THE LOGIC OF RISKY KNOWLEDGE

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I know that the last plane to L.A. leaves at 10:00 P.M.

I know that the table is $63.40 \pm .15$ inches long.

You know perfectly well that it is not raining outside.

Mary knows that John loves her.

We all know that the proportion of male births is between 50.3% and 50.8%.

Since we just obtained data in the 0.01 region, we know that H_0 is false.

The specific gravity of pure copper is known.

The mass of the electron is known.

BUT NOT

I know that the last plane to LA will leave at exactly 10:00.

I know that the table is 63.40 inches long.

You know that it will rain Saturday, since we are planning a picnic.

I know that the wheel will land on red.

We don t really know the conductivity of copper, since our value might be in error.

What is the Logic of Rational Knowledge?

What is the Logic of Scientific Knowledge?

What is the Logic of Inductive Knowledge?

A Prior question: **What do we mean by ``The Logic of ?**

Assumptions

1. Some statements are **ACCEPTED.**

2. Acceptance is based on **EVIDENTIAL PROBABILITY.**

3. Evidential Probability

   (a) Domain: $\mathcal{L} \times \mathcal{P}(\mathcal{L})$
   Statements
   Bodies of Evidence

   (b) Range:
   Intervals $[p, q]$

Accept S, given E if and only if The lower probability value of S, relative to E, is greater than $1 - \epsilon$

- **Body of Evidence: Γ_δ**
  Risk of error: δ

- **Body of Knowledge: Γ_ϵ**
  Risk of error: ϵ

The risk of error in evidence should be less than the risk of error in what is inferred from it: $\delta < \epsilon$.

Prob$(S, \Gamma_\delta) = [p, q]$ if and only if

$S \leftrightarrow Ta \in \Gamma_\delta$

$Ra \in \Gamma_\delta$

Statistics Tx, Rx, p, $q \in \Gamma_\delta$

All conflicting reference classes resolved.

Measure the table by procedure m of kind M.
Length is 63.40 $\pm$ 0.015 iff error of $m \leq$ 0.015.
There is no conflicting calculation of error.

All probabilities are conditional (on Γ_δ)

Conditional Probabilities are not ratios.

Objectivity:

Every probability is based on frequencies that are known in Γ_δ to hold in the world.

If S is in Γ_δ then S is in Γ_ϵ.

The logical structure of Γ_ϵ may be the same as that of Γ_δ

The parameters δ and ϵ are construed as *constants*, not as variables that ``approach 0.

Circularity? No. Regress? Yes. To justify $S \in \Gamma_{0.01}$ we need evidence in Γ_δ with $\delta < 0.01$, say 0.005. To justify $T \in \Gamma_{0.005}$ we need evidence in Γ_η where $\eta < 0.005$

Properties of Probability

1. Given a body of evidence Γ_δ, every S has a probability.

2. Probability is unique: If Prob$(S, \Gamma_\delta) = [p, q]$ and Prob$(S, \Gamma_\delta) = [r, s]$, then $p = r$ and $q = s$.

3. If $S \leftrightarrow T$ is in Γ_δ then Prob$(S, \Gamma_\delta) =$ Prob(T, Γ_δ).

4. If $Prob(S, \Gamma_\delta) = [p, q]$ then Prob$(\neg S, \Gamma_\delta) = [1 - q, 1 - p]$.

5. If S entails T, Prob$(S, \Gamma_\delta) = [p_S, q_S]$ and Prob$(T, \Gamma_\delta) = [p_T, q_T]$ then $p_T \geq p_S$.

ϵ-acceptability

D-1 $\Gamma_\epsilon = \{S : \exists p, q(\mathrm{Prob}(S, \Gamma_\delta) = [p, q]$
$\wedge\, p/geq1 - \epsilon\}$

T-1 $S \in \Gamma_\epsilon \leftrightarrow \exists p, q(\mathrm{Prob}(\neg S, \Gamma_\delta) = [p, q]$
$\wedge\, q \leq \epsilon$. Risk.

T-2 If $S \in \Gamma_\epsilon$ and $S \vdash T$ and $S \vdash T'$ then
$T \wedge T' \in \Gamma_\epsilon$. Limited adjunction.

T-3 It is possible that $S \in \Gamma_\epsilon$ and $T \in \Gamma_\epsilon$ but
$S \wedge T \notin \Gamma_\epsilon$. Adjunction fails in general.

T-4 Γ_ϵ is not deductively closed.

T-5 If Σ is not empty and contains the first order consequences of any statement in it, then Σ is closed under conjunction if and only if Σ is deductively closed.

Is **Adjunction** basic to any logic?

Is the failure of **Deductive Closure the failure of logic?**

There are general truths that hold of sets of sentences satisfying **D-1**

It is often said that **nonmonotonic logic** is the logic that holds for scientific knowledge.

Nonmonotonic logics have found interpretations in **modal logic.**

The operator $\Box$ is sometimes construed as ``It is known (or believed) that... .

We will look at modal logic for inspiration.

CLASSICAL SYSTEMS
(CHELLAS)

AXIOM:

Df $\Diamond$: $\Diamond S \leftrightarrow \neg\Box\neg S$

RULE OF INFERENCE:

RE: $$\frac{A \leftrightarrow B}{\Box A \leftrightarrow \Box B}$$

(This is the System **E**.)

Every Classical System satisfies

REP: $$\frac{B \leftrightarrow B'}{A \leftrightarrow A[B/B']}$$

where $A[B/B']$ is A with some occurrences of B replaced by B'.

NEIGHBORHOOD MODELS

A *tuple* $\mathcal{M} = \langle W, N, P \rangle$ is a neighborhood model if and only if

1. W is a set [a set of worlds];

2. $N : W \rightarrow 2^{2^W}$ is a function from the set of worlds to sets of sets of worlds [the neighborhood function; if $w \in W$, then $N(w)$ is a set of sets of worlds, i.e., i.e., a set of propositions];

3. $P : W \times \mathcal{P} \rightarrow \{0, 1\}$ is a function from the set of worlds and the set of propositional constants to the set of truth values [truth assignment function].

Three schemata are of interest to us:

M: $\Box(A \wedge B) \to (\Box A \wedge \Box B)$

C: $(\Box A \wedge \Box B) \to \Box(A \wedge B)$

N: $\Box\top$

There are three correspoonding constraints on Neighborhoods:

(m): If $S \wedge T \in N(w)$, then $S \in N(w)$ and $T \in N(w)$

(c): If $S \in N(w)$ and $T \in N(w)$, then $S \wedge T \in N(w)$

(n): $W \in N(w)$

KNOWLEDGE AS ϵ-ACCEPTABILITY

Let Γ_δ represent the set of statements that constitute our total evidence. This is what we take for granted. The subscript δ suggests that the items in this set may not be regarded as ``certain in any absolute sense, but may admit risk up to δ.

Let Γ_ϵ be the set of sentences that, given Γ_δ, we regard as **acceptable** or practically certain.

Interpret $\Box_\epsilon S$ as `S is a practical certainty or S is scientifically known:

D-2 $\Box_\epsilon S$ iff $S \in \Gamma_\epsilon$.

Γ_ϵ is not a CLASSICAL SYSTEM, because it is not closed under the rules of propositional logic.

$\Gamma^*_\epsilon = \{\Box_\epsilon S : S \in \Gamma_\epsilon\}$

$\sum_\epsilon = \{S : \Gamma^*_\epsilon \vdash_{EMN} S\}$ **is a classical system.**

T-6 $\Box_\epsilon S \leftrightarrow \exists p, q(\text{Prob}(S, \Gamma_\delta) = [p, q] \wedge p \geq 1 - \epsilon)$

Correspondingly, $\Diamond_\epsilon S$ means that S is scientifically possible:

T-7 $\Diamond_\epsilon S \leftrightarrow \exists r, s(\text{Prob}(S, \Gamma_\delta) = [r, s] \wedge r > \epsilon)$

T-8 $\Diamond_\epsilon S \leftrightarrow \neg\Box\neg S$ This is Df $\Diamond$

Proof: properties of probability 1, 2, and 4.

T-9 The rule **RE** preserves validity:

If $A \leftrightarrow B$ is valid, so is $\Box A \leftrightarrow \Box B$

Proof: Property 3 of probability and the second assmption.

T-10 $\Box_\epsilon(A \wedge B) \rightarrow (\Box_\epsilon A \wedge \Box_\epsilon B)$

This is schema M.

Proof: Property 5 of probability, and the fact that $A \wedge B$ entails A and B, or theorem 2.

T-11 $\Box_\epsilon \top$

This is schema N.

Proof: $\top \leftrightarrow$ \`$a \in \{a\}$ ; \`%($\{a\}, \{a\}, 1.0, 1.0$) , \`$a \in \{a\}$ $\in \Gamma_\delta$

T-12 $\Box_\epsilon A \wedge \Box_\epsilon B \rightarrow \Box_\epsilon(A \wedge B)$ is NOT valid in Γ_ϵ.

Proof: Let Γ_δ describe an urn with 100 balls, one of which is black. Let S be the statement that the first draw (with replacement) yields a non-black ball, and T the statement that the second draw (with replacement) yields a non-black ball. Let ϵ be 1/100. It is clear that $\Box_{0.01} S$ and $\Box_{0.01} T$ but we do not have $\Box_{0.01}(S \wedge T)$.

The logic of the $\Box_\epsilon$ operator satisfies **E**, **M**, **N**, as well as the rule **RE**. It does not satisfy **C**.

Chellas calls systems satisfying **M** monotonic. But if we look at Γ_ϵ it is in ordinary terms non-monotonic: We can have $S \in \Gamma_\epsilon$, expand the evidence Γ_δ by T and no longer have $S \in \Gamma_\epsilon$.

MONOTONIC

OR

NONMONOTONIC

?

Monotonicity

$T \subset T' \wedge T \models S \rightarrow T' \models S$
(Lukaszewicz, p. 33. But what is $T \subset T'$?)

$T \subset T' \wedge T \vdash S \rightarrow T' \vdash S$
But this is a commonplace about `proof .

The denial of

$$T \subset T' \wedge P(S,T) > 1 - \epsilon \rightarrow P(S,T') > 1 - \epsilon$$

has been called nonmonotonicity by subjectivists, but since there is only one probability function P involved, and there is no reason that the ratio $P(S \wedge T)/P(T)$ should have any particular relation to the ratio $P(S \wedge T')/P(T')$ this doesn t seem to nonmonotonicity at all.

$(A \Rightarrow B) \rightarrow ((A \wedge A') \Rightarrow C))$
This principle is false in some conditional logics; is that a help?

($\Gamma_\delta \subset \Gamma'_\delta$ and Accept S given Γ_δ) $\rightarrow$ Accept S given Γ'_δ
This is monotonicity for acceptance by way of deduction, if we construe Γ_δ and Γ'_δ as sets of premises. Its denial, for induction, is:

For some $\Gamma_\delta \subset \Gamma'_\delta$, $S \in \Gamma_\epsilon$ given Γ_δ but $S \not\in \Gamma_\epsilon$ given Γ'_δ.
I.e., given some evidence S is practically certain, but given further evidence, it is no longer practically certain.

Note that separating the premises Γ_δ from the conclusion Γ_ϵ is crucial.

MODEL THEORY

If $\mathcal{M} = \langle W, N, P \rangle$ is a minimal model, its *supplementation* $\mathcal{M}^+ = \langle W, N^+, P \rangle$ is the minimal model in which for every $\alpha \in W$, N_α^+ contains all the subsets of W that include members of N_α.

The system EMN corresponding to ϵ-acceptance is sound for supplemented minimal models in which every neighborhood in every world contains W. (Chellas)

This suggests that the logical system $\sum_\epsilon$ is the strongest system that characterizes the logical relationships within Γ_ϵ.

But it doesn t mean that there are no more useful things to be said about bodies of rational knowledbge. There may be other schemata beside **M** and **N** we could find justified, and that would require other constraints on our models than **m** and **n**.

But I have been unable to come up with any, so I ll stop here.