The determination of medium discontinuities by migration : results from microlocal analysis.

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Overview

(Depth) migration $\leftrightarrow$ inversion of a reflectivity function

Goal : study media with multipathing, different types of imaging

Overview

1. Phase space localization of singularities
2. Modeling
3. Imaging of constant source/offset data
4. Imaging with full data
5. Angle common image gathers

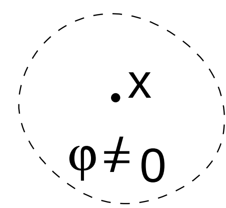
1. Phase space localization of singularities

Wave front set

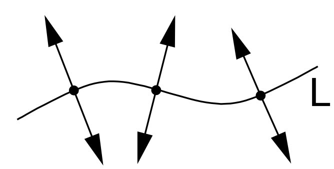
Location and orientation of events, wave fronts.

Let $f(x)$ be a function of $x \in \mathbb{R}^n$. The wave front set WF(f) is a subset of $\mathbb{R}^n \times \mathbb{R}^n$, that contain positions and directions of singularities. Directions: if (x, k_x) in WF(f) then line $(x, \lambda k_x), \lambda > 0$ in WF(f).

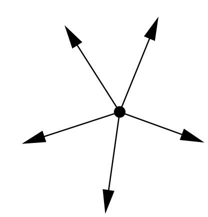
To determine whether (x, k_x) in WF(f)



1. Localize in x, consider ϕf
2. Fourier transform $x \to k_x$ of ϕf
3. Look at decay of Fourier transform in a small cone around k_x
   - strong decay: smooth at (x, k_x)
   - otherwise: singularity at (x, k_x)



Examples: 1. Discontinuity along a curve
$\text{WF}(f) = \{(x, v) \,|\, x \in L, v \perp L \text{ at } x\}$
2. Point singularity $\text{WF}(\delta) = \{(0, v) \,|\, v \in \mathbb{R}^n, v \neq 0\}$



Mapping of singularities

E.g. mapping of events by a migration operator.

Consider operator F mapping $g \mapsto f$; $f(x) = \int F(x,y) g(y)\,\mathrm{d}y$.
Assume F a Fourier integral operator

$$F(x,y) = \int A(x,y,\theta) \mathrm{e}^{\mathrm{i}\Phi(x,y,\theta)}\,\mathrm{d}\theta$$

Then F maps $\mathrm{WF}(g)$ to $\mathrm{WF}(f)$

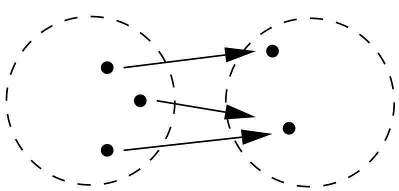
1. Compute WF-set of F

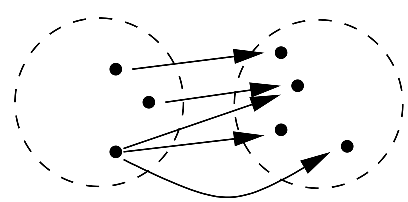
$$\mathrm{WF}(F) \subseteq \{(x,y,\nabla_x\Phi,\nabla_y\Phi) \,|\, (x,y,\theta) \text{ in set } \nabla_\theta\Phi = 0\}.$$

2. Compute canonical relation

$$\Lambda'_F = \{[(x,k_x),(y,k_y)] \,|\, (x,y,k_x,-k_y) \in \mathrm{WF}(F)\}$$

3. Map $\mathrm{WF}(g)$ via canonical relation to get $\mathrm{WF}(f)$, by "set mapping"

function 

relation 

2. Modeling

Green's function

Wave equation with smooth wave speed $c_0(x)$

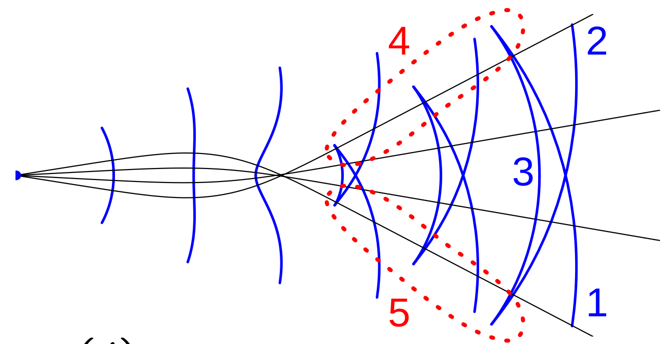
$$(c_0(x)^{-2}\partial_t^2 - \Delta)u(x,t) = g(x,t), \;\; u|_{t<0} = 0.$$

Green's function for solution

$$u(x,t) = G_0 g(x,t) = \int_{\mathbb{R}^n}\int_0^t G_0(x,x_0,t-t_0)g(x_0,t_0)\,\mathrm{d}x_0\,\mathrm{d}t_0.$$

Singularities propagate along rays

For longer times and complex media **caustics** and **multiple wave fronts** develop.



Contributions from smooth wave fronts $G_0^{(j)}$:
Multiple traveltimes ${T_1}^{(j)}(x,x_0)$, amplitudes $A^{(j)}(x,x_0)$, KMAH-index $\sigma^{(j)}(x,x_0)$, $j = 1,2,\ldots$

$$G_0^{(j)}(x,x_0,t) = \frac{1}{2\pi}\int A^{(j)}(x,x_0,\omega)\,\mathrm{e}^{\mathrm{i}\omega(t-{T_1}^{(j)}(x,x_0))}\,\mathrm{d}\omega$$

with $A^{(j)} = (-)(-\mathrm{i}\omega)^{\frac{n-3}{2}}(-\mathrm{i}\,\mathrm{sgn}(\omega))^{\sigma^{(j)}}A_0^{(j)}(x,x_0) + \text{lower order}.$

Modeling : Born approximation

Two ingredients

- c_0, background medium, that is smooth (C^∞)
- δc, medium perturbation, contains the discontinuous.

Incoming wave field with source g, assume $g(x,t) = \delta(t)\delta(x-s)$.

$$u_{\text{inc}} = G_0 g.$$

Reflected wave field

$$u_{\text{refl}} = G_0 \left(\frac{\delta c}{2c_0^3} \partial_t^2 G_0 g \right)$$

Define reflectivity $f = \frac{2\delta c}{c_0(x)^3}$.
Forward map from f to data, denoted by F

$$F : f \mapsto d(s,r,t) = \int_0^t \int_{\mathbb{R}^n} G_0(r,x,t-t')\partial_t^2 G_0(x,s,t') f(x) \, \mathrm{d}x \, \mathrm{d}t',$$

for source pos. s, receiver pos. r, time t. Aim : reconstruct f.

F is a Fourier integral operator

Assumption

- There are no rays that graze acquistion surface and enter region of interest.
- There are no direct rays s to r, over time t that enter the region of interest and satisfy (s, r, t) in acquisition set.

Theorem (Rakesh '88, Ten Kroode et al. '98) Then the operator $F : f \mapsto d$ is a Fourier integral operator. The canonical relation Λ'_F contains all

$$[(s, r, t, k_s, k_r, -\omega), (x, k_x)]$$

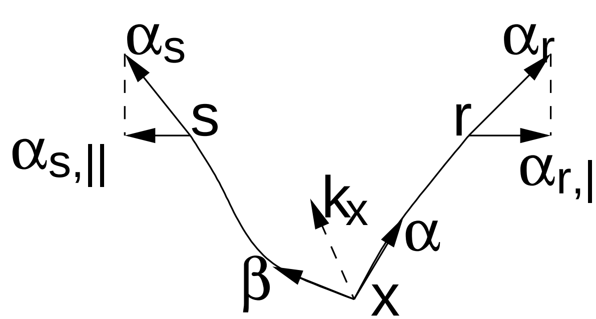
such that, with $\alpha_s, \alpha_r, \alpha, \beta$ unit vectors in the ray directions,

rays connect x and s and x and r

$$k_s = \omega c(s)^{-1} \alpha_{s,\|}$$

$$k_r = \omega c(r)^{-1} \alpha_{r,\|}$$

$$k_x = \omega c(x)^{-1} (\alpha + \beta).$$



Restricted forward map

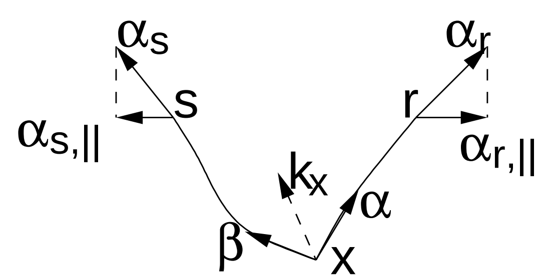
Restricted data reconstruction problems

- from constant source data $d_{\text{source}}(r,t)$ (for some given s)
- from constant offset data $d_{\text{offset}}(m,t)$, for some given offset $h = r - s$, where m is the midpoint $m = \frac{r+s}{2}$

If $[(s,r,t,k_s,k_r,-\omega),(x,k_x)] \in \Lambda'_F$, then

$$[(r,t,k_r,-\omega),(x,k_x)] \in \Lambda'_{\text{source}} \text{ if } s \text{ as given,}$$

$$[(m,t,k_m,-\omega),(x,k_x)] \in \Lambda'_{\text{offset}} \text{ if } h = r - s \text{ as given,}$$
$$\text{with } k_m = k_s + k_r.$$



Additional assumption

- (offset) matrix $\frac{\partial}{\partial(x,\alpha,\beta)}(s-r)$ has maximal rank.

Generalized Radon transform

Define two way traveltime functions

$$T^{(i,j)}(x,s,r) = T_1^{(i)}(x,r) + T_1^{(j)}(x,s),$$

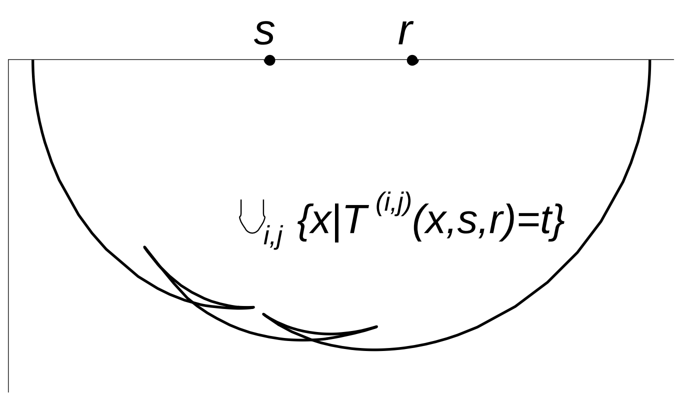
Non-caustic contributions

$$d(s,r,t) \approx \sum_{i,j} \int \int_{\mathbb{R}} A^{(i,j)}(x,s,r,\omega) \mathrm{e}^{\mathrm{i}\omega(T^{(i,j)}(x,s,r)-t)} f(x)\,\mathrm{d}x\,\mathrm{d}\omega$$

$$\approx \sum_{i,j} \int A_0^{(i,j)}(x,s,r)\,\mathrm{Hilb}^{\sigma^{(i)}+\sigma^{(j)}}\,\partial_t^{n-1} \underbrace{\delta(t - T^{(i,j)}(x,s,r)) f(x)\,\mathrm{d}x}_{\text{integration over isochrons}}$$

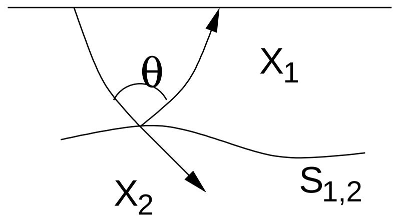
$$+ \text{lower order}$$

Generalized Radon transform : Integration over isochrons (plus amplitude factors, derivatives, Hilbert transform).



Modeling with piecewise smooth media

Regions X_1, X_2, and interfaces $S_{1,2}$.



Conversion of incoming to reflected rays
by pseudodifferential operator (y coordinate in the interface)

$$\begin{aligned} u_{\mathrm{refl}}|_{S_{1,2}} &= R(y, -\mathrm{i}\partial_y, -\mathrm{i}\partial_t) u_{\mathrm{inc}}|_{S_{1,2}} \\ &= \int\int R(y, k_y, -\omega)\hat{u}_{\mathrm{inc}}(k_y, \omega)\,\mathrm{d}k_y\,\mathrm{d}\omega. \end{aligned}$$

to highest order R is the normalized reflection coefficient $r(x, \theta)$.

Singularities of solution, apart from tangent rays (head waves), given by

$$u_{\mathrm{refl}} = G_{0,X_1 \leftarrow S_{1,2}}\Big(R(y, -\mathrm{i}\partial_y, -\mathrm{i}\partial_t) G_{0,S_{1,2} \leftarrow X_1} g\Big).$$

Reconstruct

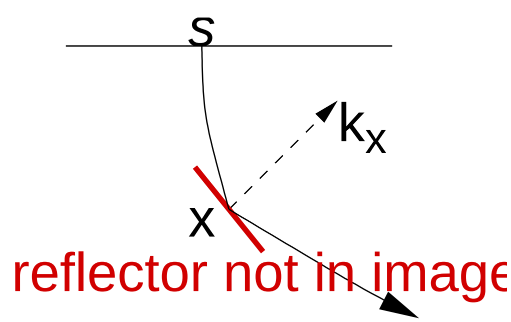
$$r(x, \theta)(\text{sing. fun. of } S_{1,2})(x)$$

for some θ depending on c_0, x and data.

3. Imaging of constant source/offset data

Partial reconstruction only

Some (x, k_x) not mapped to data

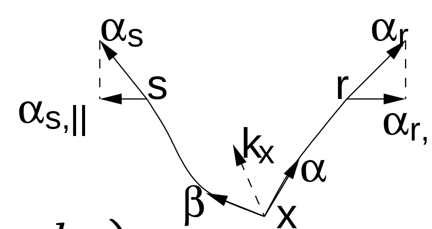


Let ψ be a smooth cutoff (tapering function) on acquisition set $\psi = \psi(s, r, t)$

$$\psi(s, r, t) = \begin{cases} 0 \text{ for } (s, r, t) \text{ outside acquisition set,} \\ \text{smoothly going to 1 inside acquisition set,} \end{cases}$$

needed to avoid edge effects in migration.

Connect (s, x, k_x) to (s, r, t) by rays



$\Rightarrow \Psi_{\text{source}}(s, x, k_x)$ smooth "cutoff" function of (x, k_x),

Non-zero where (x, k_x) illuminated ("observable"). Similar we have a smooth cutoff function $\Psi_{\text{offset}}(h, x, k_x)$.

Adjoint mapping of singularities

For any operator F that maps $f(x)$ to $g(y)$, we have

$$[(y,k_y),(x,k_x)] \in \Lambda'_F \Leftrightarrow [(x,k_x),(y,k_y)] \in \Lambda'_{F^*}.$$

Thus F^* maps observable singularities back to their original position.

If $[(y,k_y),(x,k_x)] \in \Lambda'_F$ and $[(y,k_y),(x',k'_x)] \in \Lambda'_F$, then

$$\begin{array}{c}(x,k_x)\\ \text{sing. in } f\end{array} \xrightarrow{F} \begin{array}{c}(y,k_y)\\ \text{sing. in } d\end{array} \xrightarrow{F^*} \begin{array}{c}(x',k'_x)\\ \text{sing. in image}\end{array}.$$

Possible kinematic artifacts when $(x',k'_x) \neq (x,k_x)$.

$\Rightarrow$ **Injectivity conditions** for absence of artifacts. For each (y,k_y) there must be at most one (x,k_x).

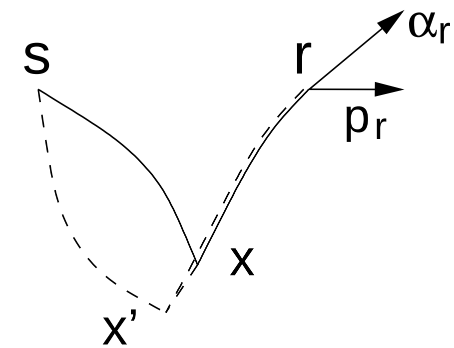
$\Rightarrow$ **Imaging equations** to determine position of singularities in image from the rays (i.e. Λ'_F) and knowledge of singularities in data.

Imaging equations (constant source)

Migration formula: Modification of adjoint by factors leaving singularities in place (amplitudes, derivatives, Hilbert transform)

$$f_{\text{source}}(s,x) = \sum_{(i,j)} \int B^{(i,j)}(x,s,r)\,\text{Hilb}^{-\sigma^{(i)}-\sigma^{(j)}}\, d(s,r,T^{(i,j)}(x,s,r))\,\text{d}r.$$

Define slowness $p_s = k_s/\omega$, $p_r = k_r/\omega$. Source slowness p_s not determined from data



In presence of multipathing (s, r, t, p_r) do not alwals determine reflection point.

Imaging equations Assume $t = T_{\text{data}}(s,r)$ is an arrival in the data. Equations for event positions in image x'.

$$T^{(i,j)}(x',s,r) = T_{\text{data}}(s,r), \quad \nabla_r T^{(i,j)} = \nabla_r T_{\text{data}}.$$

3 eqns. for 3 unknowns x' (3-D) $\rightarrow$ solutions for several (i,j)

Constant source data imaging

Assumption (traveltime injectivity)
(s, r, t, p_r) determine uniquely determine (x, α, β).

Assumption (local condition or immersivity)
The matrix $\frac{\partial k_x}{\partial(r,\omega)} = \begin{pmatrix} \omega \frac{\partial^2 T}{\partial r \partial x} & \frac{\partial T}{\partial x} \end{pmatrix}$ has maximal rank.

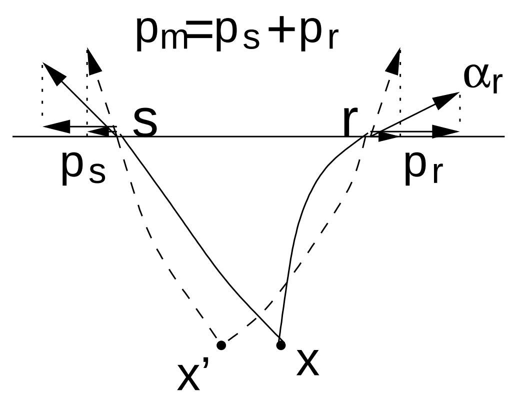
Theorem (Hansen '91) There is a microlocal inverse H_{source} such that for all f

$$H_{\text{source}} F_{\text{source}} f = \Psi_{\text{source}}(s, x, -\mathrm{i}\partial_x) f$$
$$= (2\pi)^{-n} \int_{\mathbb{R}^n} \Psi_{\text{source}}(s, x, k_x) \widehat{f}(k_x) \, \mathrm{d}k_x.$$

- Exact reconstruction of singularities where $\Psi(s, x, k_x) = 1$!
- Approximation modulo lower order error by Kirchhoff migration.

Constant offset data imaging

Offset slowness $p_h = (p_r - p_s)/2$ not determined!



Imaging equations (using traveltimes), for arrival $t = T_{\text{data}}(s, r)$ in data

$$T^{(i,j)} = T_{\text{data}}, \quad \nabla_s T^{(i,j)} + \nabla_r T^{(i,j)} = \nabla_s T_{\text{data}} + \nabla_r T_{\text{data}}.$$

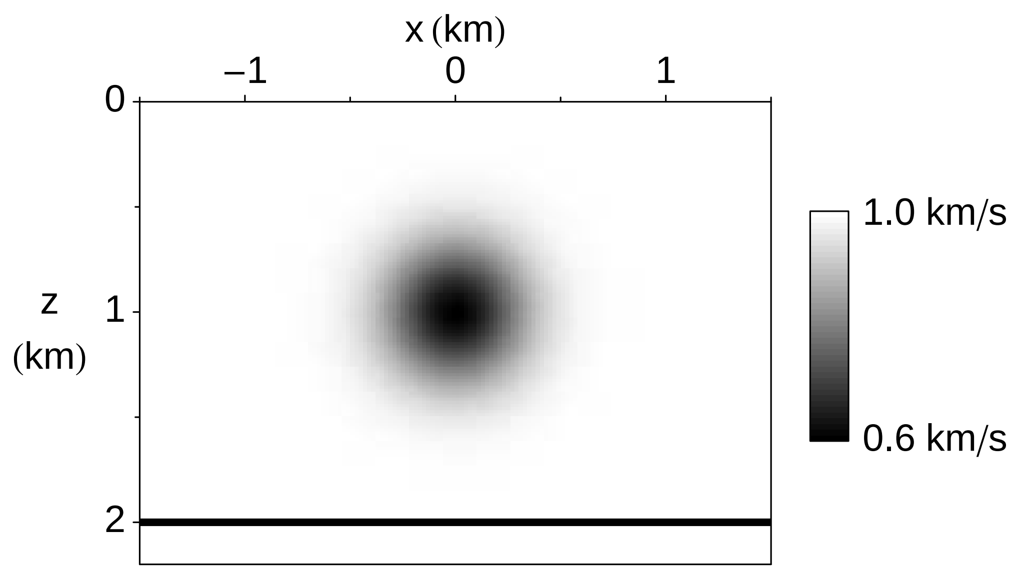
3 eqns. for 3 unknowns x' (3-D)

Inverse if
- $(s, r, t, p_m = (p_s + p_r))$ uniquely determines (x, α, β).
- Matrix $\frac{\partial k_x}{\partial(h,\omega)}$ has maximal rank.

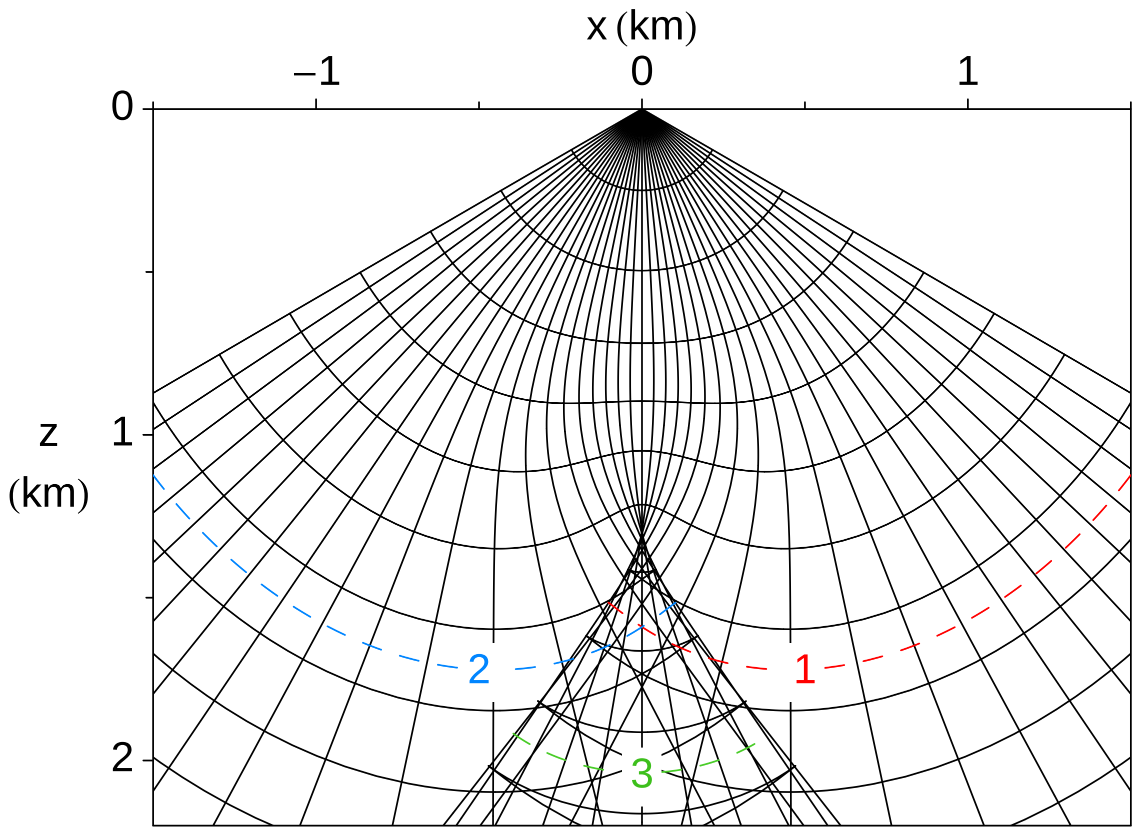
Multipathing : example

Identify kinematic artifacts by comparing migration and solving imaging equations.

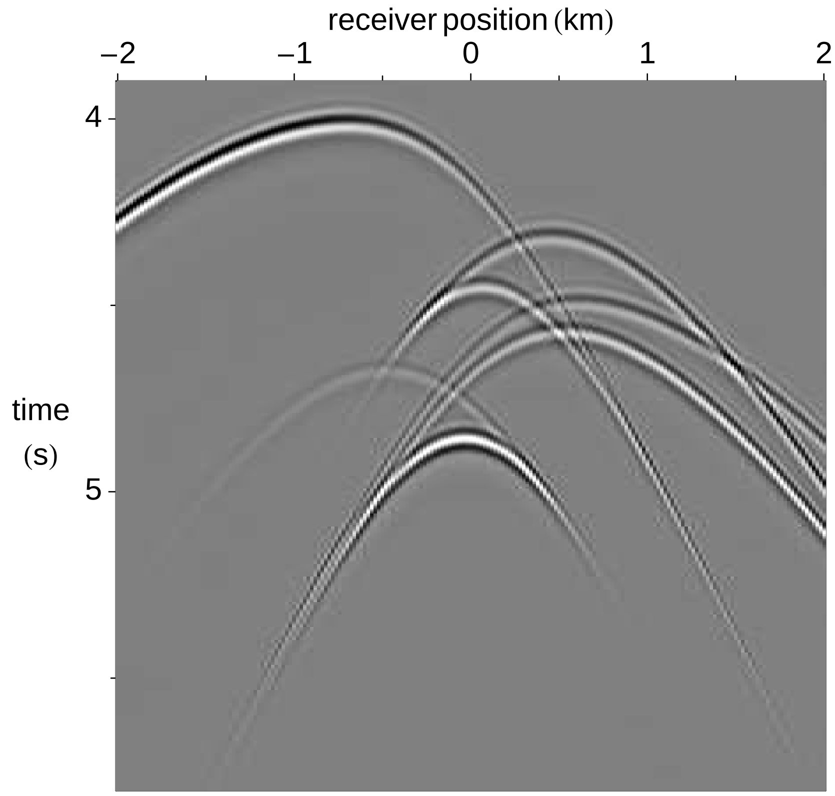
Example from (S. & Symes, '02), extending (Nolan & Symes, '96) to offset and angle imaging.



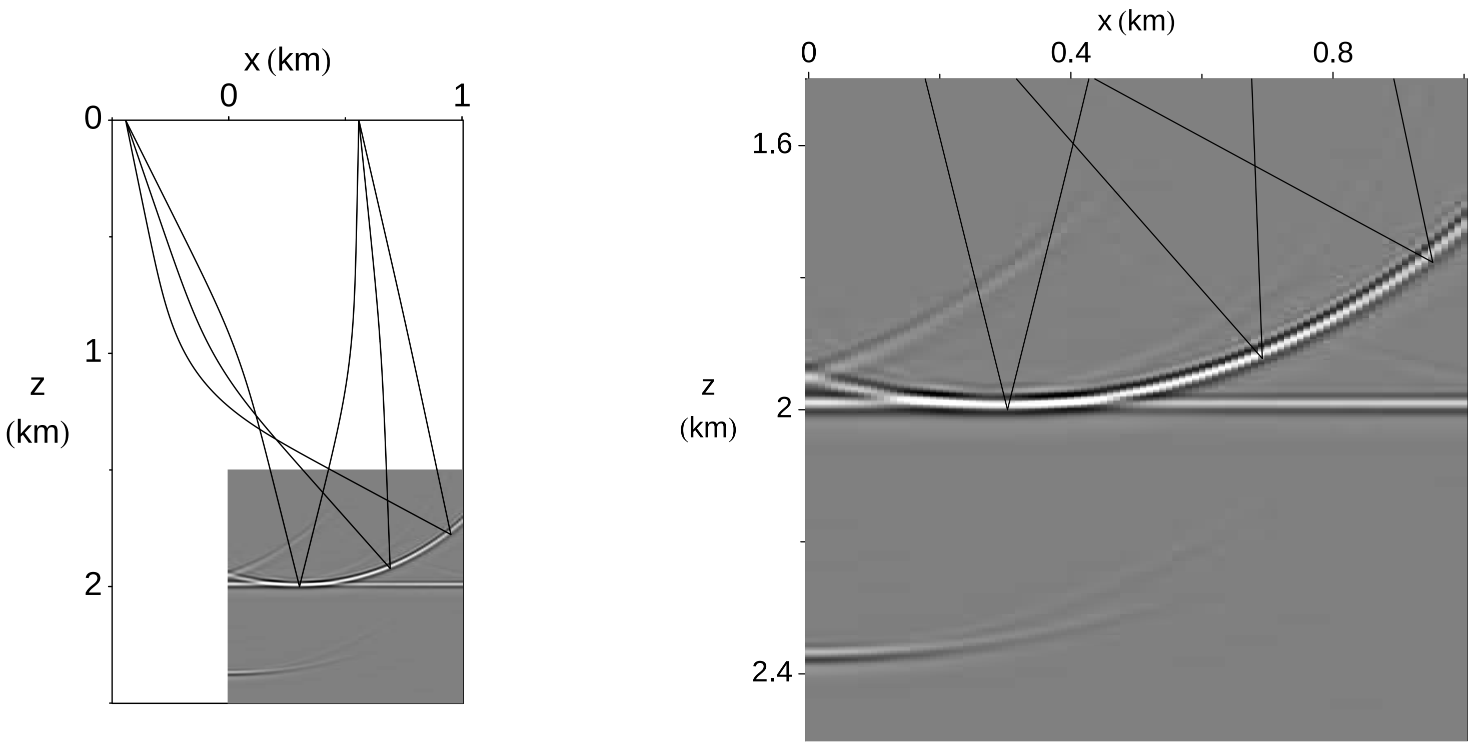
Rays and wave fronts



Data (single source)



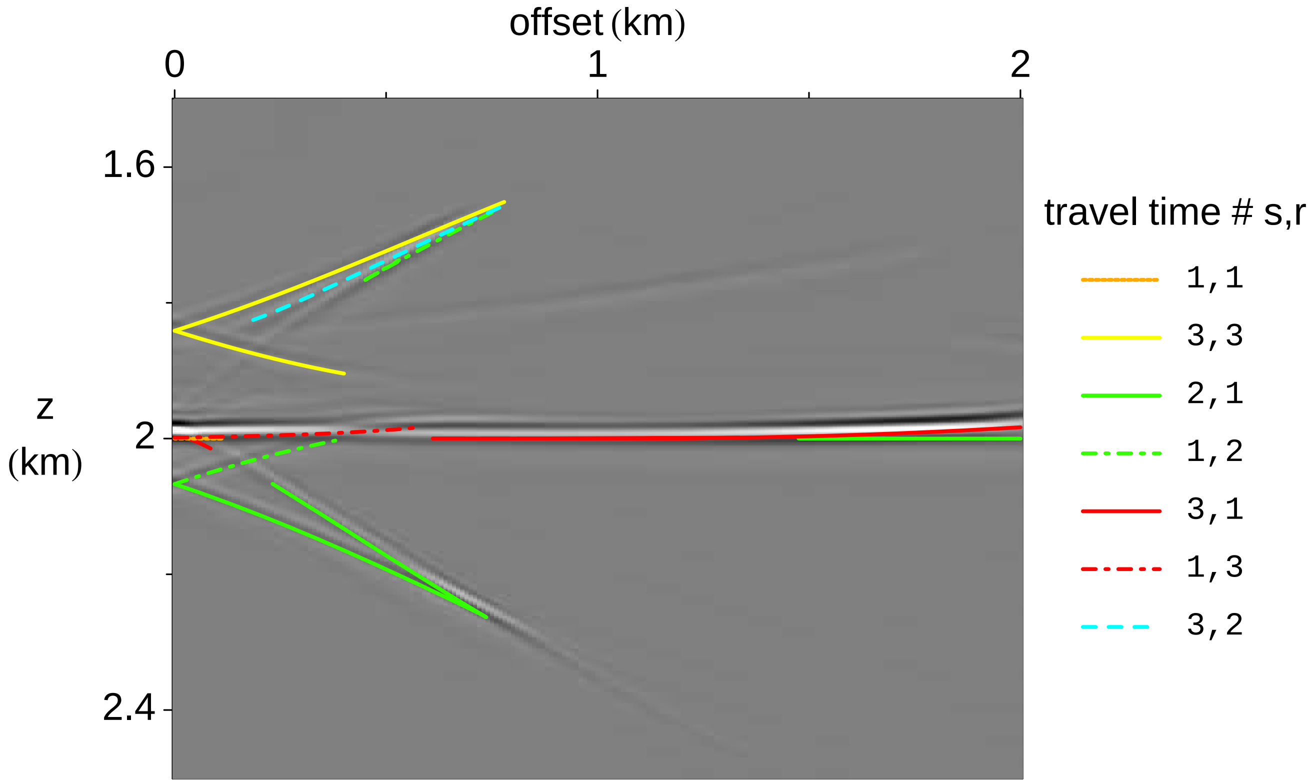
Constant offset image



Multiple ray pairs lead to both correct and incorrect events in image

Offset common image gather

Varying offset, fixed horizontal position $x_1 = 0.3$ km



With constant source/offset binning we practically have to exclude caustics.

4. Imaging with full data

Least squares

Assume source and receivers covering surface. Then data is 5-dimensional data for a 3-dimensional image.

Overdetermined problem $\rightarrow$ **least squares** : find f_{LS} that minimizes

$$\|F f_{\mathsf{LS}} - d\|^2.$$

Implies that

$$F^* F f_{\mathsf{LS}} = F^* d.$$

(1) compute normal operator $N = F^* F$
(2) Compute a (possibly regularized) inverse $\langle F^* F \rangle^{-1}$, then

$$f_{\mathsf{LS}} = \langle F^* F \rangle^{-1} F^* d.$$

Issues : - Mapping of singularities
- Amplitude factor

Injectivity condition and imaging equations

Imaging methods has access to s, r, t, p_s, p_r. Take-off directions α_s, α_r are uniquely determined.

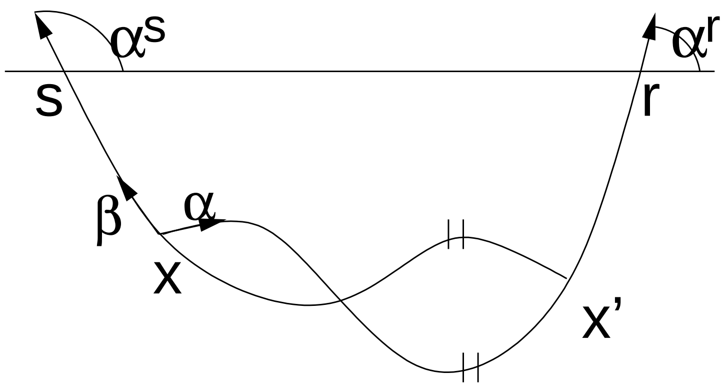
Traveltime Injectivity Condition $(s, r, t, \alpha_s, \alpha_r)$ determine uniquely (x, α, β).

Non-uniquene solutions x, and x' if

(1) x, x' on ray determined by s, α_s

(2) x, x' on ray determined by r, α_r

(3) travel time equal between
x and x' along two rays



Imaging equations : 5 eqns, 3 unknowns (3-D)

$$T^{(i,j)}(x', s, r) = T_{\text{data}}(s, r),$$
$$\nabla_s T^{(i,j)} = \nabla_s T_{\text{data}}, \qquad \nabla_r T^{(i,j)} = \nabla_r T_{\text{data}}.$$

Artifacts more rare

Normal operator

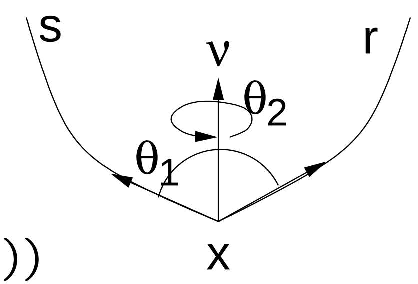
Theorem (Ten Kroode, Smit & Verdel '98) Assume Traveltime Injectivity Condition, then the operator $N = F^*F$ is a pseudodifferential operator of order $n-1$, i.e. of the form

$$Nf = (2\pi)^{-n} \int_{\mathbb{R}^n} \mathrm{e}^{\mathrm{i}x\cdot k_x} N(x, k_x) \widehat{f}(k_x)\, \mathrm{d}k_x.$$

For $N(x, k_x)$ we have

$$N(x, k_x) = B(x, \frac{k_x}{|k_x|})|k_x|^{n-1} + B(x, -\frac{k_x}{|k_x|})|k_x|^{n-1} + \text{lower order.}$$

with weight factor computed by integration over subsurface coords. scattering angle/azimuth(3-D)



$$B(x,\nu) = \frac{c_0(x)^3}{16(4\pi)^5} \int\int \mathrm{d}\theta_1\, \mathrm{d}\theta_2\, \psi(s(x,\nu,\theta), t(x,\nu,\theta), t(x,\nu,\theta)) \times \frac{\sin(\theta_1) c_0(r(x,\nu,\theta)) c_0(s(x,\nu,\theta))}{\cos(\theta_1/2)\cos(\alpha_r(x,\nu,\theta))\cos(\alpha_s(x,\nu,\theta))}.$$

Application in Kirchhoff migration

Map F^* is close to a generalized Radon transform $+$ $n{-}1$ derivatives (n dimension)

Need regularized inverse for each (x, ν).
Not infinite when $B(x,\nu) + B(x,-\nu) \to 0$,

$$\langle B(x,\nu) + B(x,-\nu) \rangle^{-1} \sim \frac{B(x,\nu) + B(x,-\nu)}{(B(x,\nu) + B(x,-\nu))^2 + \epsilon^2}.$$

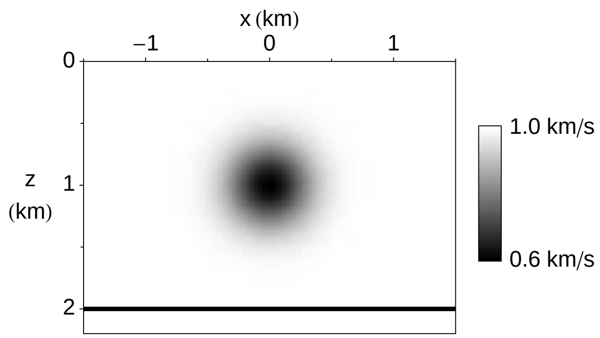
$\langle F^* F \rangle^{-1} F^*$ can be approximated by a Radon transform (to highest order away from caustic points)

$$f_{\mathsf{LS}}(x) \approx \sum_{i,j} \int\int (...) d(s, r, T^{(i,j)}(x,s,r))\, \mathrm{d}s\, \mathrm{d}r.$$

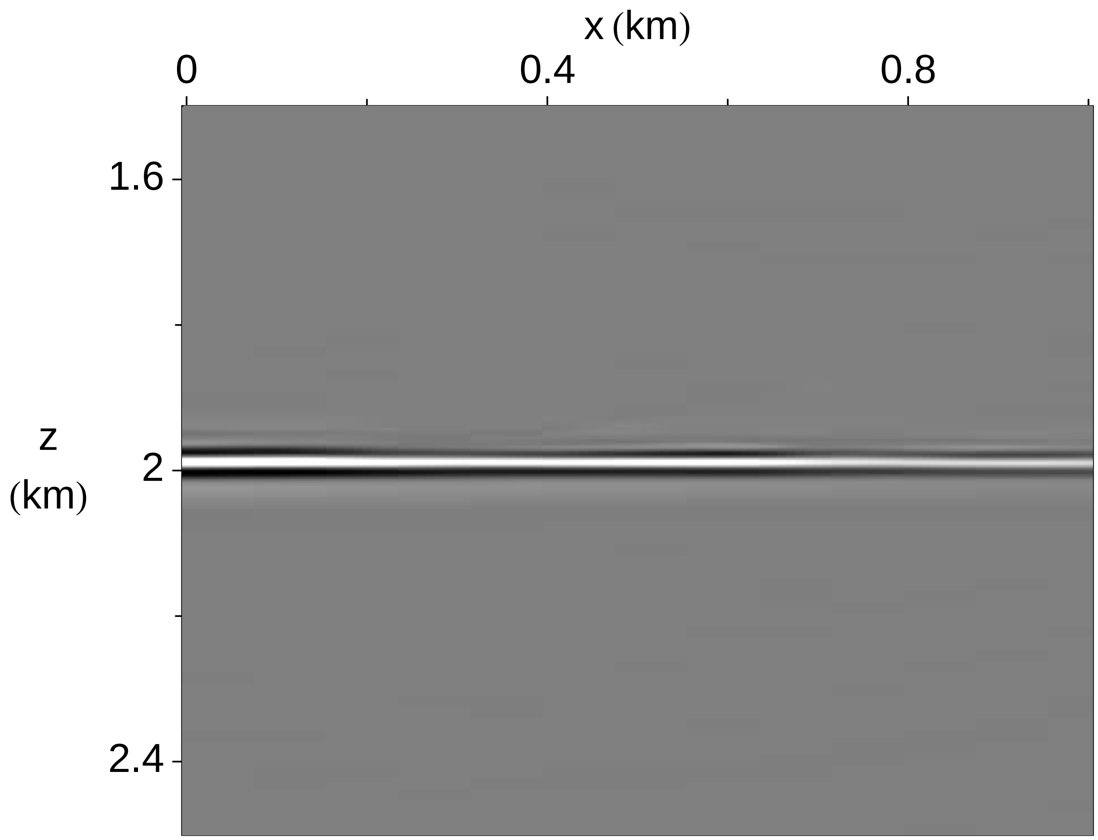
with (...) amplitudes and Hilbert transforms.

Lens example

Medium



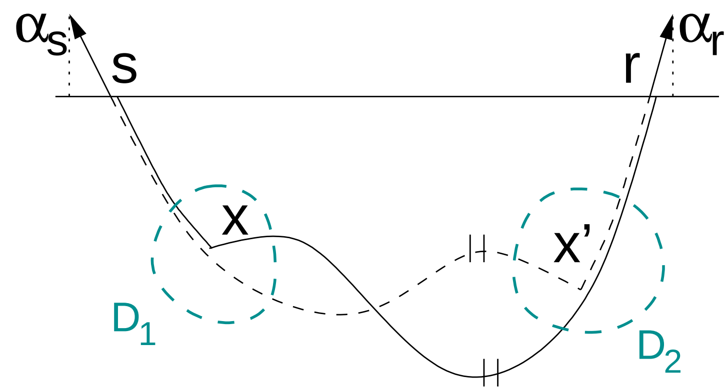
Stacked image



Artifact free as predicted.

If present, artifacts are in general less singular than image, but not always (S. '00).

Artifacts



Analysis from (S. '00).
Solutions x' and x are separated
Let

$$F_j = F \text{ restricted to } D_j, \quad j = 1, 2.$$

Then

$$(F_1 + F_2)^*(F_1 + F_2) = \underbrace{F_1^* F_1 + F_2^* F_2}_{\text{image}} + \underbrace{F_2^* F_1 + F_1^* F_2}_{\text{artifact}}.$$

Results

- **Artifacts are in general less singular than image**
  $F_2^* F_1 + F_1^* F_2$ is in general less singular than $F_1^* F_1 + F_2^* F_2$
  (2-D : explicit estimates for frequency decay)

- **In certain special situations artifacts are as strong as the image**

5. Angle common image gathers

Angle common image gathers : introduction

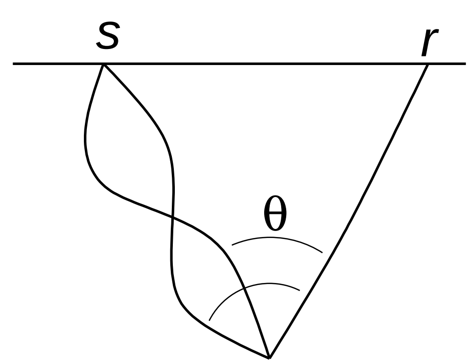
Image sets $f_{\text{source}}(s,x)$, $f_{\text{offset}}(h,x)$ not optimal

- Velocity analysis in complex media made difficult by artifacts.
  - $f_{\text{source}}(s,x)$ not independent of s, even for correct c_0 due to artifacts.
  - $f_{\text{offset}}(h,x)$ not independent of h for correct c_0.
- Identification of medium parameters at discontinuity.
  The reflected signal in general has an angle dependent reflection coefficient $R(x,\theta) \rightarrow$ useful to have image as a function of angle (AVA).

Map

all data $\longrightarrow$ set of images

$\longrightarrow$ images for each scattering angle θ

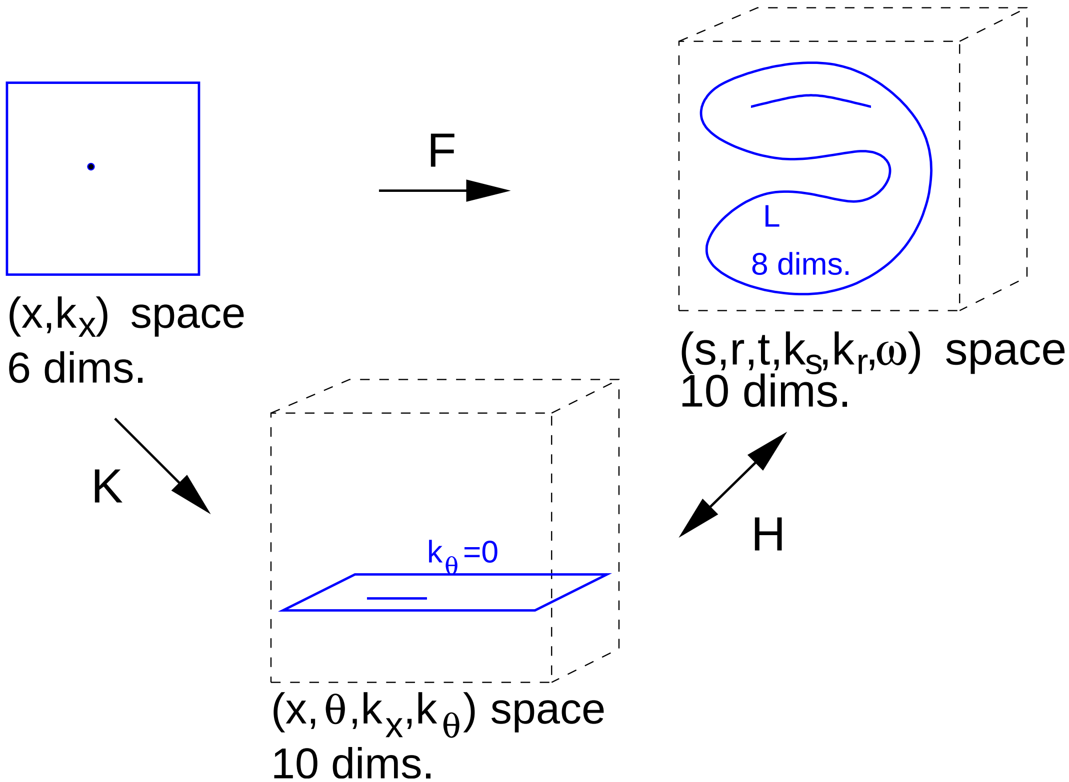


- **Generalized Radon Transform (Kirchhoff) method** Kirchhoff type integration formula, with angle binning (De Hoop et al.'94, Xu et al.'01, S. & Symes '02).

- **Wave equation** using downward continuation of data by wave equation methods, (De Bruin et al. '90, Prucha et al. '99, S. & De Hoop '01).

- Related to mathematical ideas by Guillemin ('85) (S. & De Hoop '02).

Mapping of singularities, forward map and angle transform

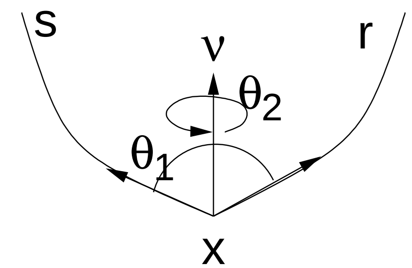


Traveltime Injectivity Condition $\rightarrow L$ has no selfintersections. Implies existence of angle migration operators.

Non-uniqueness of angle migration operators.

Kirchhoff angle CIG's

Restrict Kirchhoff integral to constant scattering angle (binning). Integration along curve in the (s, r) plane (2-D), or a surface (3-D)



$$f_{\text{angle}}(\theta, x) = \int \mathrm{d}\nu \, B(x, \nu, \theta) H^{-\sigma(x,\nu,\theta)} \\ \times d(s(x, \nu, \theta), r(x, \nu, \theta), t_s(x, \nu, \theta) + t_r(x, \nu, \theta)).$$

where B is an amplitude factor.

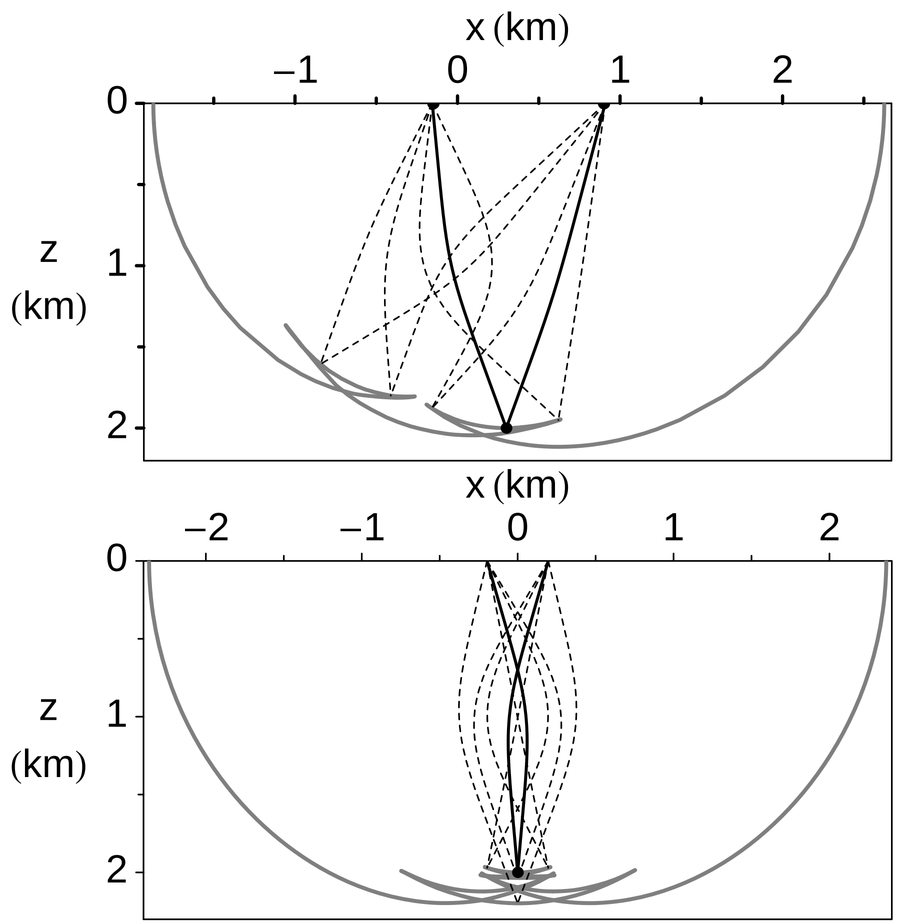
Imaging equations : Assume event at $t = T_{\text{data}}(s, r)$. Singular contribution when

$$T^{(i,j)}(x, s, r) = T_{\text{data}}(s, r),$$
$$\frac{\partial s}{\partial \nu} \frac{\partial T^{(i,j)}}{\partial s} + \frac{\partial r}{\partial \nu} \frac{\partial T^{(i,j)}}{\partial r} = \frac{\partial s}{\partial \nu} \frac{\partial T_{\text{data}}}{\partial s} + \frac{\partial r}{\partial \nu} \frac{\partial T_{\text{data}}}{\partial r}.$$

3 eqs. for 3 unknowns (3-D), like constant source/offset gathers.

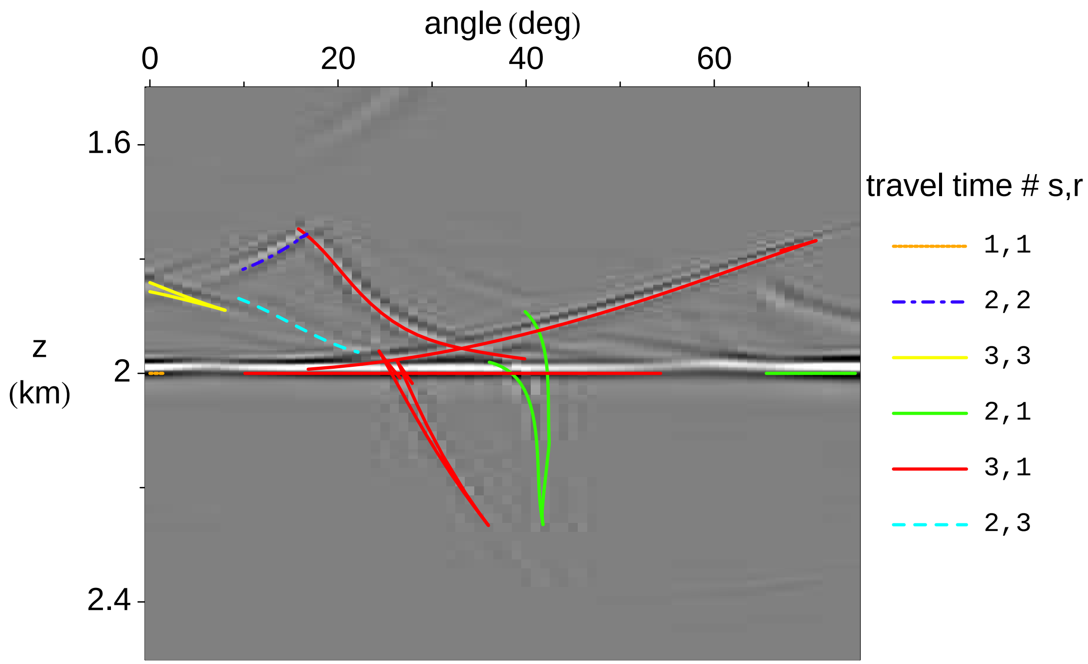
Scattering angle : example

Identify solutions to the imaging equations for the lens example for two choices of (s, r, t, p_s, p_r) (S. & Symes, '02).



Angle common image gather

horizonal
pos.
$x = 0.3$ km



Strong kinematic artifacts in Kirchhoff angle migration!
Present also in more realistic examples (Marmousi like model of Xu et al., gas lens model (Brandsberg-Dahl et al.))
Artifacts move out with angle; can sometimes be suppressed.

“Wave equation” approach

Define subsurface half offset h.

Multiply Born formula by $1 = \int_{\mathbb{R}^{n-1}} \delta(h)\,\mathrm{d}h$

$$d(s,r,t) = \int_{\mathbb{R}^n_+} \int_0^t \int_{\mathbb{R}^{n-1}\times\{0\}} G_0(r, x+h, \tilde{t}) G_0(x-h, s, t-\tilde{t}) \times \underbrace{f(x)\delta(h)}_{f_{WE}(h,x)} \,\mathrm{d}h\,\mathrm{d}t\,\mathrm{d}x.$$

So we have linear map H

$$H : f_{\mathrm{WE}} \mapsto d$$
$$f_{\mathrm{WE}}(h,x) = \delta(h) f(x)$$

Compute f_{WE} by inverting H, then

1. Imaging from $f_{\mathrm{WE}}(h,x)$.
2. Angle transform by Radon transform (slant stack) $f_{\mathrm{WE}}(h,x)$.
   H is adjoint of survey sinking + restriction to $t = 0$

Inversion of H

Double square root assumption If x is in domain of interest, and there is (s, r, t) in acquisition set such that x and s and x and r are connected by ray intervals of total time t, then the ray is nowhere tangent to horizontal, i.e. along the two rays $\left|\frac{\partial x_n}{\partial t}\right| > \epsilon$.

Main idea (S. & De Hoop '01) Possibly after applying a microlocal cutoff, H is an invertible FIO on a part of phase space, such that

$$H^* d = (\text{pseudodifferential operator}) f_{\mathsf{WE}}(h, x).$$

Results

- Precise form, including amplitudes, of the inverse of H.
- Imaging operator
- Angle transform: Consider the composition with a Radon transform

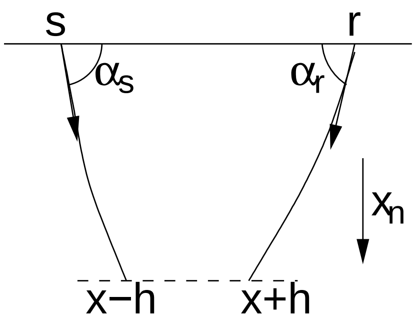
Imaging equations

Use asymptotic expression for G_0 (traveltimes)

$$H^* d(x,h) \approx \sum_{i,j} \int (\ldots) d(s,r,t - T^{(i)}(s,x-h) - T^{(j)}(r,x+h))\, \mathrm{d}s\, \mathrm{d}r.$$

with (...) operations that do not affect position of singularities.

No binning : Integration over all data for each (x,h).

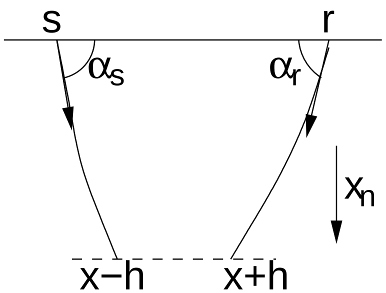


Assume event in data, traveltime $T_{\text{data}}(s,r)$.
Imaging equations

$$T^{(i)}(s,x-h) + T^{(j)}(r,x+h) = T_{\text{data}}(s,r)$$

$$\frac{\partial T^{(i)}}{\partial s}(s,x-h) = \frac{\partial T_{\text{data}}}{\partial s}(s,r), \quad \frac{\partial T^{(j)}}{\partial r}(r,x+h) = \frac{\partial T_{\text{data}}}{\partial r}(s,r).$$

Back focusing at $h = 0$



Imaging equations

$$T^{(i)}(s, x-h) + T^{(j)}(r, x+h) = T_{\text{data}}(s, r) \quad (1)$$

$$\frac{\partial T^{(i)}}{\partial s}(s, x-h) = \frac{\partial T_{\text{data}}}{\partial s}(s, r), \qquad \frac{\partial T^{(j)}}{\partial r}(r, x+h) = \frac{\partial T_{\text{data}}}{\partial r}(s, r) \quad (2)$$

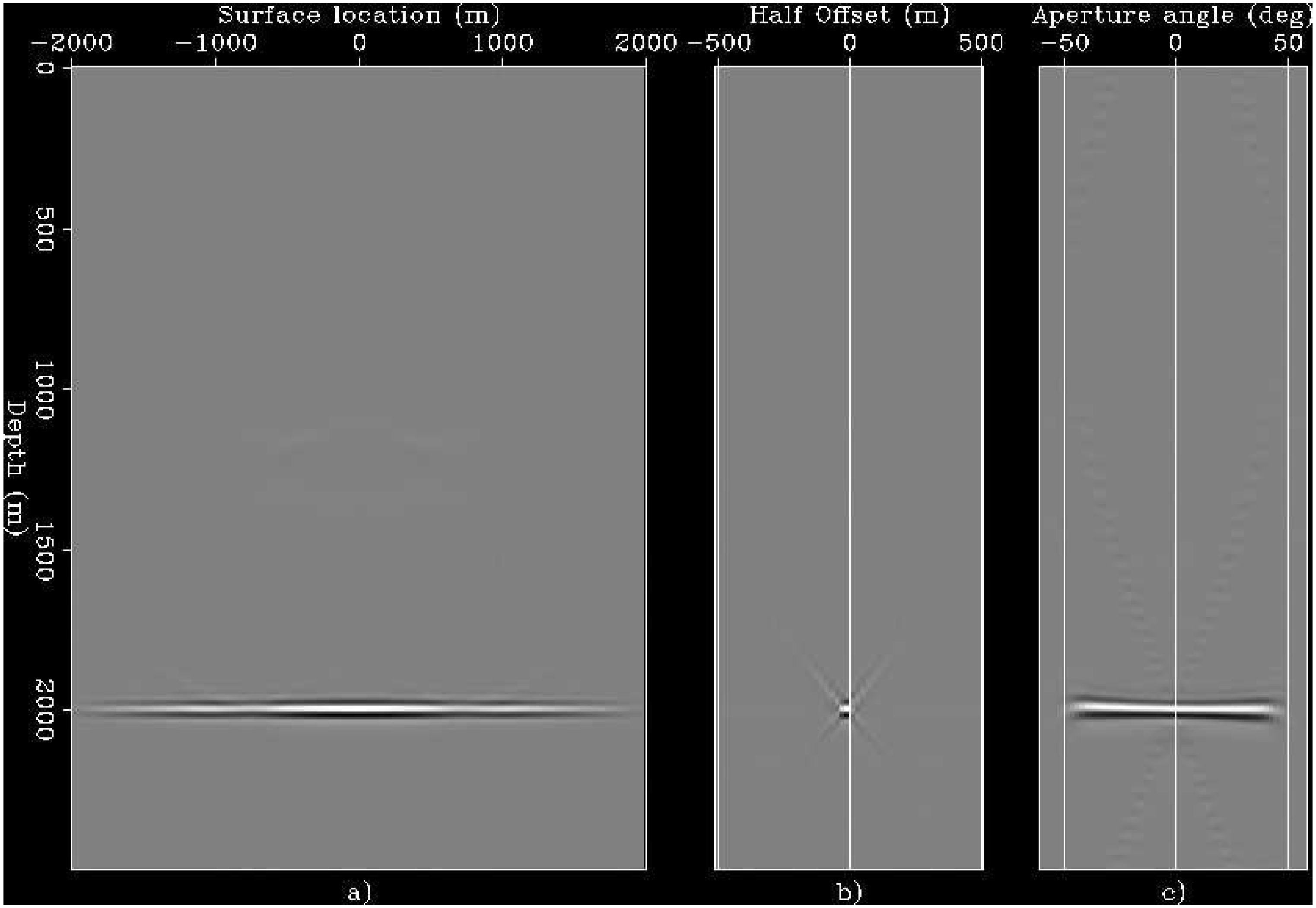
$h = 0$, $x = x_{\text{refl}}$ is a solution to (1) and (2).
This solution is **unique**

1. takeoff angle α_s determined by (2), so $x-h$ is on the (s, α_s) ray, takeoff angle α_r determined by (2), so $x+h$ is on the (r, α_r) ray. One solution for each x_n.

2. The traveltime equation (1) now determines the depth because

$$\frac{\partial}{\partial x_n}(T^{(i)}(s, x-h) - T^{(j)}(r, x+h)) > 0.$$

Wave equation lens example



(picture by Biondo Biondi, Stanford Exploration Project)

Artifact free!

Conclusions

- We presented mathematical results about imaging, or the reconstruction of reflectivity
  - Under what assumptions this is possible
  - Different imaging settings

- Multipathing leads to image artifacts in prestack migration using binning approaches (common source binning, common offset binning, or common angle binning)

- Artifacts are much more rare in the stacked image, and in prestack wave equation migration.