Reverse Time Shot-Geophone Migration and

velocity analysis, linearizing, inverse problems, extended models, etc. etc.

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Agenda:

1. Partially linearized seismic inverse problem - the working environment of seismic processing

2. Velocity Analysis = solution method for partially linearized inverse problem, based on *invertible extended model*

3. Strong lateral velocity variations $\Rightarrow$ the usual extended models of MVA aren't invertible

4. Extended model of *shot-geophone* migration is invertible, even with strong lateral velocity variations!

5. Reverse time algorithm for shot-geophone migration

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Simple acoustic model of **seismic reflection inverse problem**: given data d^{obs}, find velocity field c (a function on the *subsurface* X) so that

$$\mathcal{F}[c] = d^{\text{obs}}$$

$\mathcal{F}$ = *forward map* aka *modeling operator* aka ..., defined by

$$\mathcal{F}[c](\mathbf{x}_r, \mathbf{x}_s, t) = \frac{\partial u}{\partial t}(\mathbf{x}_r, t; \mathbf{x}_s),$$

where acoustic potential field $u(\mathbf{x}, t; \mathbf{x}_s)$ solves

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u = f(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

This problem is too hard!

Most useful progress based on these beliefs:

- If $c = v(1 + r)$ where v is smooth and r oscillatory (or even *singular*), then $\mathcal{F}[c] \simeq \mathcal{F}[v] + D\mathcal{F}[v](vr)$ (i.e. the Born approximation is accurate);

- In lots of places, the actual compressional velocity field in the Earth has this nature.

[A good math problem: exactly how is the first bullet true? Lots of numerical evidence, little mathematics.]

Partially linearized seismic inverse problem: given observed seismic data d^{obs}, find smooth *velocity* $v \in \mathcal{E}(X), X \subset \mathbf{R}^3$ oscillatory *reflectivity* $r \in \mathcal{E}'(X)$ so that

$$D\mathcal{F}[v](vr) = F[v]r \simeq d^{\text{obs}}$$

where the acoustic potential field u and its perturbation δu solve

$$\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u = f(t)\delta(\mathbf{x} - \mathbf{x}_s),$$

$$\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta u = 2r\nabla^2 u$$

plus suitable bdry and initial conditions.

$$F[v]r = \left.\frac{\partial \delta u}{\partial t}\right|_Y$$

data acquisition manifold $Y = \{(\mathbf{x}_r, t; \mathbf{x}_s)\} \subset \mathbf{R}^7$, dimn $Y \leq 5$ (many idealizations here!).

$F[v] : \mathcal{E}'(X) \to \mathcal{D}'(Y)$ is a linear map (FIO of order 1) (Rakesh 1988), but dependence on v is quite nonlinear, so this inverse problem is nonlinear.

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Extension of $F[v]$ (aka *extended model*): manifold $\bar{X}$ and maps $\chi : \mathcal{E}'(X) \to \mathcal{E}'(\bar{X})$, $\bar{F}[v] : \mathcal{E}'(\bar{X}) \to \mathcal{D}'(Y)$ so that

$$\begin{array}{ccccc} & & \bar{F}[v] & & \\ & \mathcal{E}'(\bar{X}) & \rightarrow & \mathcal{D}'(Y) & \\ \chi & \uparrow & & \uparrow & \text{id} \\ & \mathcal{E}'(X) & \rightarrow & \mathcal{D}'(Y) & \\ & & F[v] & & \end{array}$$

commutes, i.e.

$$\bar{F}[v]\chi r = F[v]r$$

(Familiar) Example: the *Convolutional Model*

- *Approximation* of P. L. model, accurate when v, r functions of z only

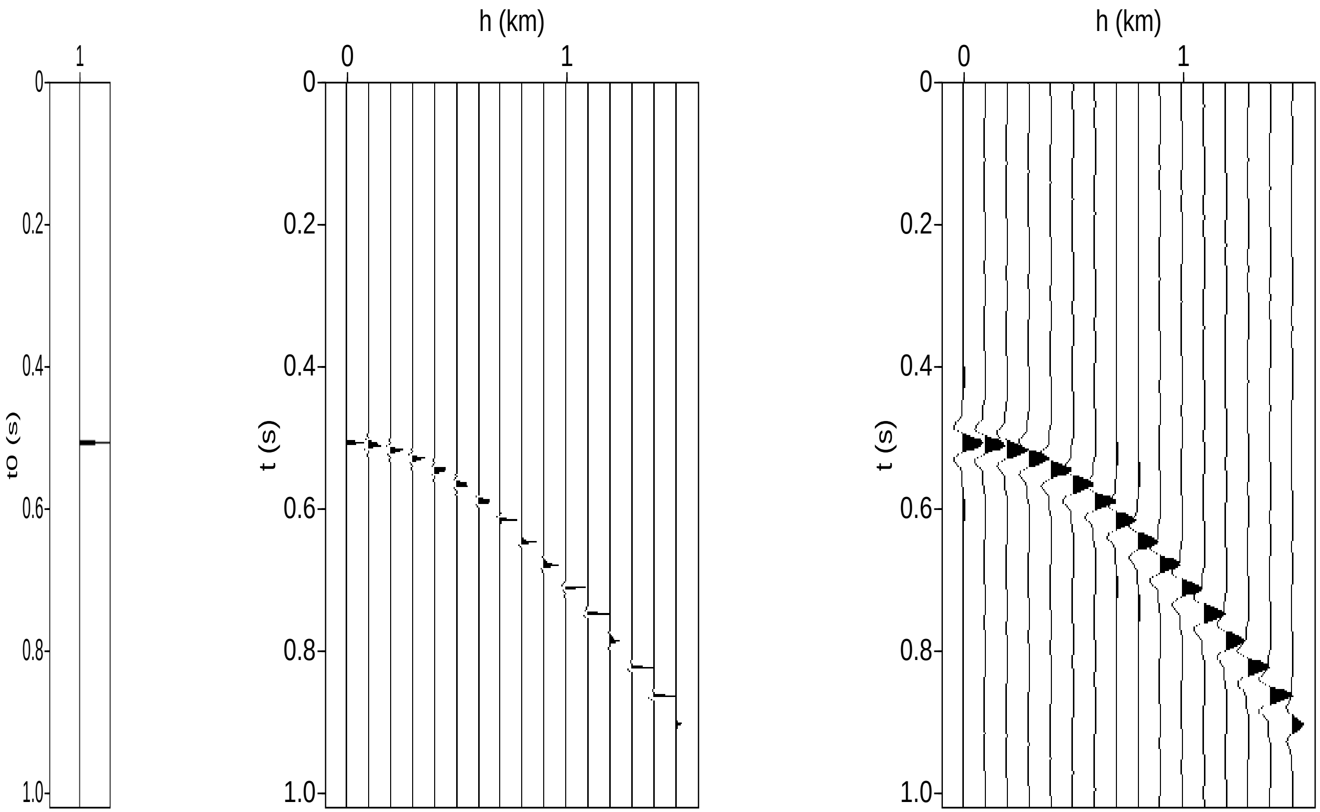
- data function of t, $h = (x_r - x_s)/2$ *half-offset*

- two-way traveltime $\tau(z,h)$, inverse $\zeta(t,h)$

- if $v =$const. then $\tau(z,h) = 2\sqrt{z^2+h^2}/v$

$$F[v]r(t,h) = \int dt'\, f(t-t')r(\zeta(t',h))$$

("inverse NMO, convolve with source")



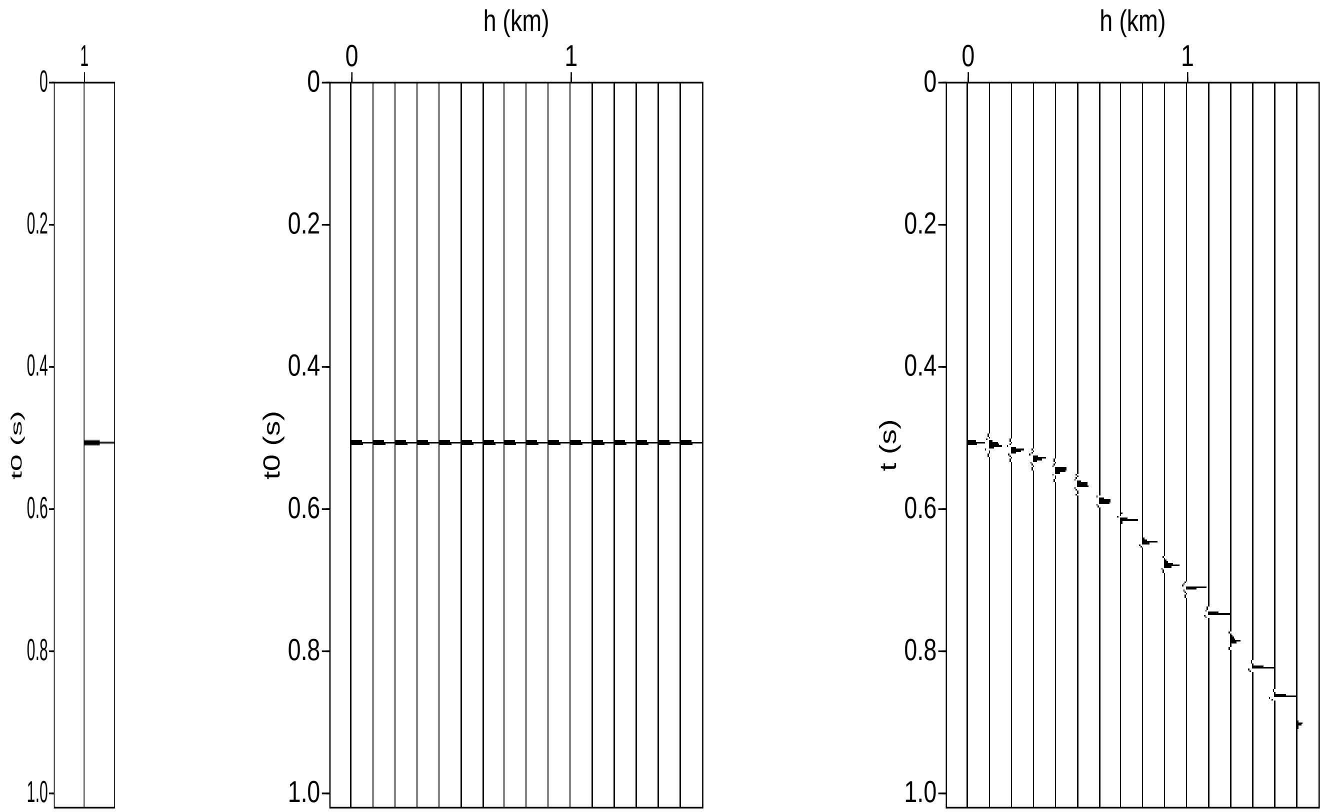
Left: $r(t_0)$. Middle: $r(\zeta(t,h))$. Right: $F[v]r(t,h)$

Factor convolutional model through extension: *replicate* r for each h

$$\chi : r(z) \mapsto \bar{r}(z,h) = r(z)$$

then apply inverse NMO and convolve with source, independently for each h

$$\bar{F}[v] : \bar{r}(z,h) \mapsto f * \bar{r}(\zeta(t,h),h)$$



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Left: $r(t_0)$. Middle: $\bar{r}(t_0, h) = \chi r$. Right: $\bar{r}(\zeta(t, h), h)$.

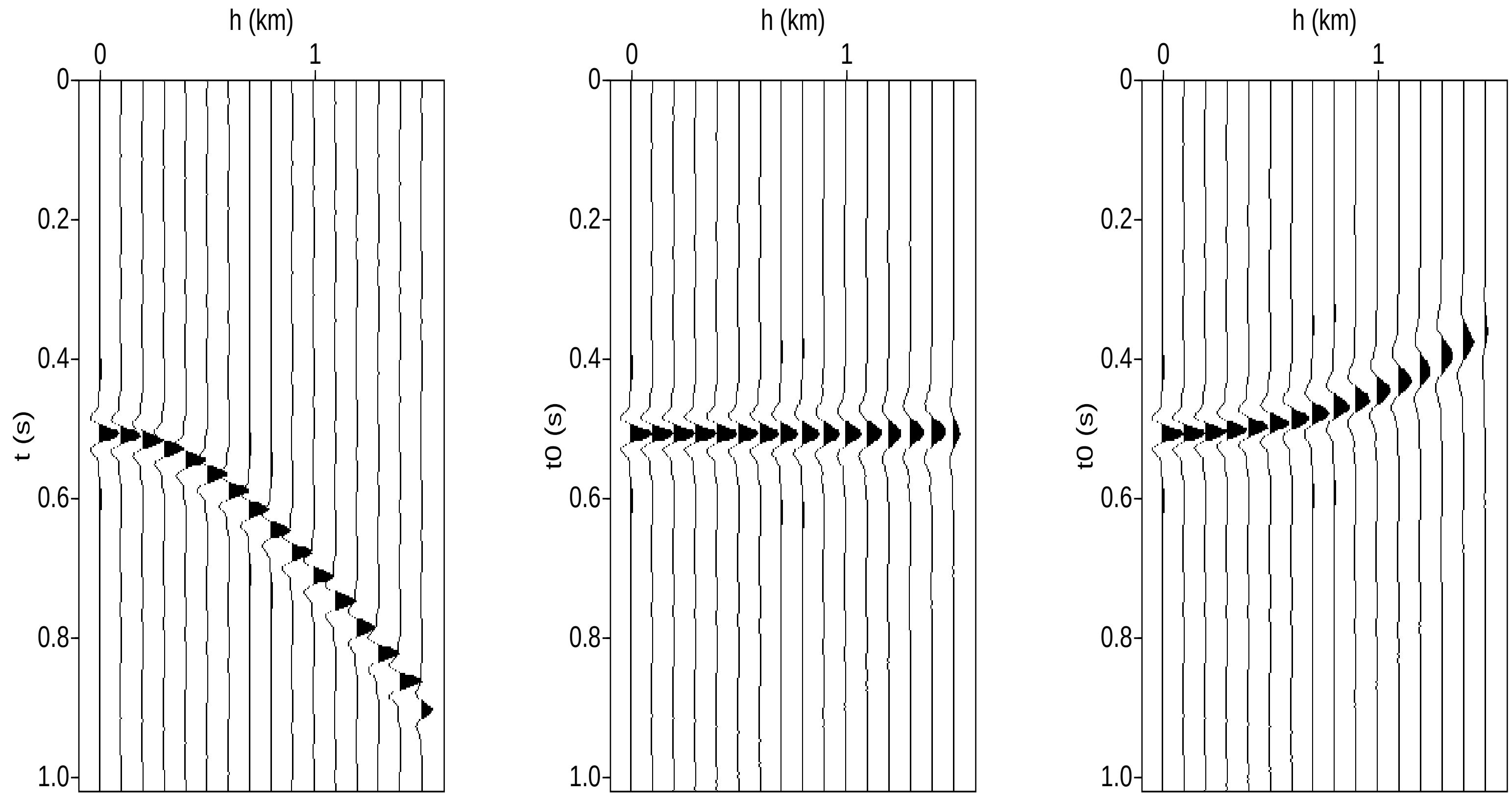
Invertible extension: $\bar{F}[v]$ has a *right parametrix* $\bar{G}[v]$, i.e.

$$I - \bar{F}[v]\bar{G}[v]$$

is smoothing.

Example: for the convolutional model, $\bar{G}[v]$ is signature decon followed by NMO, applied trace-by-trace.

NB: The trivial extension - $\bar{X} = X, \bar{F} = F$ - is virtually never invertible.



Left: $d(t,h) = F[\overset{'}{v}]r(t,h)$. Middle: $\bar{G}[\overset{'}{v}]d(t_0,h)$. Right: $\bar{G}[v_1]d(t_0,h), v_1 \neq v$

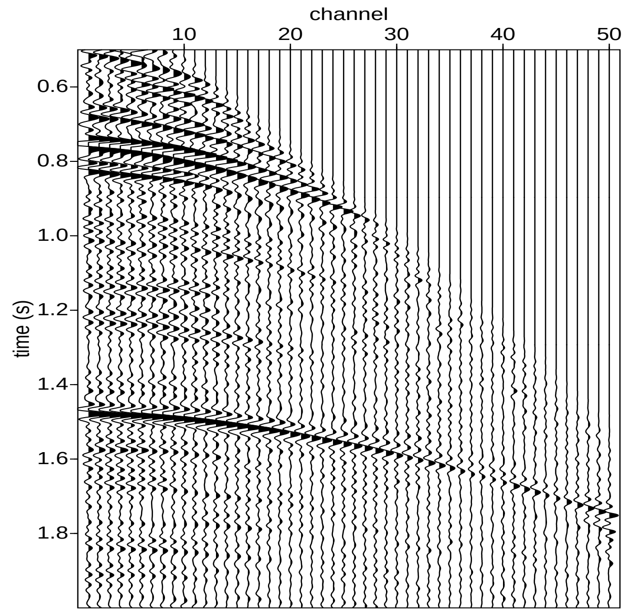
Reformulation of inverse problem: given d^{obs}, find v so that $\bar{G}[v]d^{\text{obs}} \in$ the range of χ.

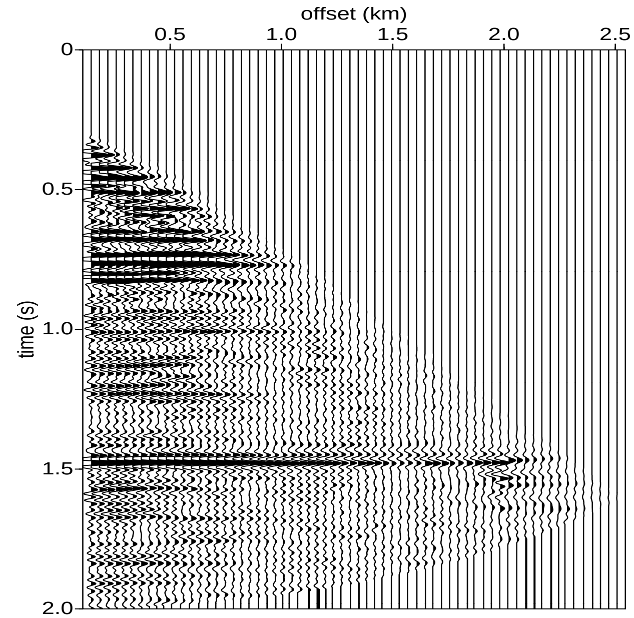
Non-Gary Proof: that is, $\bar{G}[v]d^{\text{obs}} = \chi r$ for some r, so $d^{\text{obs}} \simeq \bar{F}[v]\bar{G}[v]d^{\text{obs}} = \bar{F}[v]\chi r = F[v]r$ Q. E. D.

This is velocity analysis!

Example: Standard VA. Apply convolutional model to each midpoint in CMP-binned data. Range of $\chi = \bar{r}(z,h)$ *independent of h*, i.e. flat. SO: twiddle v so that $\bar{G}[v]d^{\text{obs}}$ shows flat events.

Caveats: in practice, be happy when $\bar{G}[v]d^{\text{obs}}$ is in range of χ except for wrong amplitudes, finite frequency effects, and obvious (!) noise.





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Left: part of survey (d^{obs}) from North Sea (thanks: Shell Research), lightly preprocessed.
Right: restriction of $\bar{G}[v]d^{\text{obs}}$ to $\mathbf{x}_m = $ const (function of depth, offset): shows rel. sm'ness in h (offset) for properly chosen v.

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The usual extended model behind Migration Velocity Analysis:

- v, r functions of all space variables

- $\chi r(\mathbf{x},\mathbf{h}) = r(\mathbf{x})$ (so $\bar{r} \in$ range of $\chi \Leftrightarrow$ plots of $\bar{r}(\cdot,\cdot,z,h)$ appear *flat*)

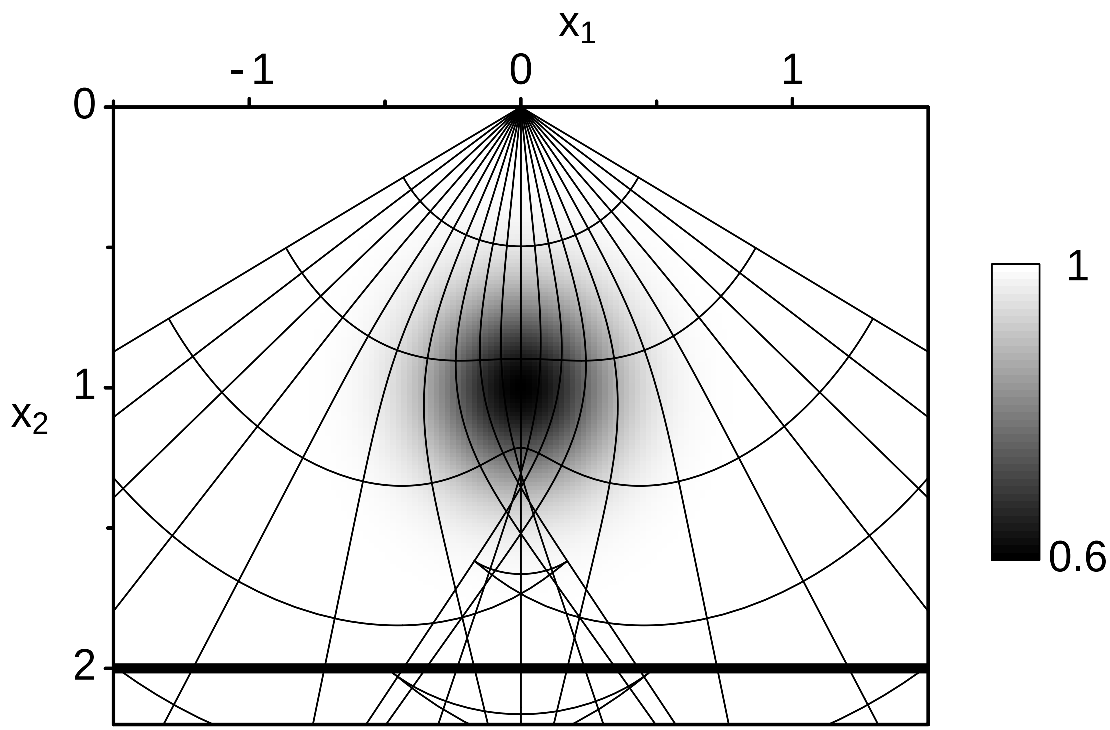
- $$\bar{F}[v]\bar{r}(\mathbf{x}_r,\mathbf{x}_s,t) = \frac{\partial^2}{\partial t^2}\int dx\,\bar{r}(\mathbf{x},\mathbf{h}) \int ds\, G(\mathbf{x}_r, t-s;\mathbf{x})u(\mathbf{x}_s,s;\mathbf{x})$$
  (recall $\mathbf{h} = (\mathbf{x}_r - \mathbf{x}_s)/2$)

NB: $\bar{F}$ is “block diagonal” - family of operators (FIOs) parametrized by $\mathbf{h}$.

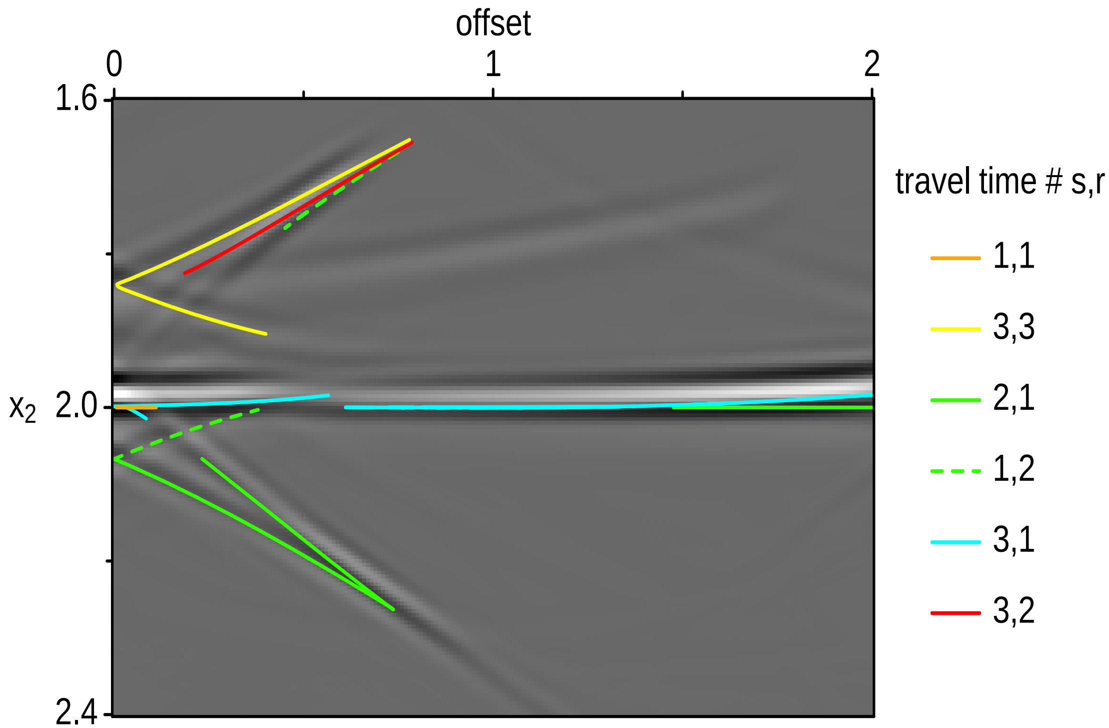
- Beylkin (1985), Rakesh (1988): if $\|\nabla^2 v\|_{C^0}$ "not too big", then the usual extension is invertible.

- $\bar{G}$ = *common offset* migration-inversion aka ray-Born inversion aka true-amplitude migration etc. etc. Usually implemented as generalized Radon transform = "weighted diffraction stack" (Beylkin, Bleistein, DeHoop,...)

- Nolan, Stolk, WWS: if $\|\nabla^2 v\|_{C^0}$ is too big, *usual extension is not invertible!*



Example: Gaussian lens over flat reflector at depth 2 km ($r(\mathbf{x}) = \delta(z-2)$).



Common Image Gather (const. x, y slice) of $\bar{G}[v]d^{\mathrm{obs}}$: not flat, i.e. not in range of χ even allowing for amplitude, finite frequency errors, and *even though velocity is correct!* [Stolk, WWS 2002]

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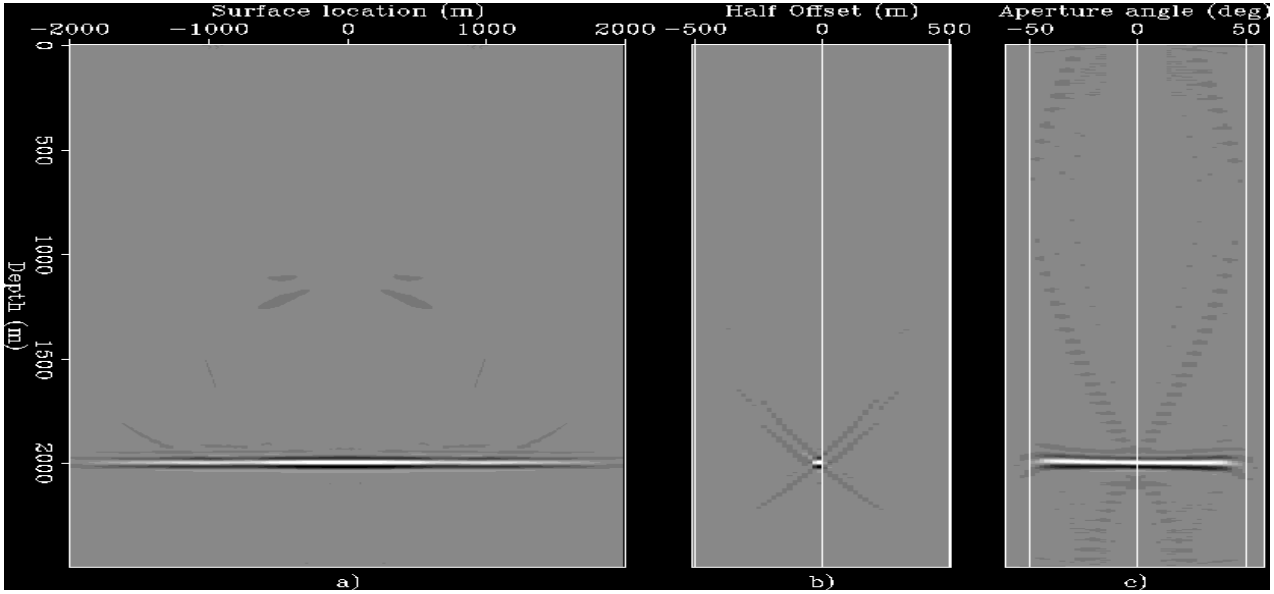
Claerbout's extended model = basis of *survey sinking* or *shot-geophone* migration:

- $\chi r(\mathbf{x},\mathbf{h}) = r(\mathbf{x})\delta(\mathbf{h})$, so $\bar{r} \in$ range of $\chi \Leftrightarrow$ plots of $\bar{r}(\cdot,\cdot,z,h)$ appear *focussed* at $\mathbf{h} = 0$

$$\bar{F}[v]\bar{r}(\mathbf{x}_r,\mathbf{x}_s,t)$$

$$= \frac{\partial^2}{\partial t^2}\int dx \int dh\, \bar{r}(\mathbf{x},\mathbf{h}) \int ds\, G(\mathbf{x}_r, t-s; \mathbf{x}+\mathbf{h}) u(\mathbf{x}_s, s; \mathbf{x}-\mathbf{h})$$

- This extension is invertible, assuming (i) $\mathbf{h}_3 = 0$ (horizontal offset only) and (ii) "DSR hypothesis": rays do not turn. Then adjoint map is equivalent modulo elliptic ΨDO factor to shot-geophone migration via DSR equation [Stolk-DeHoop 2001]



Lens data, shot-geophone migration [B. Biondi, 2002]
Left: Image via DSR. Middle: $\bar{G}[v]d$ - well-focused (at $\mathbf{h} = 0$), i.e. in range of χ to extent possible. Right: Angle CIG.

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Alternate expression for extended S-G model:

$$\bar{F}[v]\bar{r}(\mathbf{x}_r, t; \mathbf{x}_s) = \frac{\partial}{\partial t}\delta\bar{u}(\mathbf{x}, t; \mathbf{x}_s)|_{\mathbf{x}=\mathbf{x}_r}$$

where

$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{x}}\right)\delta\bar{u}(\mathbf{x}, t; \mathbf{x}_s) = \int_{\mathbf{x}+2\Sigma_d} dy\,\bar{r}(\mathbf{x}, \mathbf{y})u(\mathbf{y}, t; \mathbf{x}_s)$$

Computing $\bar{G}[v]$: instead of parametrix, be satisfied with adjoint - the two differ by a ΨDO factor, which will not affect smoothness of CIGs.

Computing the adjoint: use the *adjoint state method* (WWS, Biondi & Shan, SEG 2002).

Define *adjoint state* w:

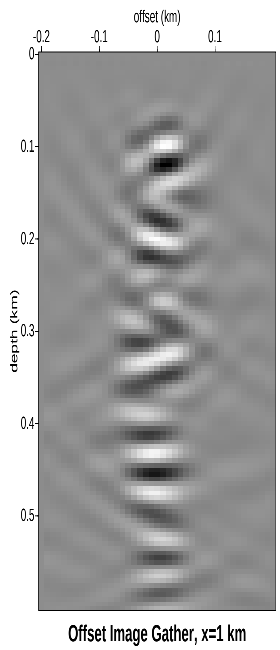
$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{x}}\right) w(\mathbf{x},t;\mathbf{x}_s) = \int dx_r\, d(\mathbf{x}_r,t;\mathbf{x}_s)\delta(\mathbf{x}-\mathbf{x}_r)$$

with $w(\mathbf{x},t;\mathbf{x}_s) = 0, t >> 0$. [This is exactly the backpropagated field of standard reverse time prestack migration, cf. Lines talk.]

Then

$$\bar{F}[v]^* d(\mathbf{x},\mathbf{h}) = \int dx_s \int dt\, u(\mathbf{x}+2\mathbf{h},t;\mathbf{x}_s) w(\mathbf{x},t;\mathbf{x}_s)$$

[This is exactly the same computation as for standard reverse time prestack, except that crosscorrelation occurs at an offset $2\mathbf{h}$].



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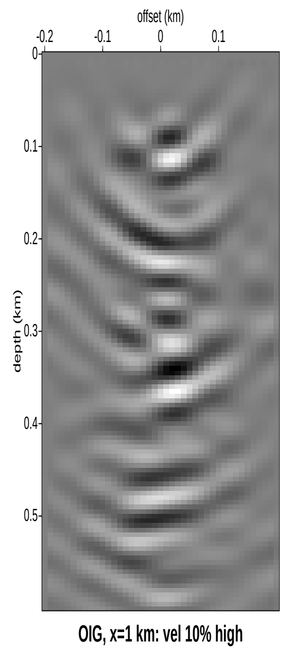

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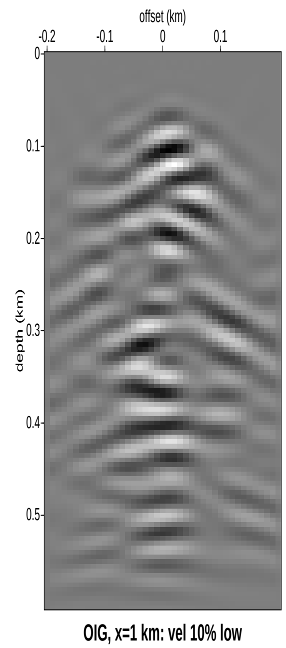

Two way reverse time horizontal offset S-G image gathers of data from random reflectivity, constant velocity. From left to right: correct velocity, 10% high, 10% low.

Some Loose Ends

- invertibility of S-G extended model only known under DSR assumption with horizontal offsets [Stolk-DeHoop, 2001]. Vertical offsets are good when DSR breaks down, eg. to image overhanging reflectors [Biondi, WWS 2002]. Current best result: data focusses only at offset $= 0$ within a limited range off offsets; focussing at large offsets not ruled out [WWS, 2002]. What actually happens?

- S-G extension amounts to construction of annihilators [cf. DeHoop]. How can one characterize globally invertible annihilator representations?

- quantification of non-membership in range of χ (DSO) - which ones yield good optimization problems locally [Stolk-WWS, IP 2003] or globally?

Conclusions

- Most of contemporary SDP related *partially linearized* seismic inverse problem
- Velocity analysis = approach to solution of PL seismic inverse problem via invertible extended models
- Usual extended models (common offset, common shot, common angle,...) are not invertible when the velocity structure is complex, due to multipathing
- The extended model of shot-geophone migration is invertible even in the presence of multipathing
- Shot-geophone migration has a reverse-time implementation

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