ABSTRACTS

Knots in Vancouver, July 19 - 23, 2004

at the Pacific Institute for the Mathematical Sciences, UBC campus.

Colin Adams, Williams College

Surfaces in Hyperbolic Knot Complements

Prime non-satellite non-torus knots are hyperbolic. We will look at how essential surfaces in their complements can inherit two-dimensional hyperbolic metrics, either as pleated or totally geodesic surfaces, and what that tells us about the knots.

Stephen J Bigelow, University of California, Santa Barbara

Beyond BMW

Jones discovered his famous polynomial while studying the Iwahori-Hecke algebra. Kauffman redefined the Jones polynomial using a skein relation, which was later used to define the Birman-Murakami-Wenzl algebra. These two algebras seem to me to be the beginning of an infinite series. I will propose a candidate for the third term of this series.

David Boyd, UBC

The A-polynomials of families of symmetric knots

Abstract: The A-polynomial $A(x, y)$ (not the Alexander polynomial) of a knot complement is an invariant that is notoriously difficult to compute. For example, the A-polynomials of some of the knots with 8 crossings are still unknown. We will describe a method for computing the A-polynomials of knots in a family of branched cyclic coverings of a 2-link X branched over one of the components of the link. The n-fold cover X_n has symmetry group containing the cyclic group Z_n. The main result is a formula for the A-polynomial of X_n which gives $A(x^n, y)$ as a product of $n\phi(2n)/2$ (if n is odd) or $n\phi(2n)/4$ (if n is even) polynomials with coefficients in $Q(2cos(\pi/n))$. These polynomials are all obtained from a single polynomial $G(x, y, w)$ called the G-polynomial of the link which is computed from a representation of the fundamental group of the link or from a triangulation of the link complement. A familiar example is the sequence of Turk's head knots with 3 strands of which the knot 8_18 is the first interesting example. Using this method we compute the A-polynomial of one of the dodecahedral knots by regarding it as a 5-fold branched cyclic covering of a certain 2-bridge link. The resulting polynomial is of degree 32×160 in (x, y)i and the largest coefficient is a 32 digit integer, so it is unlikely that this polynomial could be computed by any of the earlier methods.

Nathan Dunfield, Cal. Tech.

Does a random 3-manifold fiber over the circle?

I'll discuss the question of when a tunnel number one 3-manifold fibers over the circle. In particular, I will discuss a criterion of Brown which answers this question from a presentation of the fundamental group. I will describe how techniques of Agol, Hass and W. Thurston can be adapted to calculate this very efficiently by using that the relator comes from an embedded curve on the boundary of a genus 2 handlebody. I will then describe some experiments which strongly suggest the answer to the question: Does a random tunnel-number one 3-manifold fiber over the circle? If time allows, I'll discuss how we're trying to prove that what is observed is really correct. (Ongoing joint work with Dylan Thurston)

Stavros Garoufalidis, Georgia Tech

On the Generalized Hyperbolic Volume Conjecture

The hyperbolic volume conjecture states that the n-th Jones polynomial of a knot evaluated at an n-th complex root of unity grows exponentially, and that the rate of growth is proportional to the volume of the knot. We will present positive and negative evidence of this conjecture for a class of knots. Our analysis involves studying asympotics of solutions of difference equations with a parameter. Depending on the shape of the $SL(2, \mathbb{C})$ character variety of a knot, we will present positive and negative evidence.

Herman Gluck, U Pennsylvania

The Gauss linking integral on the 3-sphere and in hyperbolic 3-space

We introduce here explicit integral formulas for linking, twisting, writhing and helicity on the 3-sphere and in hyperbolic 3-space.

These formulas, like their prototypes in Euclidean 3-space, are geometric rather than just topological, in the sense that their integrands are invariant under orientation-preserving isometries of the ambient space.

They are obtained by developing and then applying a steady-state version of classical electrodynamics in these two spaces, including an explicit Biot-Savart formula for the magnetic field and a corresponding Ampere's law contained in Maxwell's equations.

The Biot-Savart formula leads, in turn, to upper bounds for the helicity of vector fields and lower bounds for the first eigenvalue of the curl operator on subdomains of the 3-sphere and hyperbolic 3-space.

This is joint work with Dennis DeTurck.

Cameron Gordon, University of Texas, Austin

Knots with unknotting number 1

After surveying some of the methods that have been used to show that a knot has unknotting number greater than 1, particularly those related to Dehn surgery, we will describe recent joint work with John Luecke showing that if K is a knot with unknotting number 1 and S is an essential toric 2-suborbifold of K, then, generically, any unknotting arc for K can be moved off S. The examples for which this is not possible are precisely the doubly composite knots with unknotting number 1 constructed by Eudave-Munoz, and certain other closely related knots. The proof uses our recent classification of non-integral toroidal Dehn surgeries on hyperbolic knots. In particular, combining this with earlier results, and the recent work of Ozsvath and Szabo, the knots with unknotting number 1 and at most 10 crossings are completely determined. Other applications include an algorithm for determining which (generic) Conway algebraic knots have unknotting number 1, and a proof that having unknotting number 1 is invariant under mutation.

Robion Kirby, University of California, Berkeley

Nearly symplectic 4-manifolds

This concerns work with David Gay showing that an orientable, smooth, 4-manifold has a closed 2-form which would be a symplectic form except that it vanishes identically on a 1-dimensional submanifold.

Lou Kauffman, UIC

Virtual Knot Theory

This talk will discuss topics and problems in virtual knot theory. Virtual knot theory is the study of knots in thickend surfaces taken up to homeomorphisms of the surface and stablilization by the addition and subtraction of empty handles. There is a remarkably simple diagrammatic system for virtual knots consisting in adding an extra "virtual" crossing that is neither under, nor over, and adding corresponding moves to the Reidemeister moves. This talk will discuss some or all of the following topics: Virtual knots with unit Jones polynomial and related conjectures, Alexander and self-linking invariants of virtuals, biquandle invariants, generalizations of the Jones polynomial, relationships of virtual knot theory with quantum gravity in dimensions 2 +1. We may also disxcuss "rotational virtual knot theory", a modification similar in spirit to regular isotopy for classical knots. All quantum link invariants extend to invariants of rotational virtuals, and there is a rich field of phenomena to be studied here.

Sergei Matveev, Chelyabinsk

3-manifold recognizer

I will describe a partial efficient algorithm for recognition of 3-manifolds and demonstrate the corresponding computer program. Some application will be presented.

Kunio Murasugi, University of Toronto

Genus of a Montesinos knot and a characterization of fibred Montesinos knots

This is joint work with M. Hirasawa, Gakushuuin Univ. Tokyo. In this talk I discuss how to characterize fibred Montesinos knots K. We should note the condition that each constituent 2-bridge knot or link be fibred is neither necessary nor sufficient for K to be fibred. I also discuss briefly on tunnel number 1 Montesinos knots.

Jozef Przytycki, George Washington University

Khovanov homology: categorification of the Kauffman bracket skein module

Rachel Roberts, Washington University

Foliated hyperbolic 3-manifolds containing no $\mathbb{R}$-covered foliation

We discuss a family of hyperbolic 3-manifolds which contain taut foliations but no $\mathbb{R}$-covered foliation. We ask whether these 3-manifolds have left-orderable fundamental groups.

Lev Rozanski

Matrix factorization and link homology

The talk is based on a joint work with M. Khovanov (math.QA/0401268). We extend the construction of Khovanov's categorification from the Jones polynomial to the $SU(N)$ Jones-HOMFLY polynomial. To a link in S^3 we associate a complex of graded vector spaces up to a homotopy, such that its graded Euler characteristic is equal to the Jones-HOMFLY polynomial of the link. We use this complex in order to construct a (projective) 4-d TQFT for link cobordisms.

Our construction is based on the Murakami-Ohtsuki-Yamada formula for the Jones-HOMFLY polynomial in the same way as the original Khovanov's construction was based on Kauffman's bracket. The MOY paper presents the $SU(N)$ polynomial as an alternating sum over the polynomial invariants of special 3-valent graphs. We associate graded vector spaces to these graphs and then define the differentials between the spaces related to graphs which differ only locally.

Our tool is the category of matrix factorizations, first introduced by D. Eisenbud in relation to hypersurface singularities, and recently reappearing as a convenient way to describe the boundary conditions in 2-d topological Landau-Ginzburg theories. We associate a particular matrix factorization to an elementary open graph and then define the vector space of a closed 3-valent graph as a cohomology of the tensor product of matrix factorizations of its constituent elementary graphs.

Colin Rourke, University of Warwick

Klyachko, simple homotopy and the second homotopy group

A fundamental paper by Anton Klyachko proves the Kervaire conjecture for torsion-free groups. His methods however prove far more than this and I shall talk about two applications to low dimensional topology coauthored with Marshall Cohen and Max Forester.

(1) Suppose that L is a connected CW complex with torsion-free fundamental group and that K is constructed by attaching a 1-cell followed by a 2-cell. If inclusion induces a surjection on π_1, then it is simple homotopy equivalence.

(2) In the same situation as (1), if the 2-cell is attached by an amenable t-shape, then π_2 changes by extension of scalars. (Note: amenable t-shapes include all shapes of total index 1 in t.)

There is a common diagramatic core to both applications and this leads to information on the following oustanding conjecture:

Suppose that a group H is obtained from a torsion-free group G by adding a generator t and a relation with amenable t-shape then H is also torsion-free.

J. Hyam Rubinstein, University of Melbourne

Approximating the length of minimal Euclidean Steiner trees

IDEAL TRIANGULATIONS - geometric structures and proper essential surfaces

This is joint work with Ensil Kang (Chosun University). We have been studying when an ideal triangulation admits an angle structure. A necessary and sufficient condition (obstruction) has been found to solve this first step in Casson's approach to finding hyperbolic structures by solving the hyperbolic gluing equations. Another topic is the use of spun normal surfaces to represent proper essential surfaces. We have an existence result, for when incompressible and boundary incompressible surfaces can be isotoped or homotoped to be spun normal. Using this, one gets interesting algorithms to decide whether a given boundary slope for a given knot or link can be represented by such a surface. Finally this also leads to a fast algorithm to decide if a closed or proper surface is incompressible.

Dan Silver, University of South Alabama

see Susan Williams below

Dev Sinha, University of Oregon

Homotopy methods in knot theory

Building on ideas of Bott and Goodwillie, we have started to forge a new connection between knot theory and algebraic topology by passing from a knot to its induced map on completed configuration spaces. In the simplest non-trivial case (looking at configurations of three points) we recover the simplest non-trivial finite-type knot invariant in a novel geometric manner. In this talk I will recount this work and report on progress made in further cases.

De Witt Sumners, Florida State University

DNA Topology: Experiments and Analysis

Cellular DNA is a long, thread-like molecule with remarkably complex topology. Enzymes which manipulate the geometry and topology of cellular DNA perform many important cellular processes (including segregation of daughter chromosomes, gene regulation, DNA repair, and generation of antibody diversity). Some enzymes pass DNA through itself via enzyme-bridged transient breaks in the DNA; other enzymes break the DNA apart and reconnect it to different ends.

In the topological approach to enzymology, circular DNA is incubated with an enzyme, producing an enzyme signature in the form of DNA knots and links. By observing the changes in DNA geometry (supercoiling) and topology (knotting and linking) due to enzyme action, the enzyme binding and mechanism can often be characterized. This minicourse will discuss topological models for DNA strand passage and exchange in site-specific DNA recombination, and use of the spectrum of DNA knots to infer bacteriophage DNA packing in viral capsids.

Dylan Thurston, Harvard

How efficiently do 3-manifolds bound 4-manifolds?

It known since 1954 that every 3-manifold bounds a 4-manifold. Thus, for instance, every 3-manifold has a surgery diagram. There are many proofs of this fact, including several constructive ones, but they generally produce exponentially complicated 4-manifolds. Given a 3-manifold M of complexity n, we show how to construct a 4-manifold bounded by M of complexity

$O(n^2)$ for reasonable definitions of "complexity". (For instance, one notion of complexity is the number of tetrahedra in a triangulation of M .) It is an open question whether this quadratic bound can be replaced by a linear bound.

Peter Teichner, UC Berkeley

Milnor invariants via Whitney towers

In joint work with Rob Schneiderman, we give a geometric interpretation of Milnor's mu-invariants of a link L in terms of certain 2-complexes, embedded in the 4-ball and with boundary L. These 2-complexes, the Whitney towers, are made out of iterating the attachment of a Whitney disk to a surface. In the talk we shall explain how to get an intersection invariant for a Whitney tower which generalizes intersection numbers of surfaces in a 4-manifold (giving the linking numbers of L). Our intersection invariant takes values in a certain graded abelian group generated by trees, modulo the well known AS and IHX relations, which will thus be explained in terms of 4-dimensional topology. It turns out that rationally, this is exactly the (leading term of the tree part of the) Kontsevich integral. Applying a purely algebraic mapping to the intersection invariant, Milnor's mu-invariants arise. This was shown for the Kontsevich integral by Habegger and Masbaum, but we give an independent geometric proof in terms of a Whitney tower-grope duality theorem.

Vladimir Turaev, Strasbourg

Minicourse: Virtual Strings

A virtual string is a scheme of self-intersections of a closed curve on a surface. It can be presented by an arrow diagram consisting of a circle and several oriented chords. I shall discuss algebraic invariants of strings as well as two equivalence relations on the set of strings: homotopy and cobordism. Connections between virtual strings and virtual knots will be also discussed.

Susan Williams and Dan Silver, University of South Alabama

Minicourse: Applications of Symbolic and Algebraic Dynamics to Knot Theory

Many classical knot invariants can be computed and understood from the perspective of symbolic and algebraic dynamics. These include Alexander polynomials, Fox colorings and torsion numbers. Basic techniques will be explained with an emphasis on examples. We will conclude with a discussion of Lehmer's Question, a difficult open problem in number theory that can be approached from these ideas.
