by preferences

Preference representation and NMR

4. From belief change to preference change

a. revision of beliefs about preferences by preferences

A: I’d like to have a Berliner Weisse, please
B: with green syrup or with red syrup?
A: no syrup please, thanks

B’s beliefs about A’s preferences

before

	green > red > pure
or	red > green > pure
or	red ~ green > pure

→

after

	pure > green > red
or	pure > red > green
or	pure > red ~ green

Preference representation and NMR

4. From belief change to preference change

b. XXXX of preferences by facts

Preference representation and NMR

4. From belief change to preference change

b. XXXX of preferences by facts

[from a discussion with K. Konczak]

A: would you prefer to give your talk on monday or tuesday?
B: well, rather on tuesday
A: I just learned that the pope is visiting the lab
on monday (so that he can attend talks on monday)
B: then I prefer to give the talk on monday

Preference representation and NMR

4. From belief change to preference change

b. XXXX of preferences by facts

did the preference change?

→ depends on the granularity of the language

pope not in the language

tuesday
> monday

pope in the language

(tuesday, pope)
> (monday, pope)
> (tuesday, ¬pope)
> (monday, ¬pope)

Preference representation and NMR

4. From belief change to preference change

b. XXXX of preferences by facts

did the preference change?

→ depends on the granularity of the language

pope not in the language

tuesday
> monday

pope in the language

(tuesday, pope)
> (monday, pope)
> (tuesday, ¬pope)
> (monday, ¬pope)

focusing on the most normal situations

Preference representation and NMR

4. From belief change to preference change

b. XXXX of preferences by facts

did the preference change?

⇒ depends on the granularity of the language

pope not in the language

monday
> tuesday

pope in the language

(tuesday, pope)
> **(monday, pope)**
> (tuesday, ¬pope)
> (monday, ¬pope)

after learning that the pope is visiting the lab on monday

Preference representation and NMR

4. From belief change to preference change

c. ‘temporal change of preferences’

sushis > walk

↓ 3 plates of sushis later

walk > sushis

did preference change?

Preference representation and NMR

4. From belief change to preference change

c. 'temporal change of preferences'

did the preference change?

⇒ depends once again on the granularity of the language!

$\neg$ full: sushis > walk

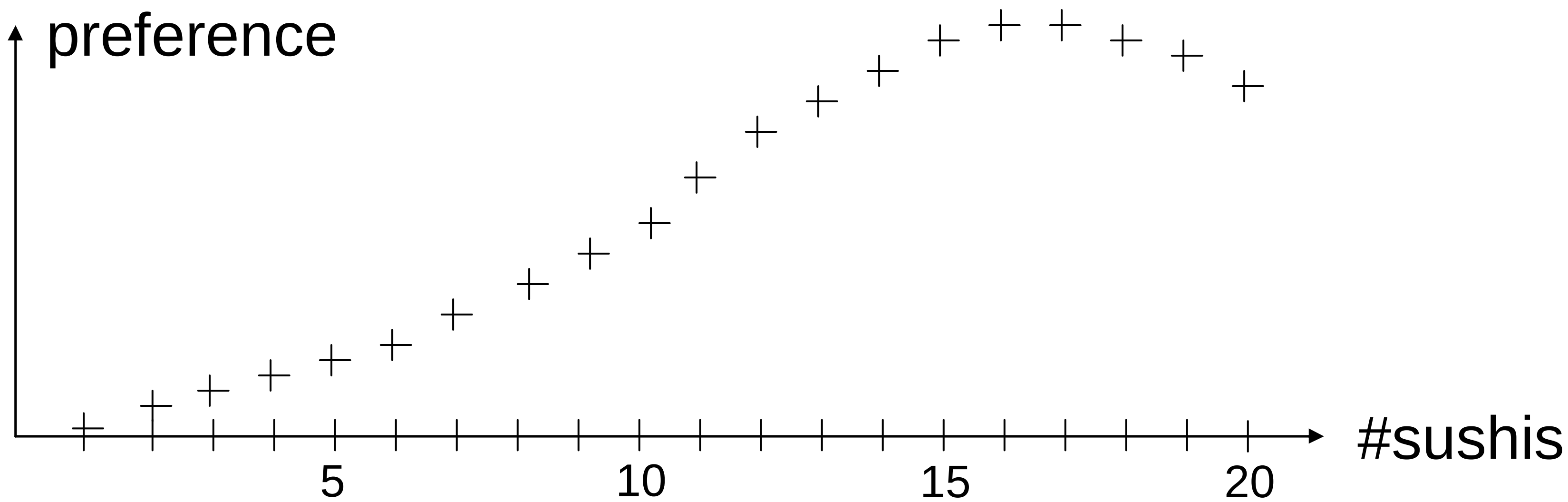
full: walk > sushis

Preferences seem to be much more static than beliefs

- About the meaning of preference
- The need for compact representations + the role of logic
- A brief survey on propositional logical languages for preference representation
- Preference representation and NMR
- **Other issues**

Logical representation of more sophisticated preferences

1. Variables with numerical domains (or even continuous)



but prefers a few sushis less if there is green tea ice-cream on the menu

using fuzzy (ordinal or cardinal) quantities / quantifiers

Logical representation of more sophisticated preferences

1. Variables with numerical domains (or even continuous)

Extending existing languages?

Probably easier for

- (functional) weights
- distances

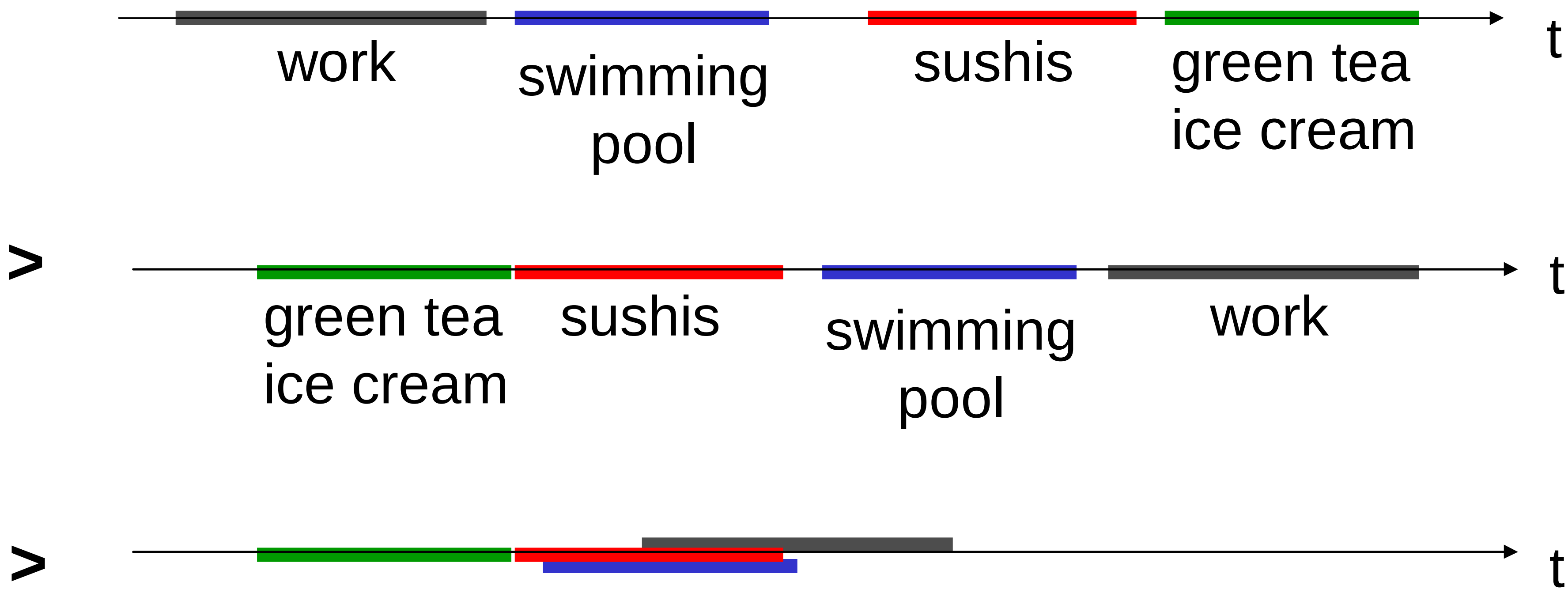
than with

- priorities
- conditionals
- ceteris paribus statements

Logical representation of more sophisticated preferences

2. Temporal preferences

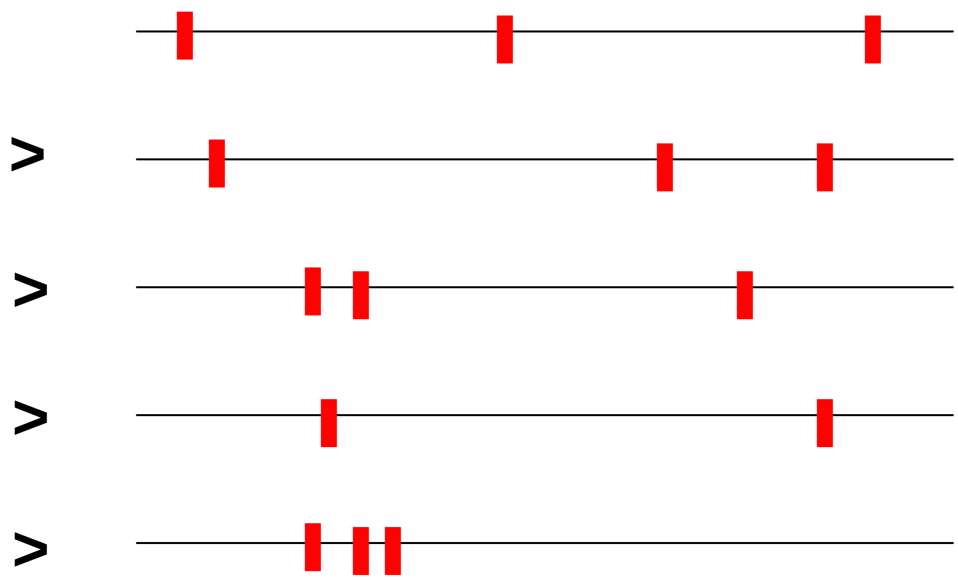
cf. [Delgrande, Schaub & Tompits, KR2004]



Logical representation of more sophisticated preferences

2. Temporal preferences

I'd like to have three coffee breaks today but with some regularity



Logical representation of more sophisticated preferences

3. Integrating ordinal and cardinal preference: compact representation of fuzzy relations over propositional domains

$\mu_P : 2^{VAR} \times 2^{VAR} \rightarrow [0,1]$

$\mu_P(\omega,\omega') \in [0,1]$ degree to which x is at least as good as y

some assumptions that may be imposed (or not)
such as

transitivity $\mu_P(\omega,\omega'') \geq \min(\mu_P(\omega,\omega'), \mu_P(\omega',\omega''))$

Logical representation of more sophisticated preferences

3. Integrating ordinal and cardinal preference: compact representation of fuzzy relations over propositional domains

$\mu_P(\omega,\omega') \in \{0,1\}$	⇒	(partial) weak order
$\mu_P(\omega,\omega') \in \{0,1\}$ $\mu_P(\omega,\omega') + \mu_P(\omega',\omega) \geq 1$	⇒	complete weak order
$\mu_P(\omega,\omega') = \mu_P(\omega,\omega'')$ $(= u(\omega))$ for all ω,ω',ω''	⇒	utility function

Logical representation of more sophisticated preferences

3. Integrating ordinal and cardinal preference: compact representation of fuzzy relations over propositional domains

Can existing representation languages for ordinal / cardinal preferences be integrated / extended so as to represent fuzzy relations over alternatives?

Logical representation of more sophisticated preferences

4. Epistemic preferences

cf. Isaac Levi ’s epistemic utilities

> preference relation over belief states
u set of belief states $\rightarrow \Re$

- can be action-directed

  - *I’d like to know where the nearest sushi place is*

  - *I ’d like to know if there is already sugar in my coffee*

  - *John wants to know whether Mary still loves him*

Logical representation of more sophisticated preferences

4. Epistemic preferences

> preference relation over belief states

u set of belief states $\rightarrow \mathfrak{R}$

- can be action-directed
- or not

- *I'd like to know why the British drive left*

- *but I'd prefer to know who won Roland-Garros*

Logical representation of more sophisticated preferences

4. Epistemic preferences

> preference relation over belief states

u set of belief states $\rightarrow \Re$

- can be action-directed
- or not

- *I don't want to learn whether I passed the exam or not before I'm back from my holiday*

- *I learn that I passed the exam*
  > I keep on ignoring whether I passed the exam
  > I learn that I failed the exam

Logical representation of more sophisticated preferences

5. Preferences involving other agents

- preferences about others' epistemic state

John would prefer the fishy man behind him keep on ignoring his credit card secret code

Mary would like John to know that she loves him but before all she does not want Peter to learn about that

Mary would like John to have a not-too-strong belief that she loves him (and prefers a state where John does not have any clue to a state where he is fully sure that she loves him).

Logical representation of more sophisticated preferences

5. Preferences involving other agents

- preferences about others' epistemic state
- preferences about others' preferences

John prefers a state where Mary prefers to marry him to a state where she prefers to marry Peter

Logical representation of more sophisticated preferences

5. Preferences involving other agents

- preferences about others' epistemic state
- preferences about others' preferences

COMPACT REPRESENTATION ?

Going further than compact representation

1. **Bridging preference representation, elicitation, and optimization**

Going further than compact representation

1. Bridging preference representation, elicitation, and optimization

2. Integrating preference representation languages with uncertainty representation languages
   $\Rightarrow$ decision under uncertainty

Going further than compact representation

1. **Bridging preference representation, elicitation, and optimization**

2. **Integrating preference representation languages with uncertainty representation languages**
   $\Rightarrow$ **decision under uncertainty**

3. **Logical preference representation + social choice**
   a. preference representation & merging

   - aggregating logically-expressed individual preferences (existing approaches to merging $\Rightarrow$ only for simple preference representation languages
   - logical view of manipulation and strategyproofness [Everaere, Konieczny & Marquis, KR2004]

Going further than compact representation

1. Bridging preference representation, elicitation, and optimization

2. Integrating preference representation languages with uncertainty representation languages
   $\Rightarrow$ decision under uncertainty

3. Logical preference representation + social choice
   a. preference representation & merging
   b. application to fair division
   c. application to vote

Going further than compact representation

1. Bridging preference representation, elicitation, and optimization

2. Integrating preference representation languages with uncertainty representation languages
   $\Rightarrow$ decision under uncertainty

3. Logical preference representation + social choice
   a. preference representation & merging
   b. application to fair division
   c. application to vote

3. Logical preference representation + fair division (+ combinatorial auctions)

A = {1,…, N} set of agents
G = {g_1, …, g_p} set of indivisible goods

Find a fair division

$$D: G \rightarrow A$$

given
- some constraints on feasible divisions
- the preferences of the agents
- some fairness of efficiency criteria

Needs compact preference representation!

3. Logical preference representation + fair division

$$\geq : 2^G \rightarrow A$$

Dependencies (non-additivity of $\geq$)

additivity

A, B, C disjoints subsets of G and A > B
$\Rightarrow (A \cup C) > (B \cup C)$

{coffee} ??? {cookie}

3. Logical preference representation + fair division

$$\geq : 2^G \rightarrow A$$

Dependencies (non-additivity of $\geq$)

~~additivity~~

~~A, B, C disjoints subsets of G and A > B~~
~~$\Rightarrow (A \cup C) > (B \cup C)$~~

{coffee} > {cookie}

{coffee, tea} ??? {cookie, tea}

3. Logical preference representation + fair division

$$\geq : 2^G \rightarrow A$$

Dependencies (non-additivity of $\geq$)

additivity

A, B, C disjoints subsets of G and A > B
$\Rightarrow (A \cup C) > (B \cup C)$

{coffee} > {cookie}

{coffee, tea} < {cookie, tea}

positive synergy between *tea* and *cookie*
and/or negative synergy between *tea* and *coffee*

Going further than compact representation

1. Bridging preference representation, elicitation, and optimization

2. Integrating preference representation languages with uncertainty representation languages
   $\Rightarrow$ decision under uncertainty

3. Logical preference representation + social choice
   a. preference representation & merging
   b. application to fair division
   c. application to vote