Signal Analysis and Imaging Group
Department of Physics
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Regularized Migration/Inversion

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This doc ->
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Outline

- Motivation and Goals
- Migration/inversion - Evolution of ideas and concepts
- Quadratic versus non-quadratic regularization
  - two examples
- The migration problem
  - RLS migration with quadratic regularzation
  - Examples
  - RLS migration with non-quadratic regularization
  - Examples
- Summary

Motivation

To go beyond the resolution provided by the data (aperture and band-width) by incorporating *quadratic* and *non-quadratic* regularization terms into migration/inversion algorithms

This is not a new idea…

Evolution of ideas and concepts

Migration with Adjoint Operators ***[Current technology]***

RLS Migration (Quadratic Regularization) ***[Not in production yet]***

RLS Migration (Non-Quadratic Regularization) ***[??]***

→

Resolution

Evolution of ideas and concepts

Migration with Adjoint Operators

1 ***RLS Migration (Quadratic Regularization)***

2 ***RLS Migration (Non-Quadratic Regularization)***

→

Resolution

Evolution of ideas and concepts
Two examples

LS Deconvolution → *Sparse Spike Deconvolution*

LS Radon Transforms → *HR Radon Transforms*

Quadratic Regularization

Stable and Fast Algorithms

Low Resolution

Non-quadratic Regularization

Requires Sophisticated Optimization

Enhanced Resolution

Deconvolution

Quadratic versus non-quadratic regularization

$$W\,r = s$$

Convolution / Cross-correlation / Deconv

Convolution $$W r = s$$

Cross-correlation $$\tilde{r} = W' s$$

Deconvolution $$\tilde{r} = W' W r = R r$$

$$r = R^{-1} \tilde{r}$$

$$= R^{-1} W' s$$

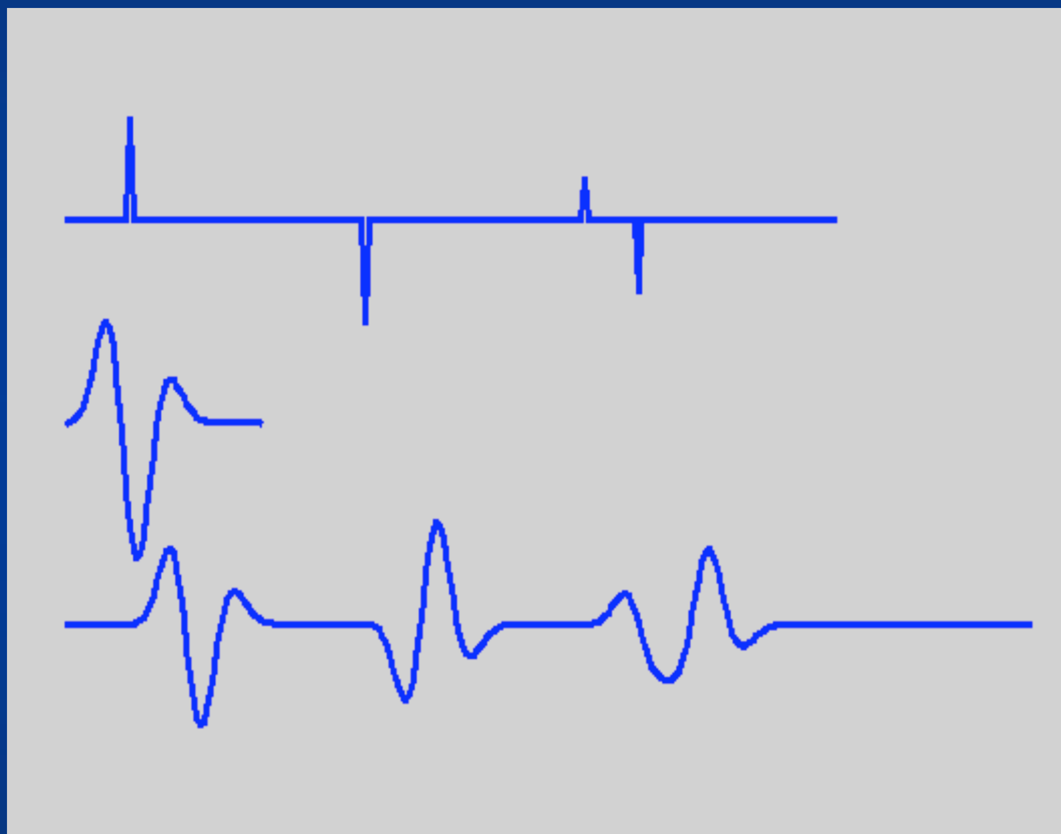
Quadratic and Non-quadratic Regularization

Cost $J = \| W r - s \|^2 + \lambda R(r)$

Quadratic $R = \| r \|^2, \quad r = (W'W + \lambda I)^{-1} W' s$

Non-quadratic $R = \rho(r), \quad r = (W'W + \lambda Q(r))^{-1} W' s$

$$\rho(r) = \sum_i \ln\left(1 + \frac{r_i^2}{\sigma^2}\right)$$



r

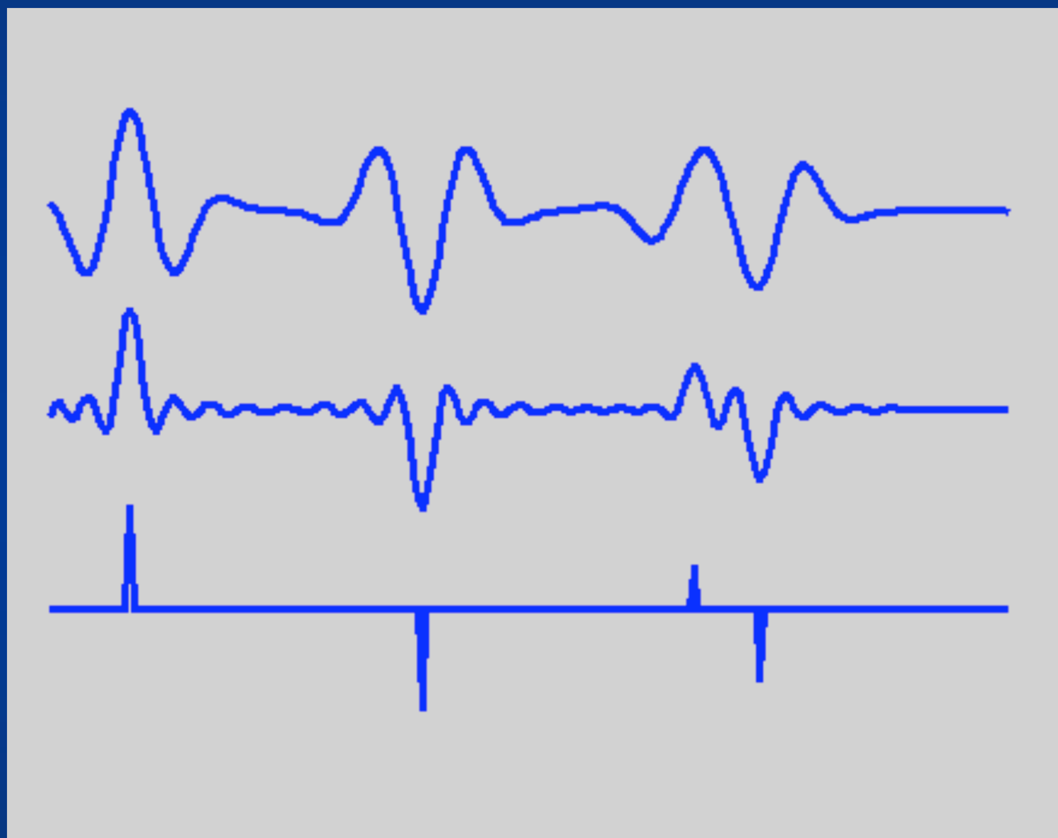
W

$s = W\,r$

Adj

LS

HR



$$\tilde{r} = W's$$

$$r = (W'W + \lambda I)^{-1} W's$$

$$r = (W'W + \lambda Q(r))^{-1} W's$$

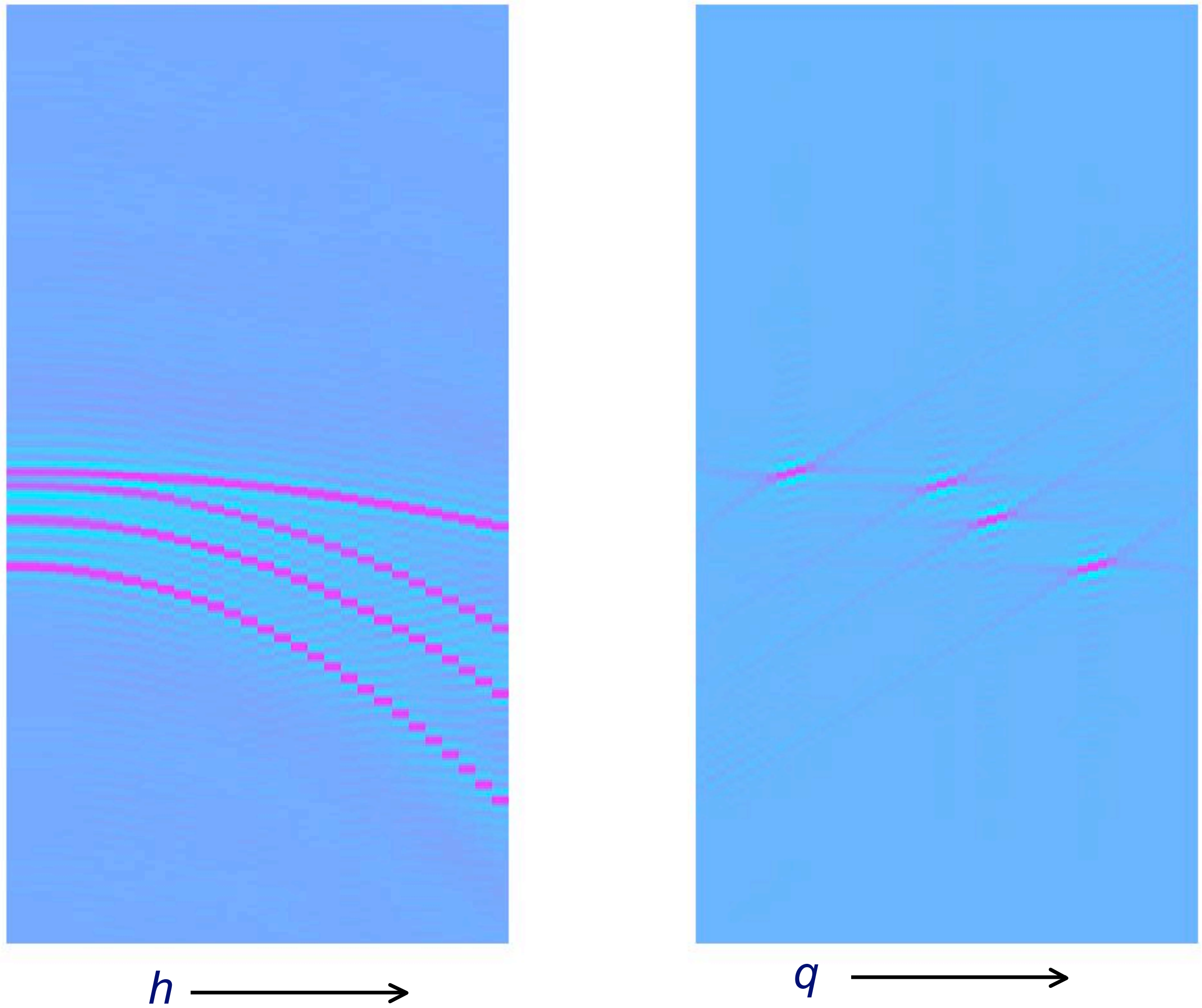
Radon Transform

Quadratic versus non-quadratic regularization

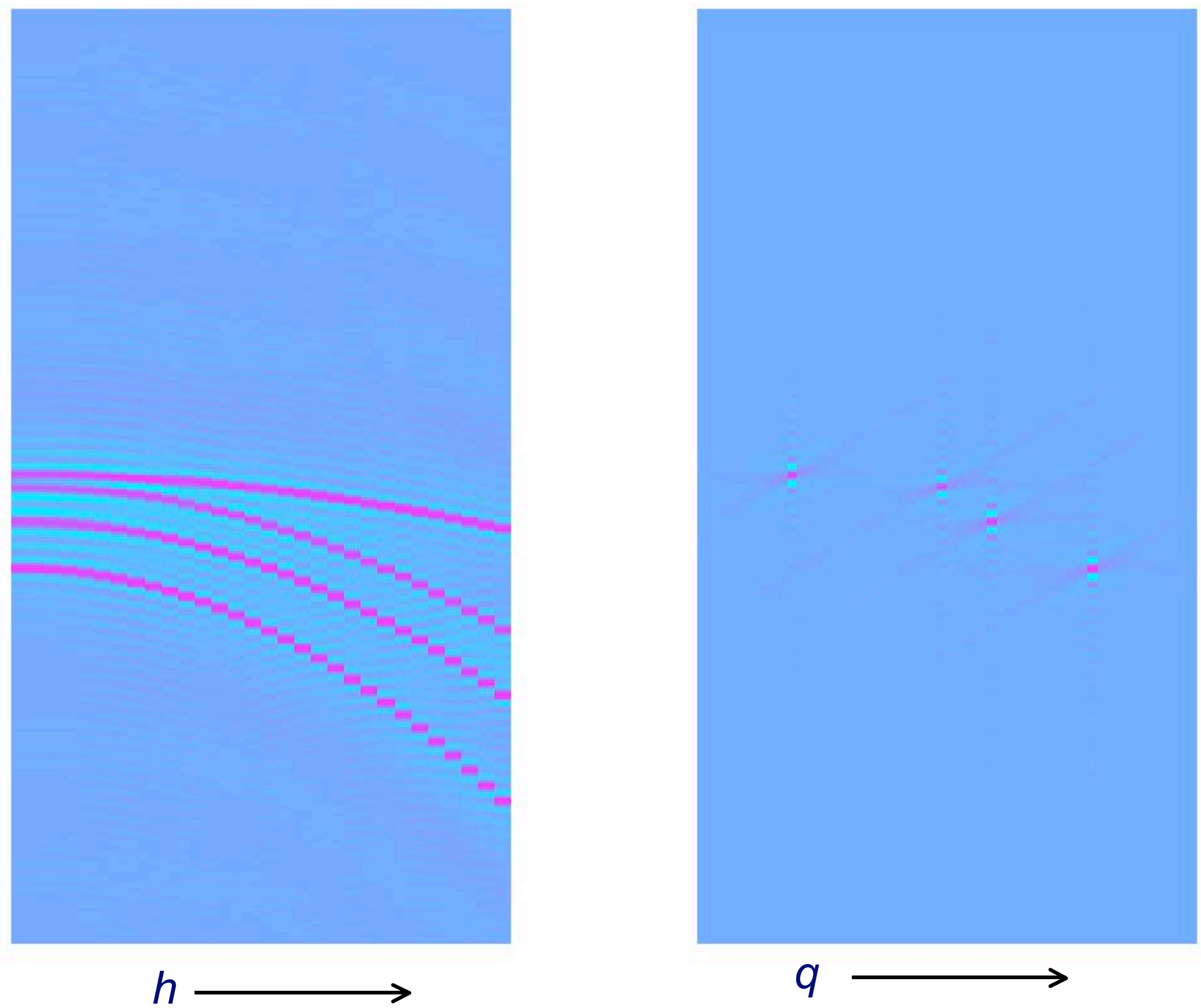
$$\int m(t - \tau(x,p), p)\,dp = d(t,x)$$

$$or$$

$$Lm = d$$



LS Radon



HR Radon

Modeling / Migration / Inversion

Modeling $Lm = d$

Migration $\tilde{m} = L'd$

$$< m, L'd > = < d, Lm >$$

De-blurring problem $\tilde{m} = L'Lm = Km$

$$m = K^{-1}\tilde{m}$$

$$= K^{-1}L'd$$

Back to migration

Inducing a sparse solution via non-quadratic regularization appears to be a good idea (at least for the two previous examples)

Q. *Is the same valid for Migration/Inversion?*

Back to migration

Inducing a sparse solution via non-quadratic regularization appears to be a good idea (at least for the two previous examples)

Q. *In the same valid for Migration/Inversion?*

A. *Not so fast… First, we need to define what we are inverting for…*

Migration as an inverse problem

$$W\ L\ m = d$$ *Forward*

$$\tilde{m} = L'\,W'\,d$$ *Adjoint*

$$d = d(s, g, t)$$ *Data*

$$m = m(x, z)$$ *"Image" or*

$$m = m(x, z, p)$$ *Angle Dependent Reflectivity*

Migration as an inverse problem

- *There is No general agreement about what type of regularization should be used when inverting for m=m(x,z)… Geology is too complicated…*
- *For angle dependent images, m(x,z,p), we attempt to impose horizontal smoothness along the “redundant” variable in the CIGs:*

$$J = \| W\,(L\,m \square d)\,\|^2 + \square \| D_p m(x,z,p)\,\|^2$$

Kuehl, 2002, PhD Thesis UofA

url: cm-gw.phys.ualberta.ca~/sacchi/saig/index.html

Quadratic Regularization Migration Algorithm

Features:

DSR/AVP Forward/Adjoint Operator (Prucha et. al, 1999)

PSPI/Split Step

Optimization via PCG

2D/3D (Common Azimuth)

MPI/Open MP

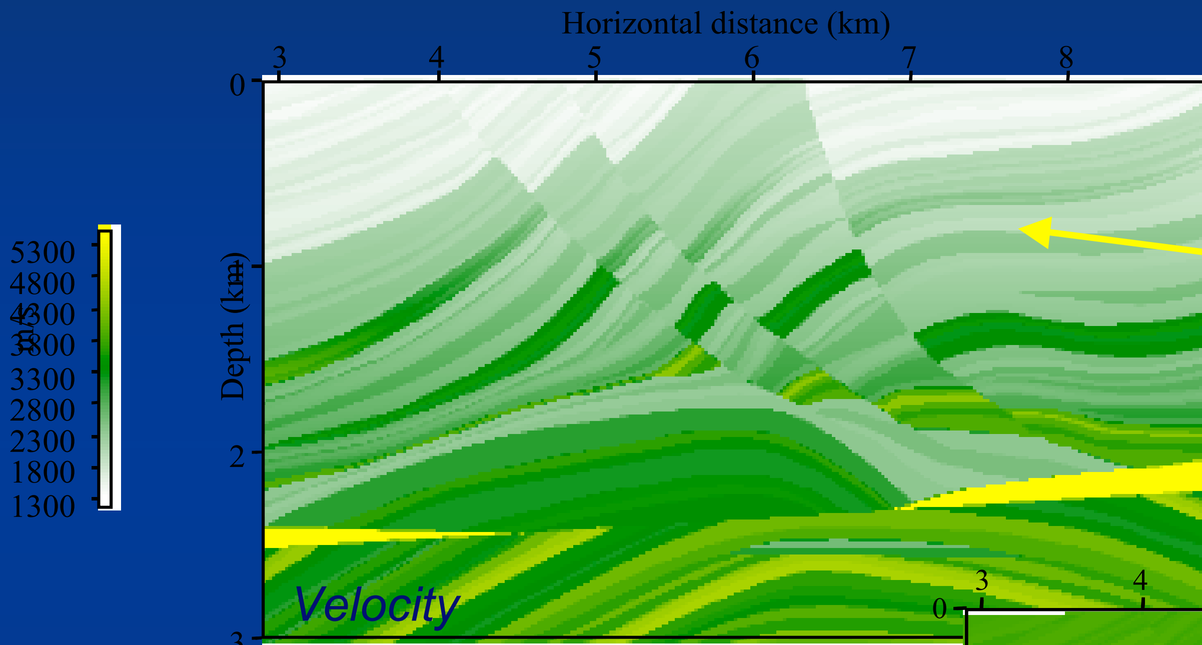
$$J = \| W(Lm - d) \|^2 + \lambda \| D_p m(x,z,p) \|^2$$

Kuehl and Sacchi, Geophysics, January 03 [Amplitudes in 2D]

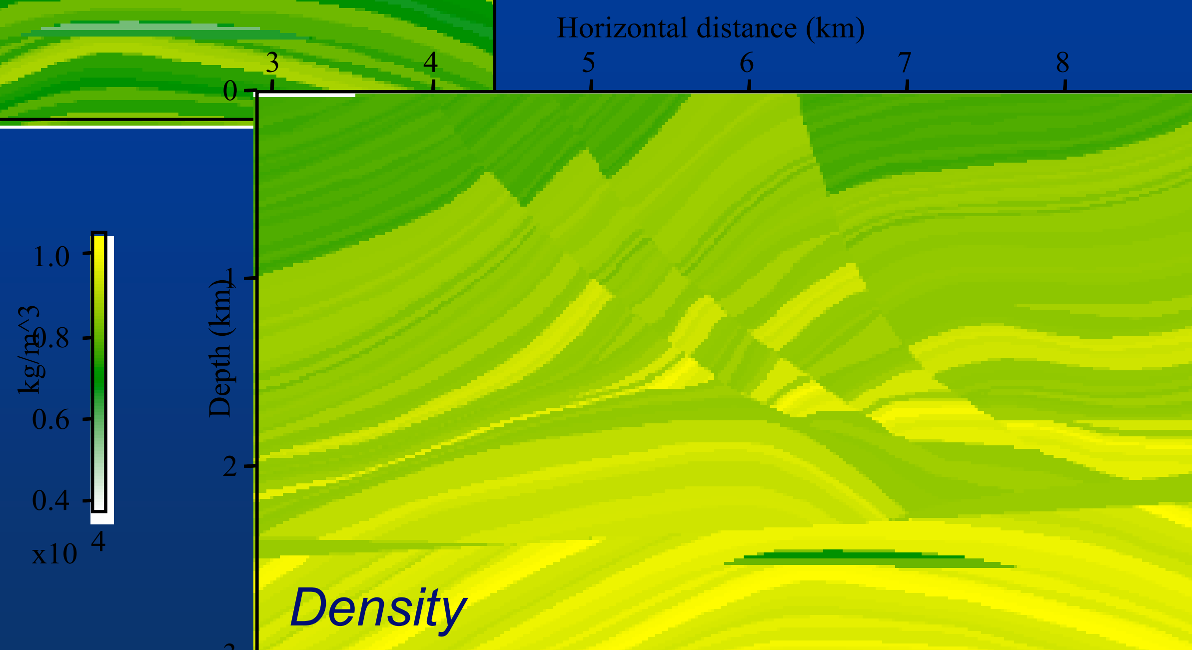
2D - Synthetic data example

Marmousi model

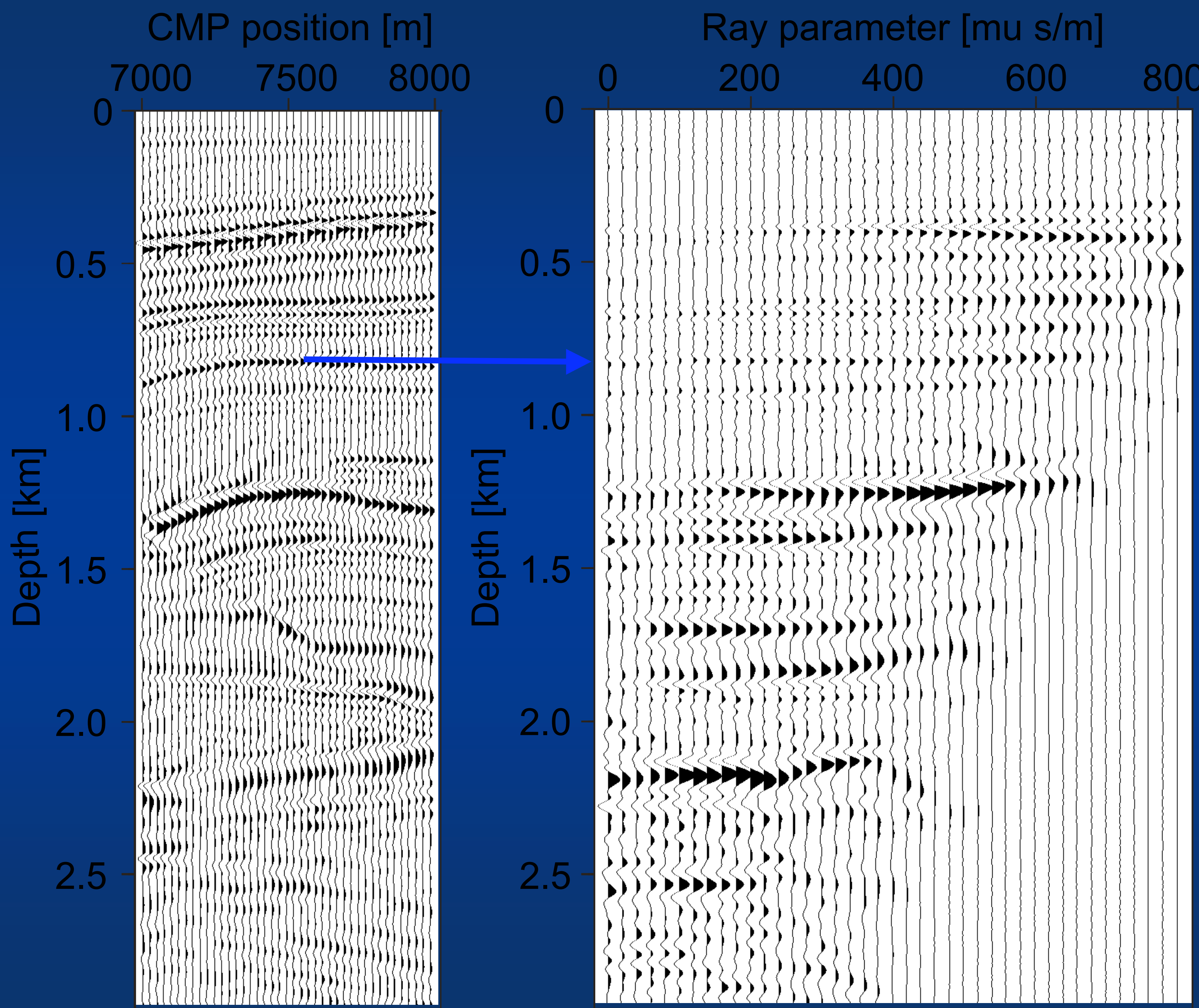
(From H. Kuehl Thesis, UofA, 02)



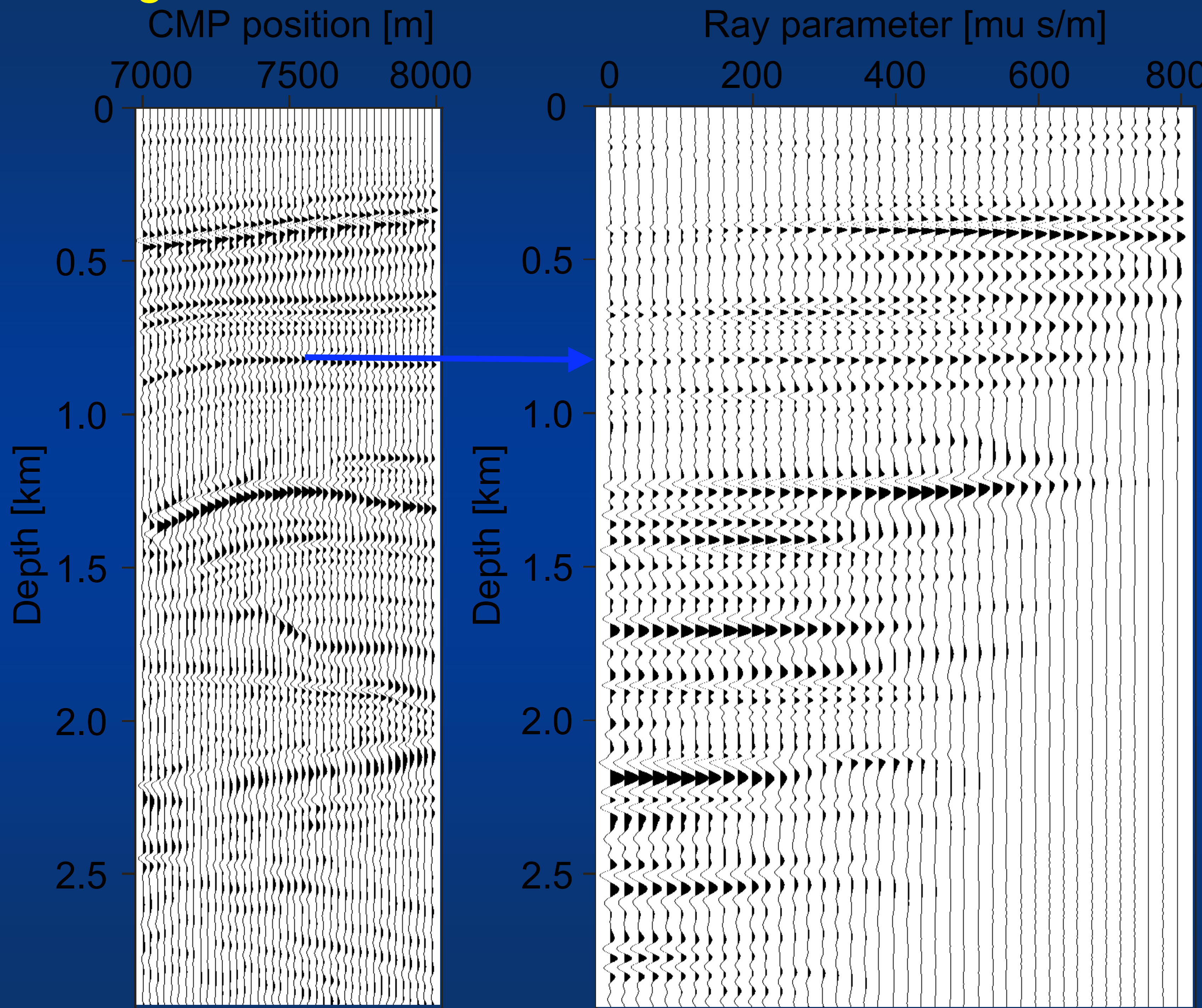
AVA target

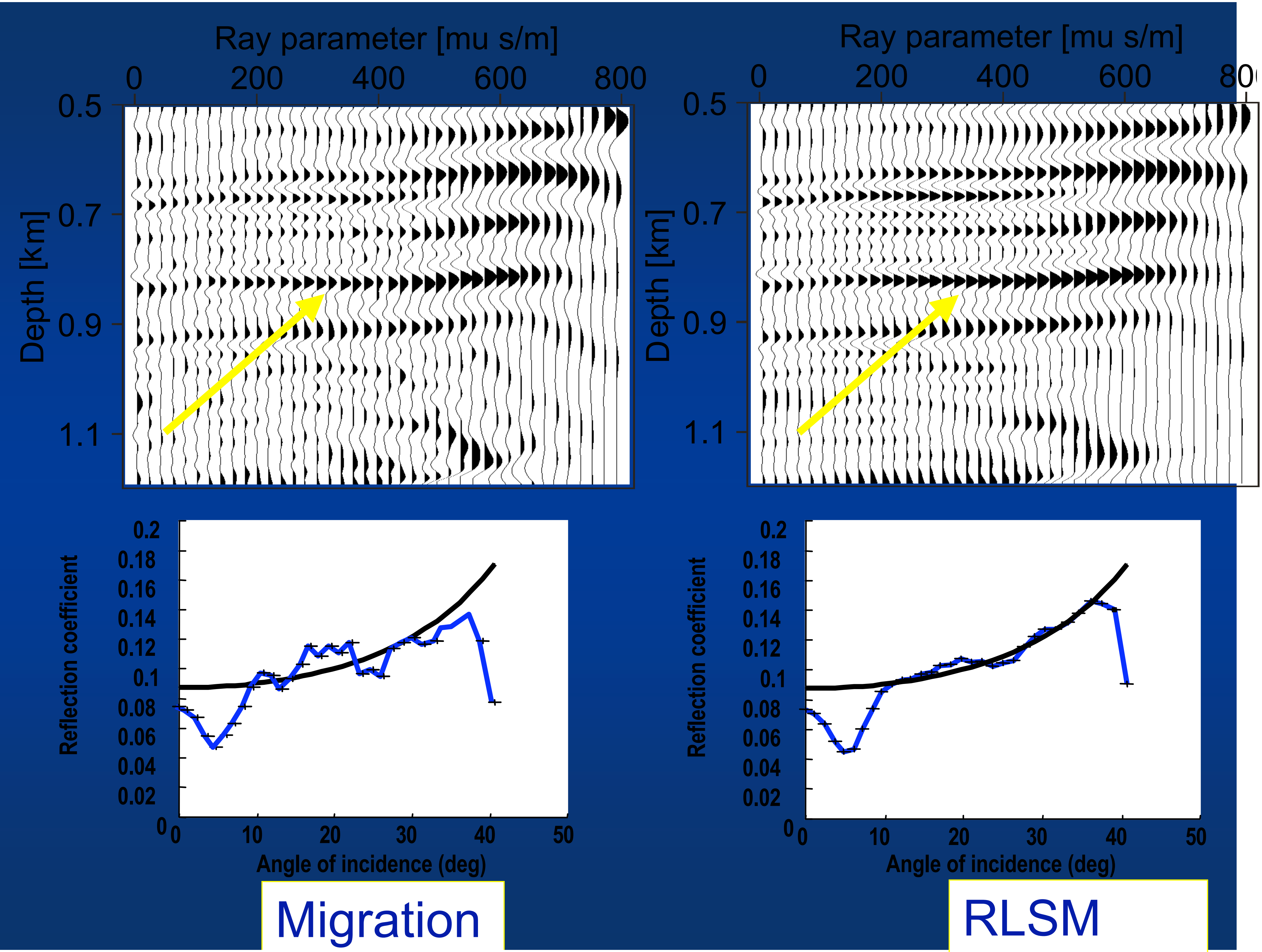


Migrated AVA

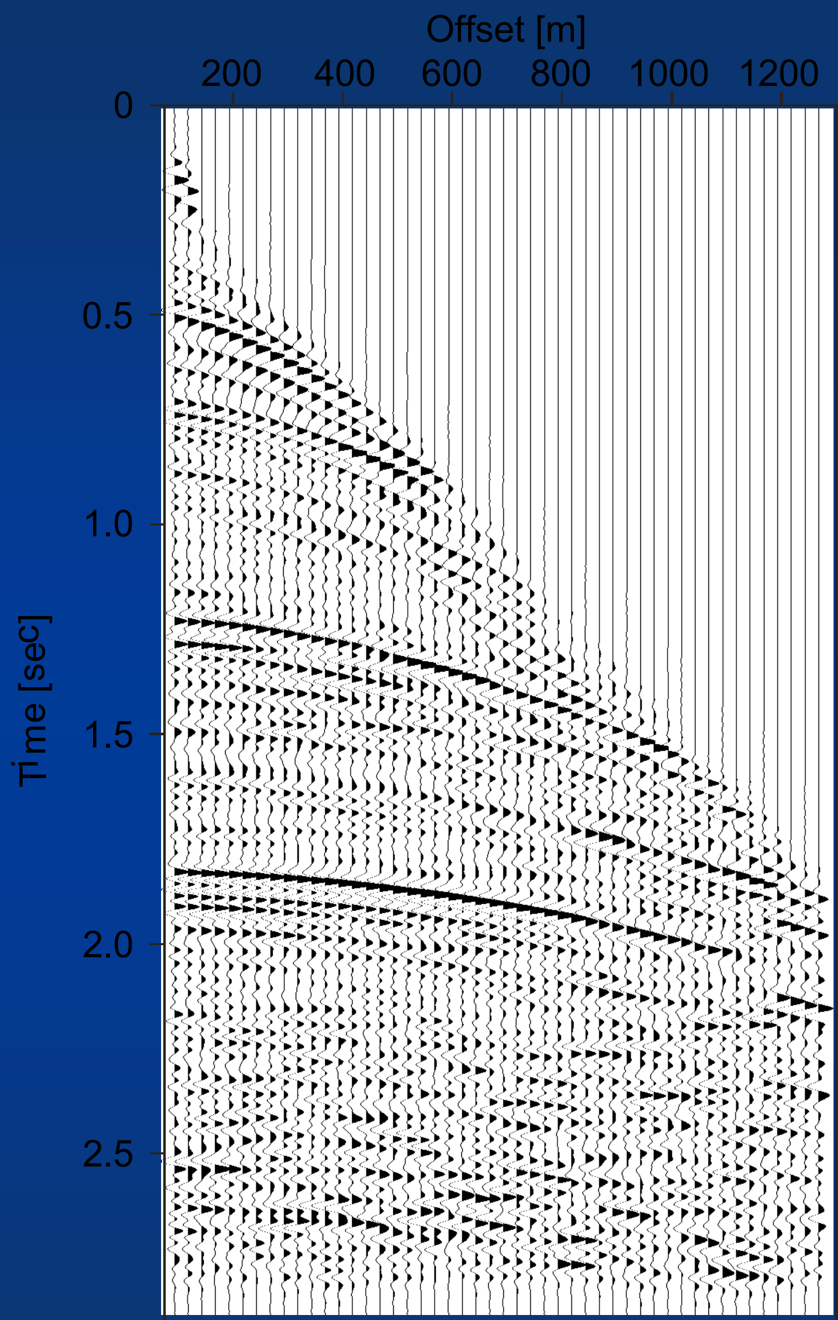


Regularized Migrated AVA

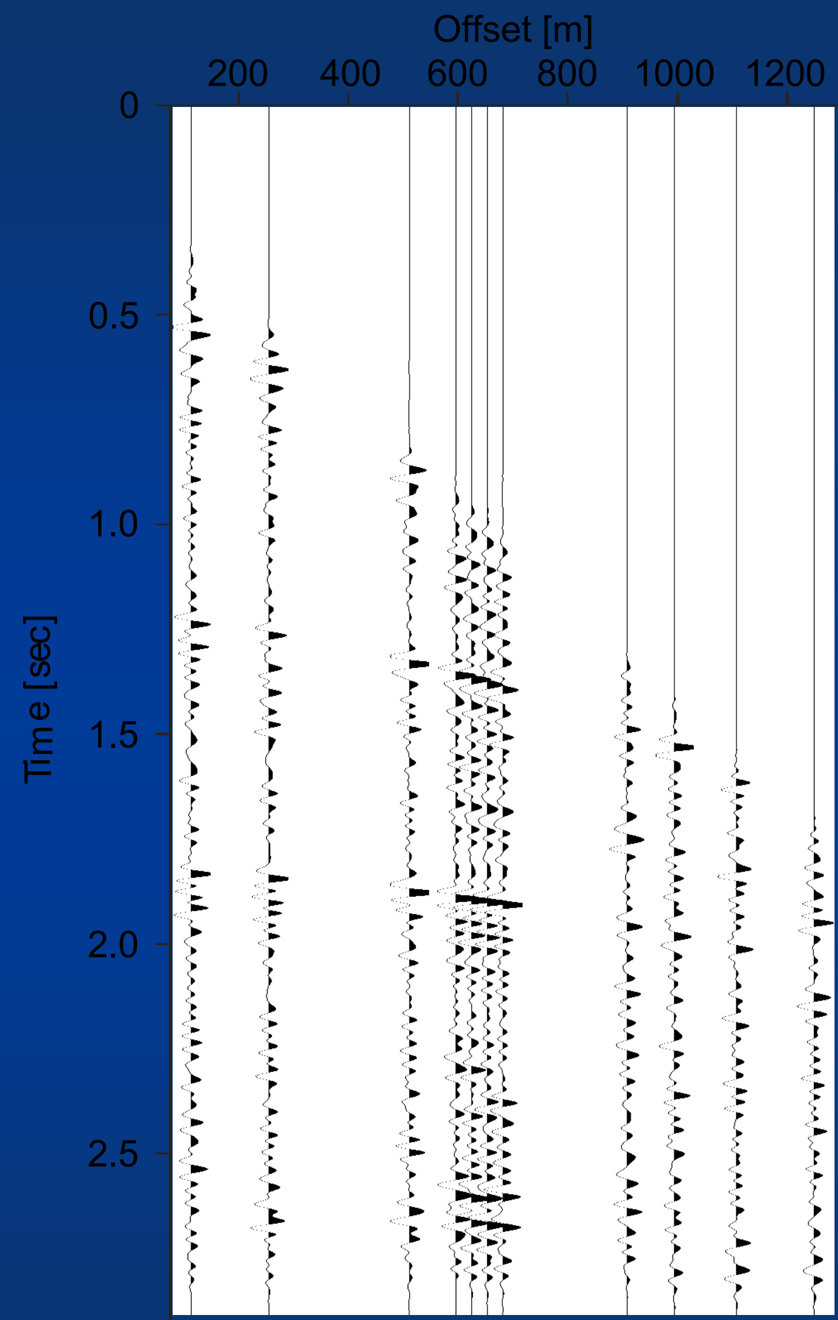




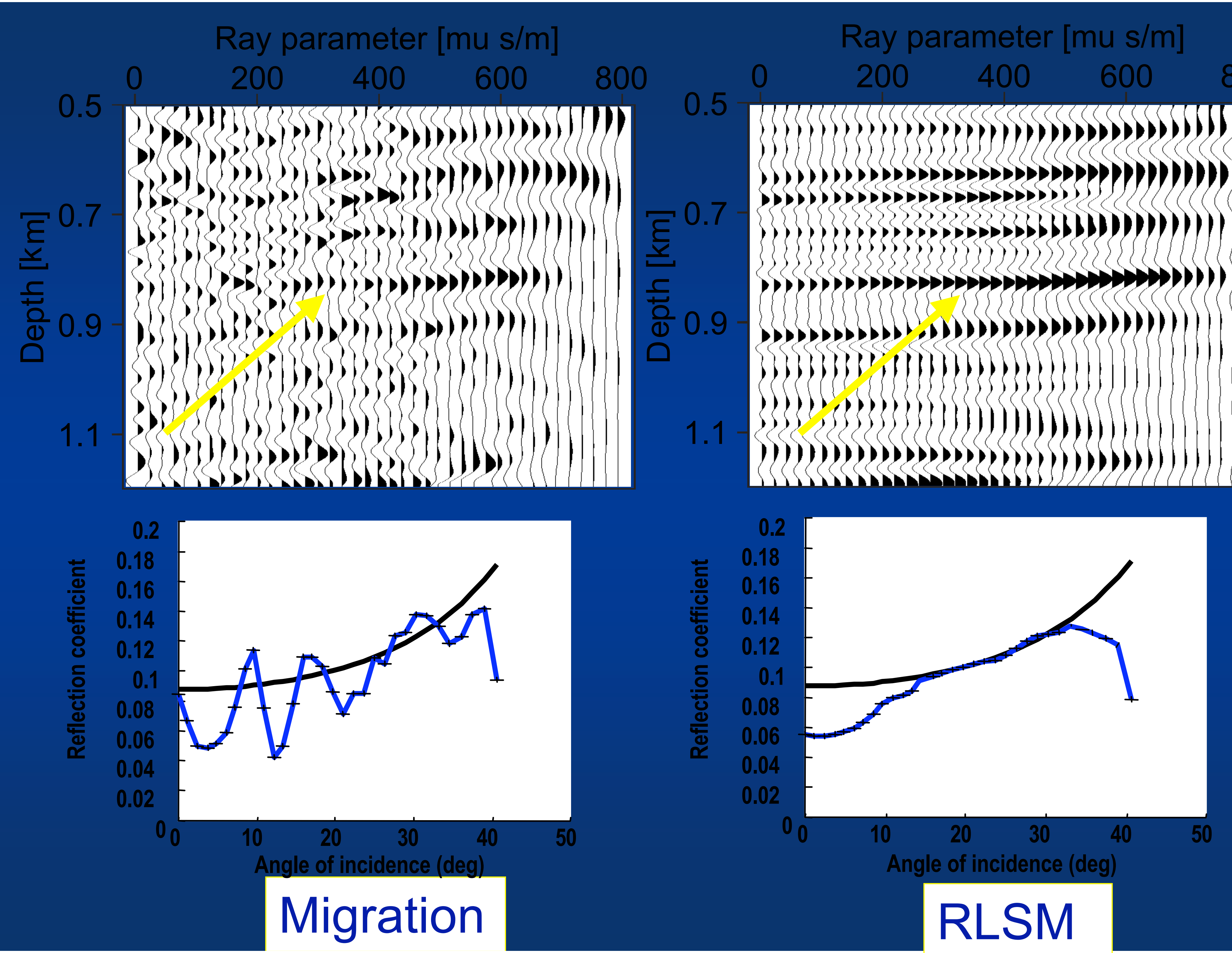
Ray parameter [mu s/m]
0
200
400
600
800
Depth [km]
0.5
0.7
0.9
1.1
Ray parameter [mu s/m]
0
200
400
600
Depth [km]
0.5
0.7
0.9
1.1
Reflection coefficient
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
Angle of incidence (deg)
Reflection coefficient
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
Angle of incidence (deg)
Migration
RLSM



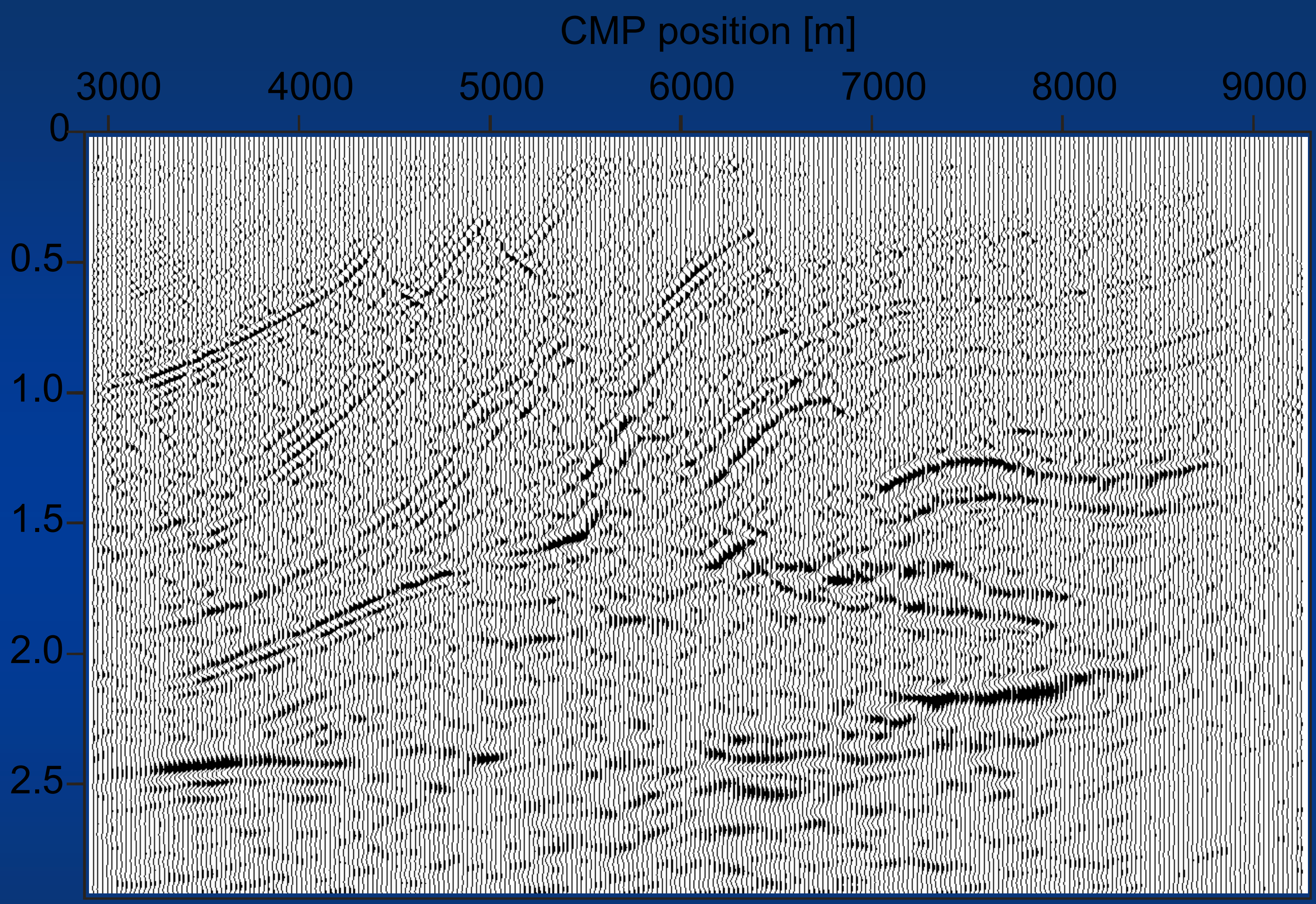
Complete CMP



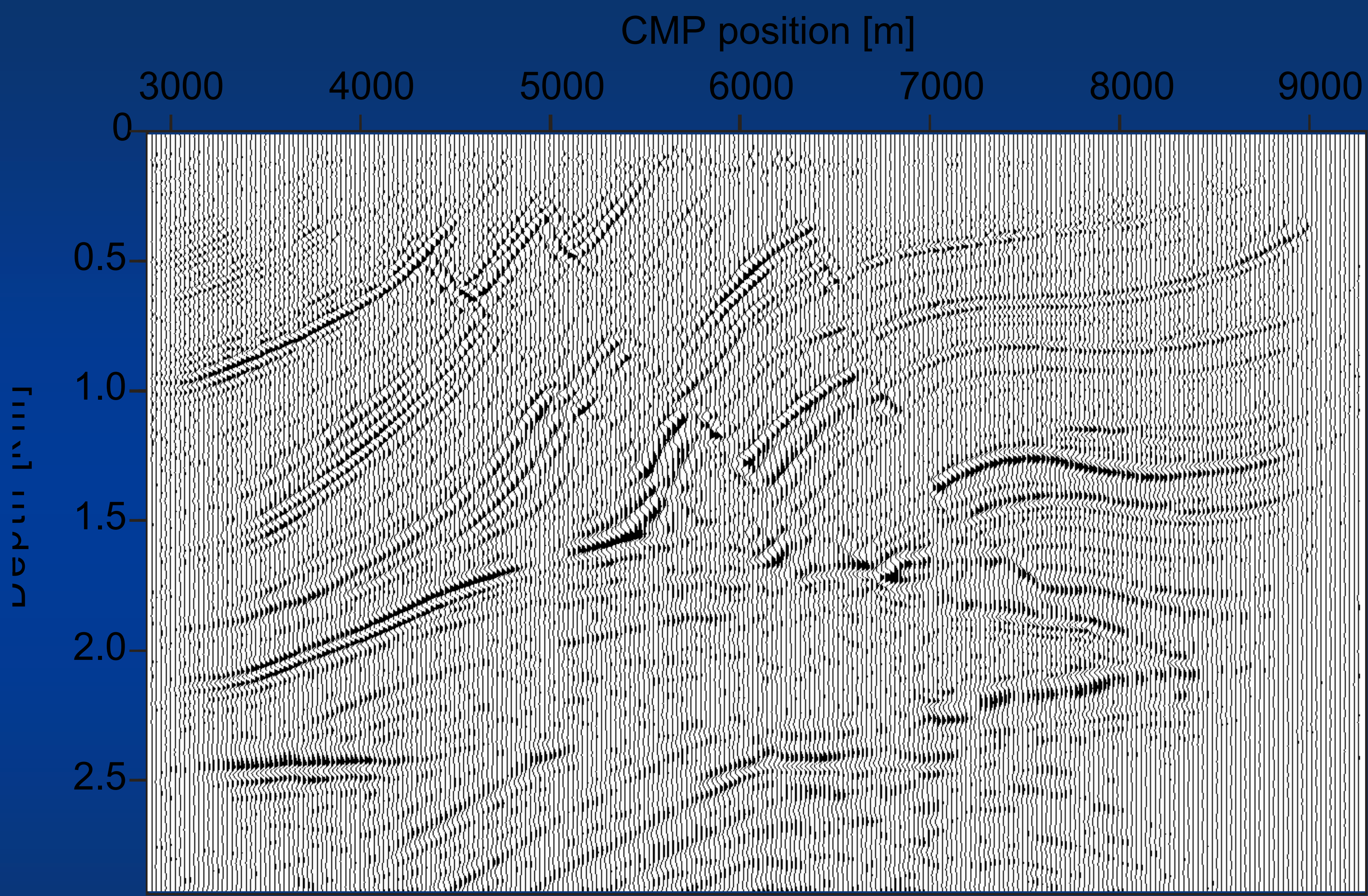
Incomplete CMP (30 %)



Ray parameter [mu s/m]
0 200 400 600 800
Depth [km]
0.5 0.7 0.9 1.1
Ray parameter [mu s/m]
0 200 400 600 80
Depth [km]
0.5 0.7 0.9 1.1
Reflection coefficient
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2
Angle of incidence (deg)
0 10 20 30 40 50
Reflection coefficient
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2
Angle of incidence (deg)
0 10 20 30 40 50
Migration
RLSM



Constant ray parameter image (incomplete)



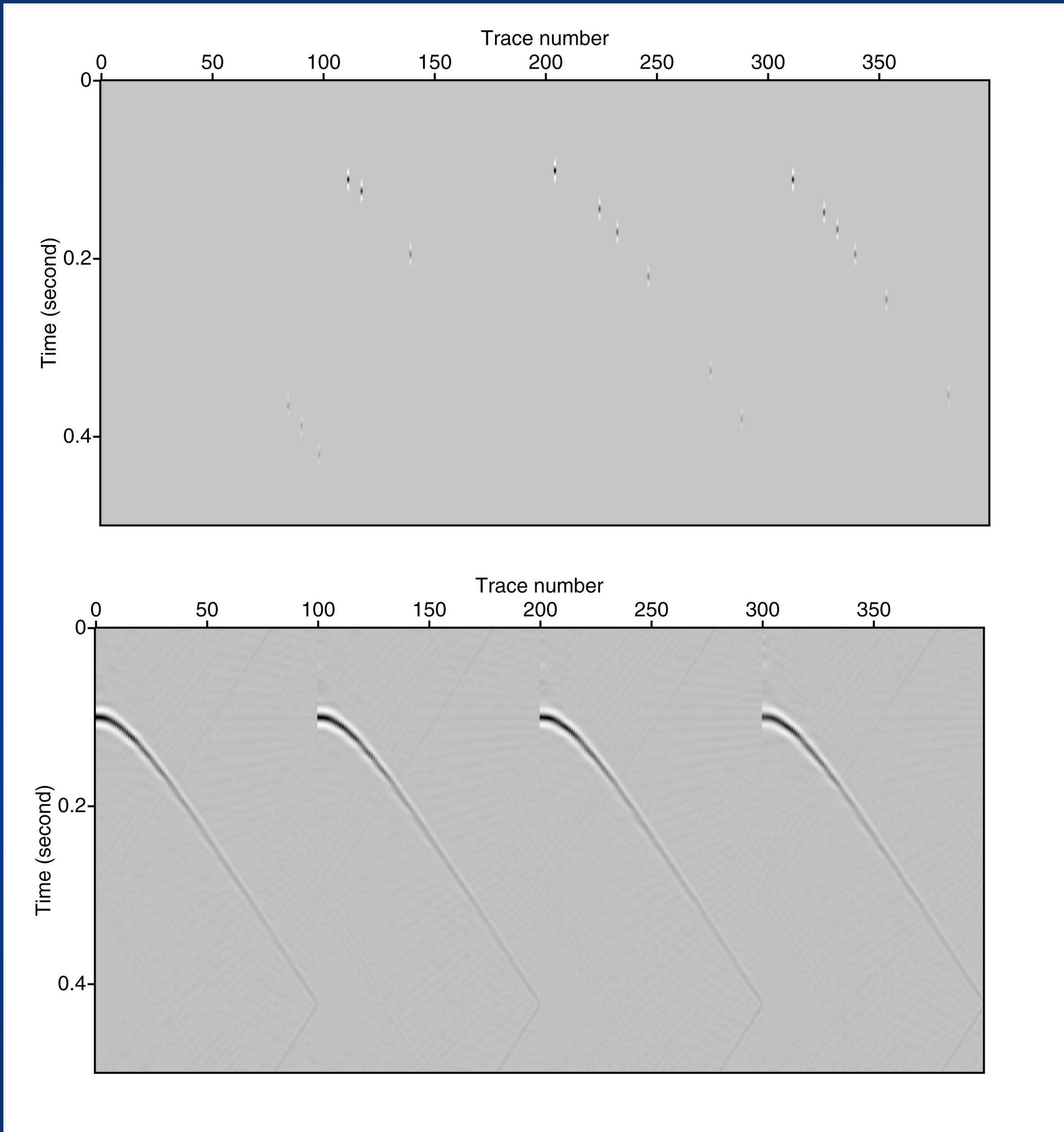
Constant ray parameter image (RLSM)

3D - Synthetic data example
(J Wang)

Parameters for the 3D synthetic data

x-CDPs : 40
Y-CDPs : 301
Nominal Offset: 50
dx=5 m
dy=10 m
dh=10 m

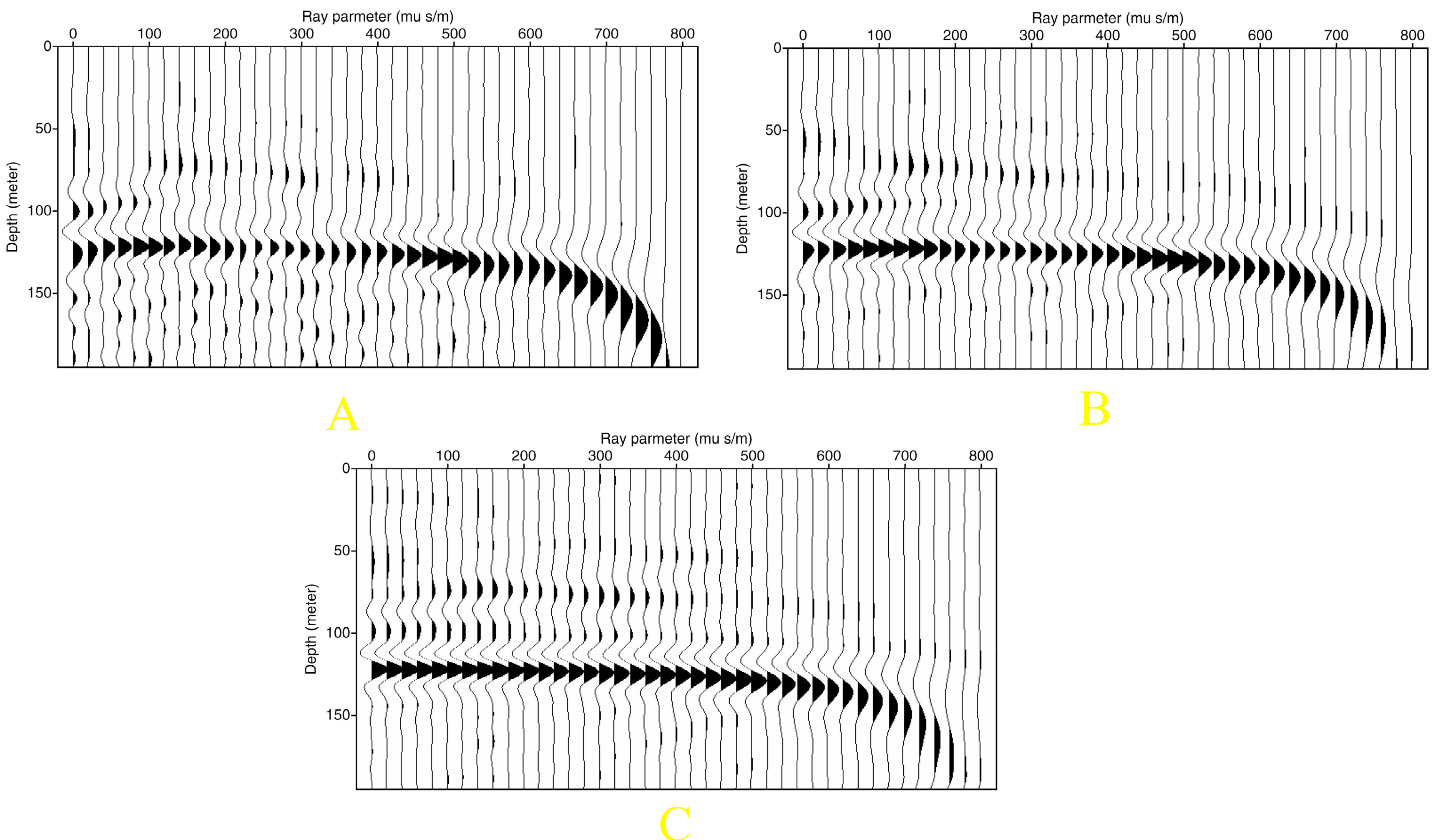
90% traces are randomly removed to simulate a sparse 3D data.



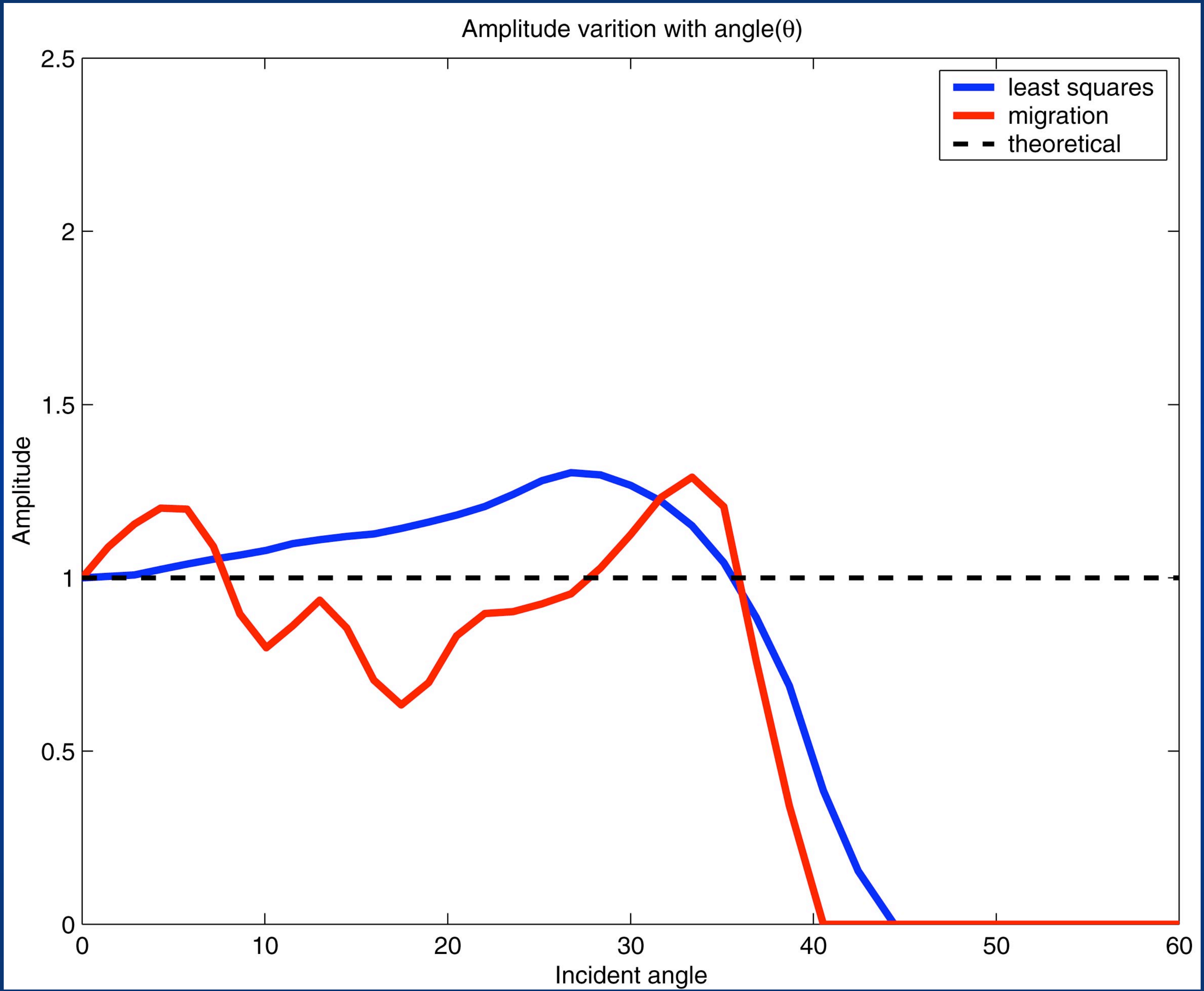
Incomplete Data

4 adjacent CDPs

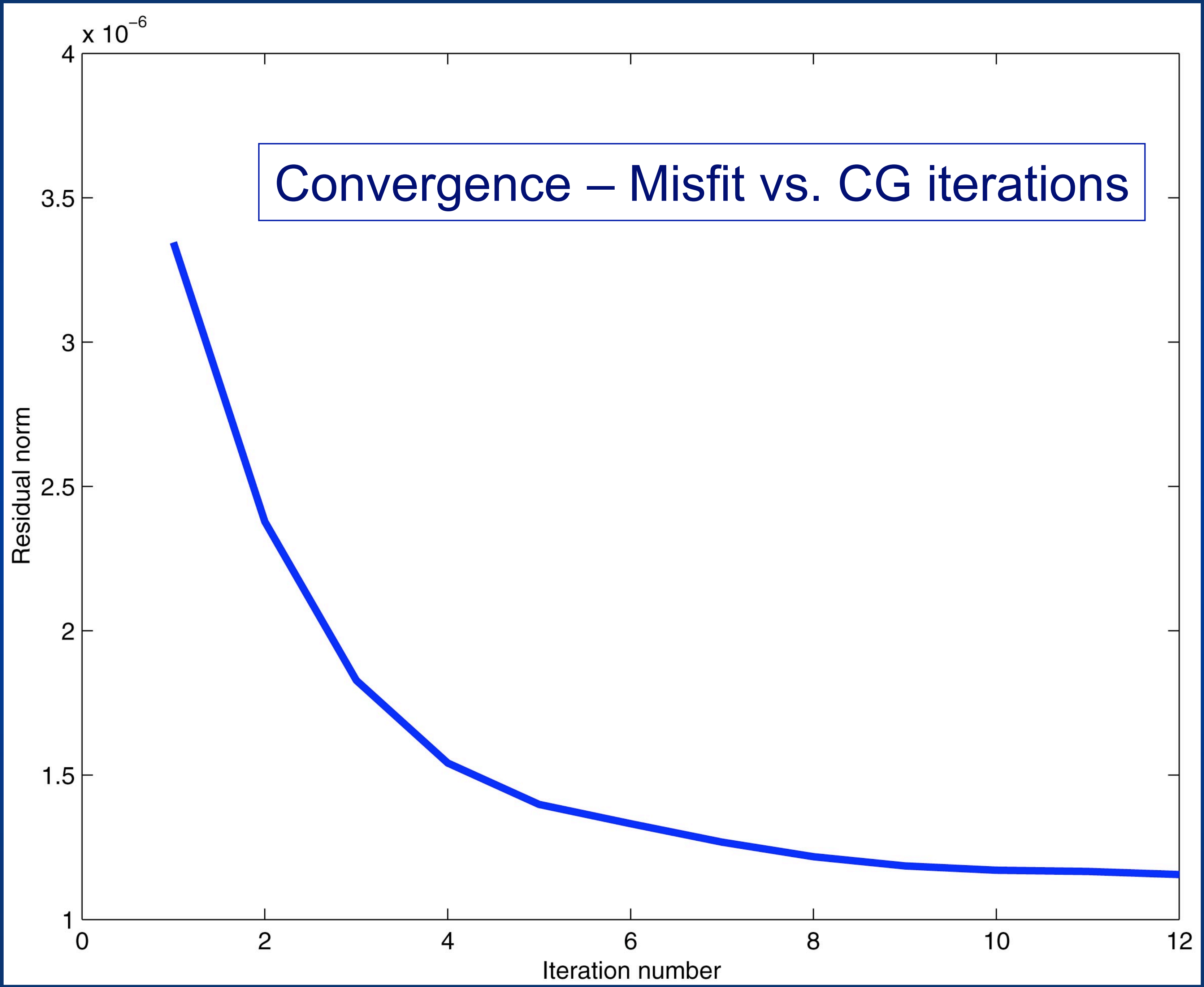
Reconstructe data (after 12 Iterations)



A: Iteration 1 B: Iteration 3 C: Iteration 12

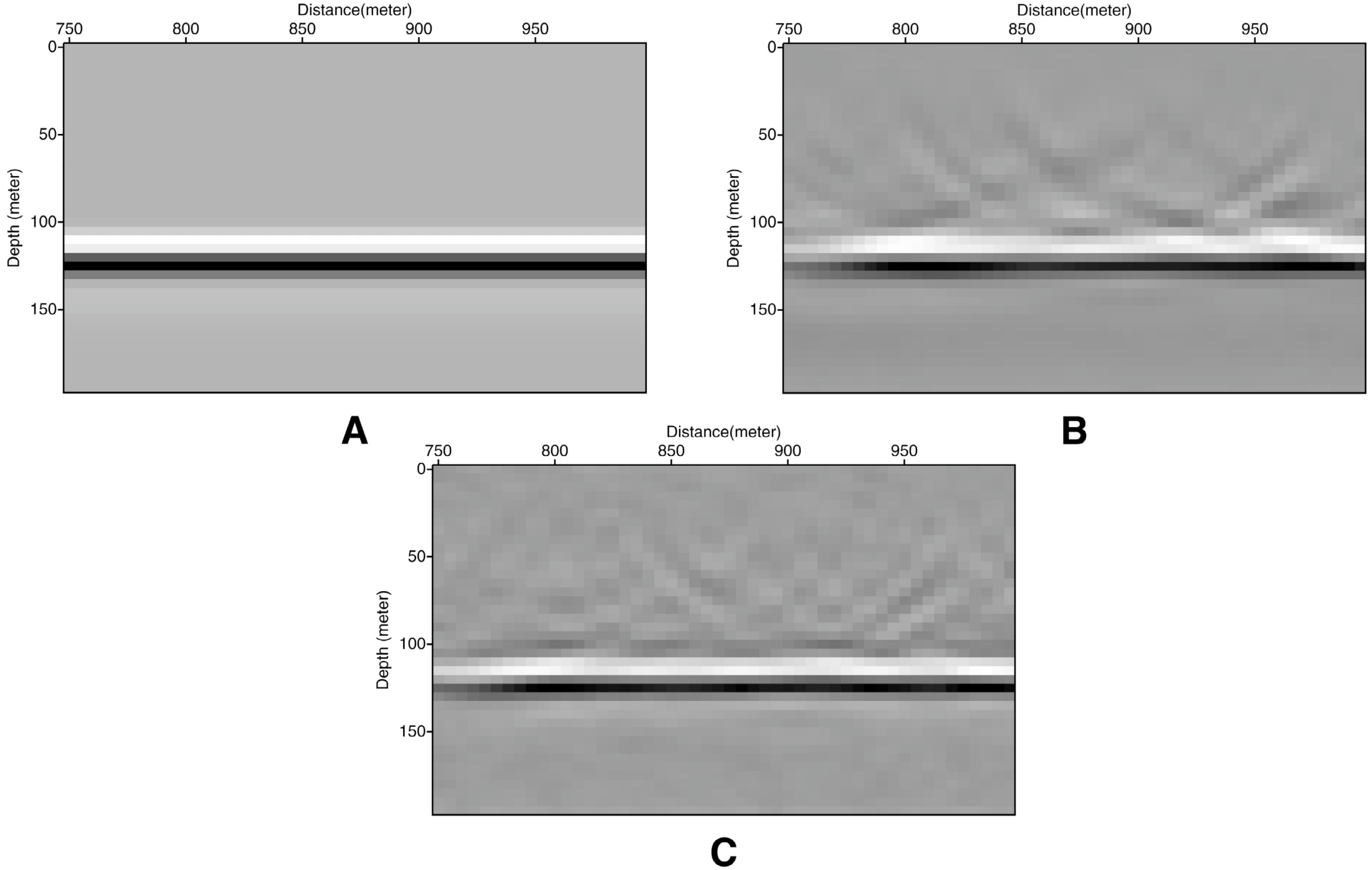


At y=950 m, x=130 m



Convergence – Misfit vs. CG iterations
x 10^-6
4
3.5
3
2.5
2
1.5
1
Residual norm
0
2
4
6
8
10
12
Iteration number

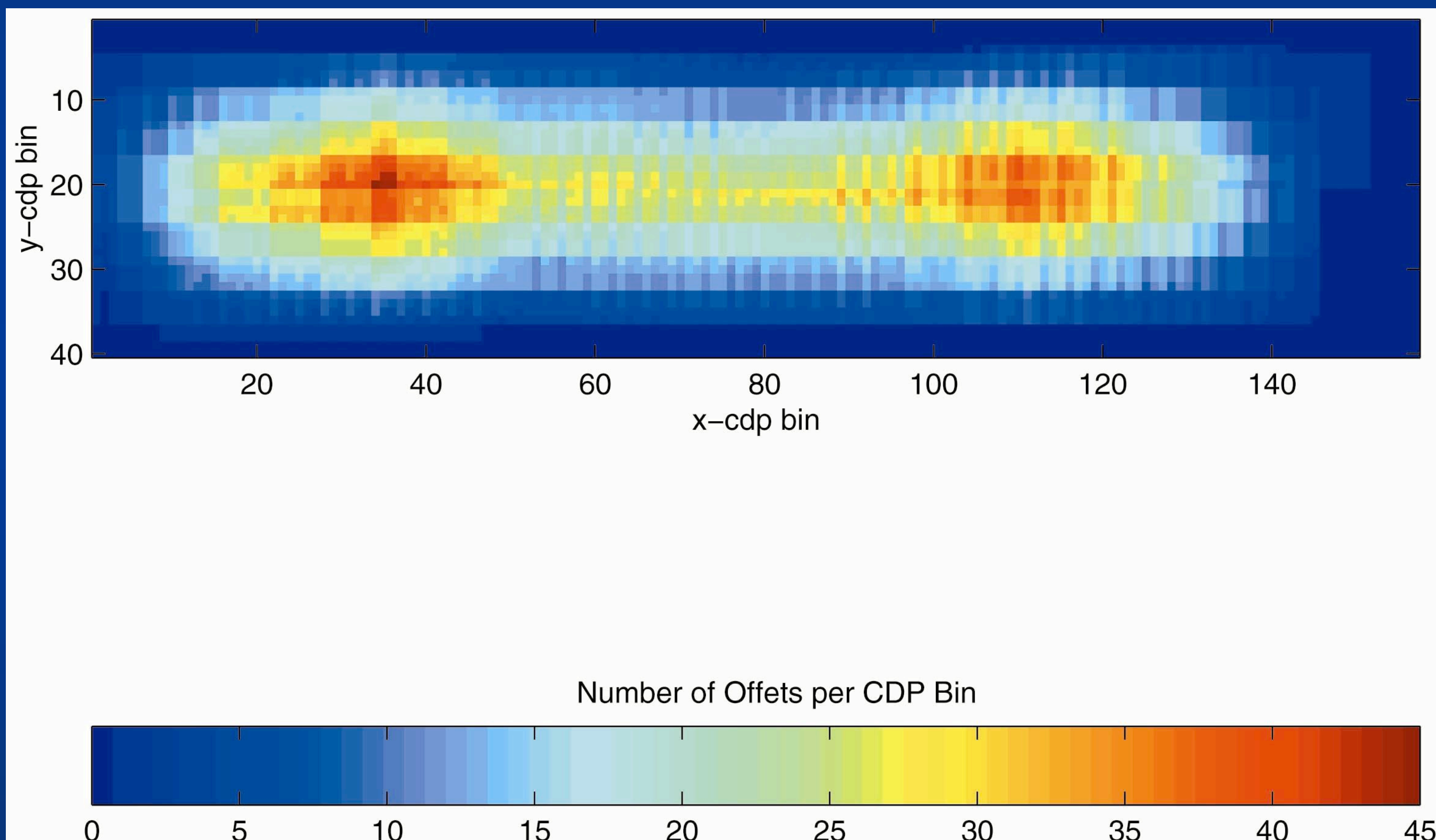
Stacked images comparison (x-line 30)



A complete data B. Remove 90% traces C. least-squares

Field data example

ERSKINE (WBC) orthogonal 3-D sparse land data set



dx=50.29m

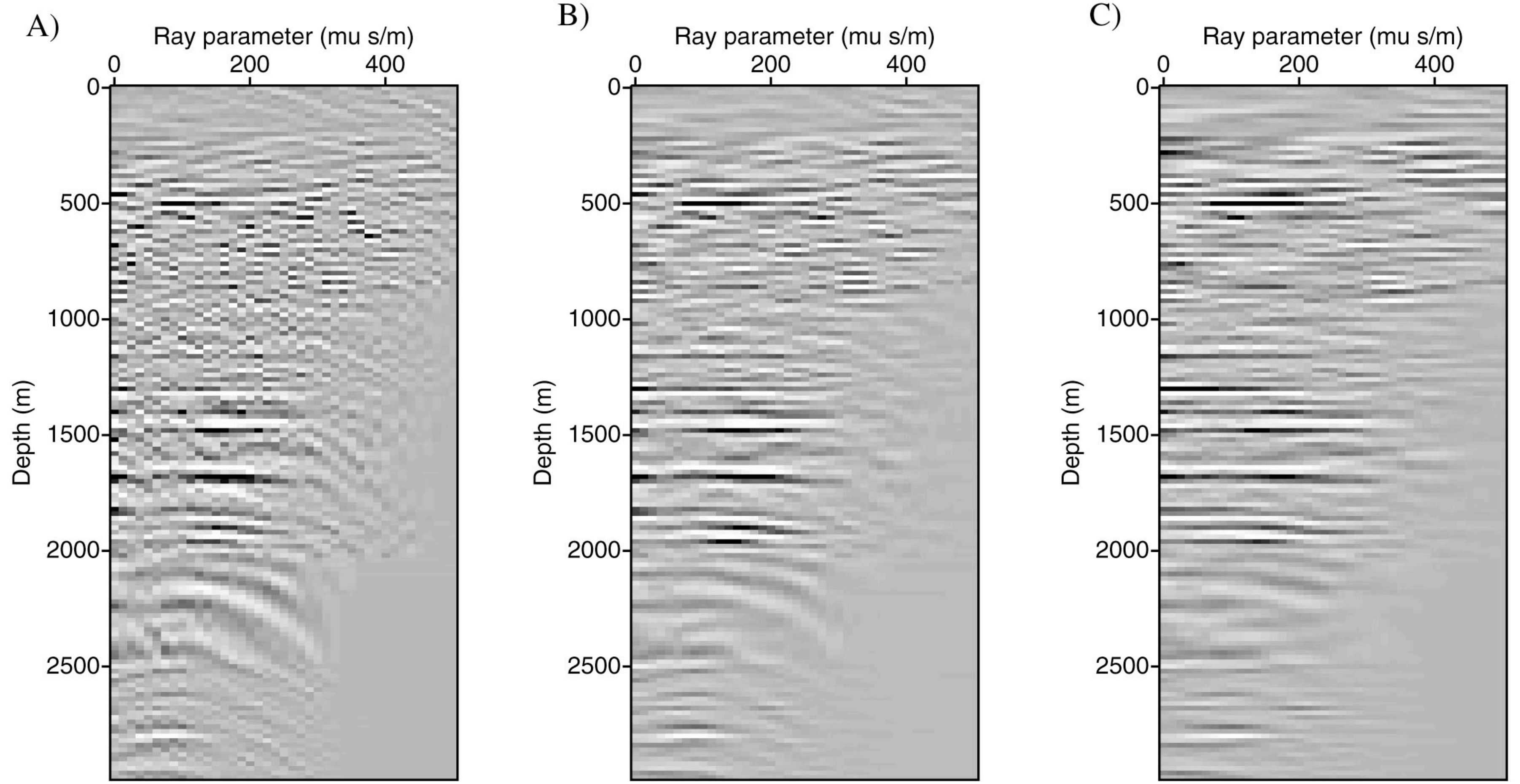
dy=33.5m

Comment on Importance of W

$$J = \| W\,(L\,m \Box d)\|^2 + \Box\| D_p m(x,z,p)\|^2$$

CIG at x-line #10, in-line #71

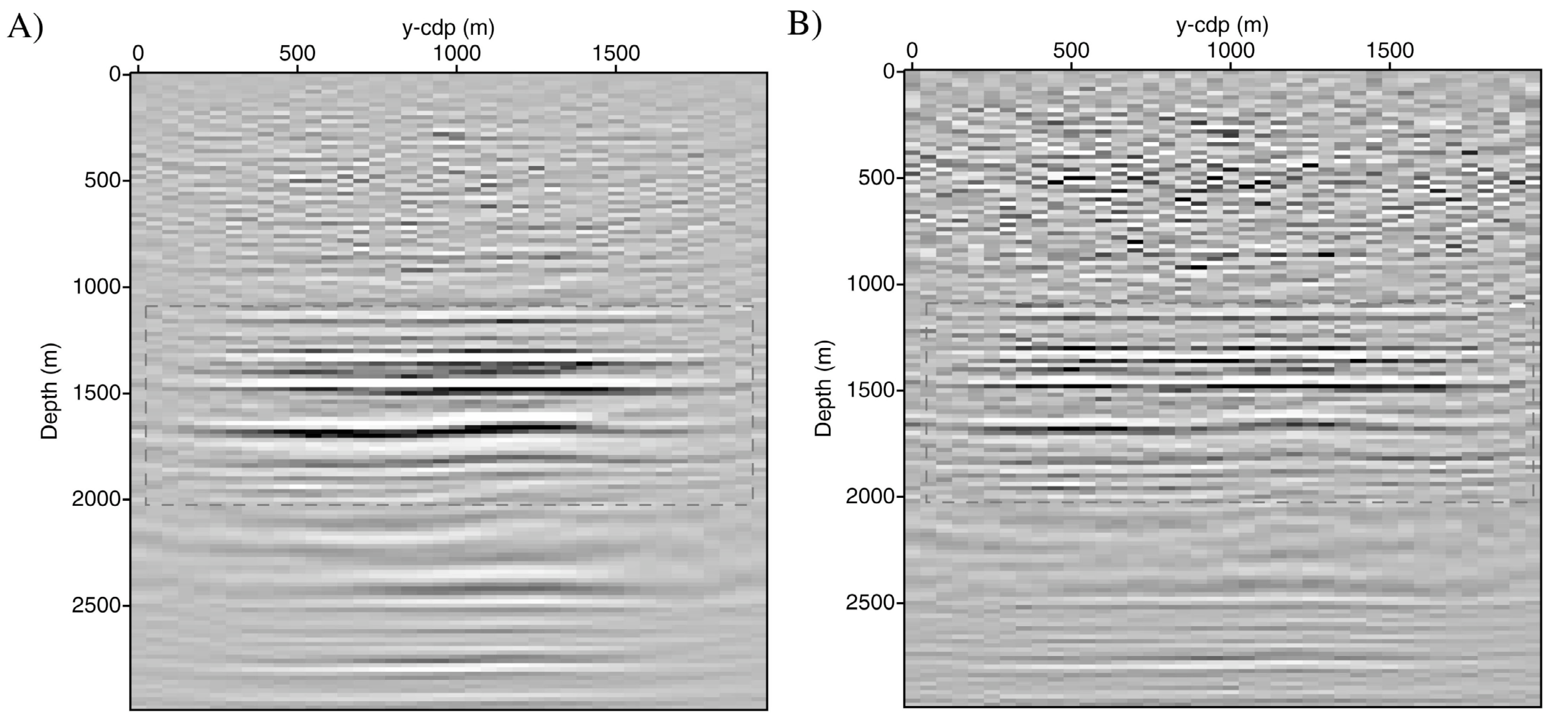
Iteration 1 | Iteration 3 | Iteration 7



Cost Optimization

$$J = || W\,(L\,m \,\square\, d)\,||^2 + \square|| D_p m(x,z,p)\,||^2$$

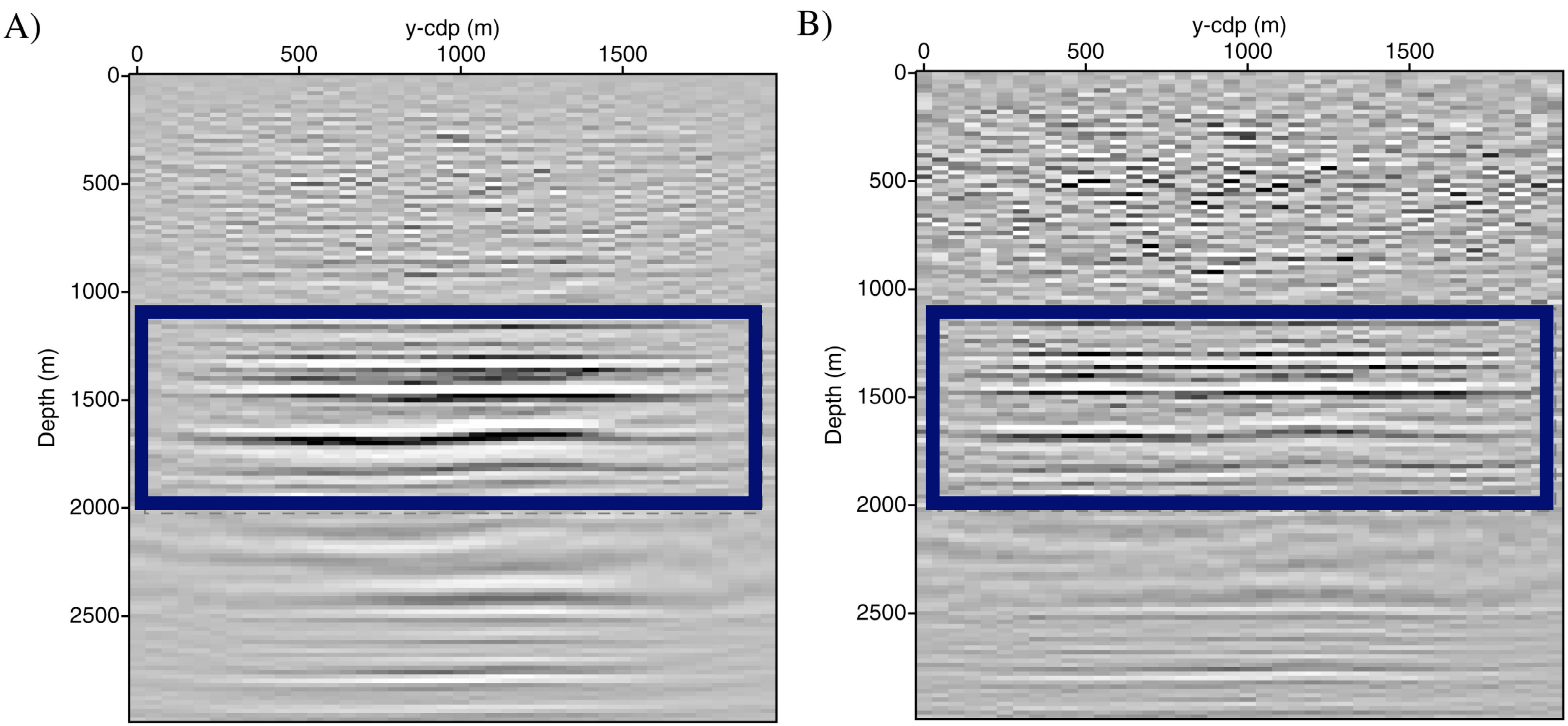
Stacked image, in-line #71



Migration

RLS Migration

Stacked image, in-line #71



Migration

RLS Migration

Image and Common Image Gather (detail)

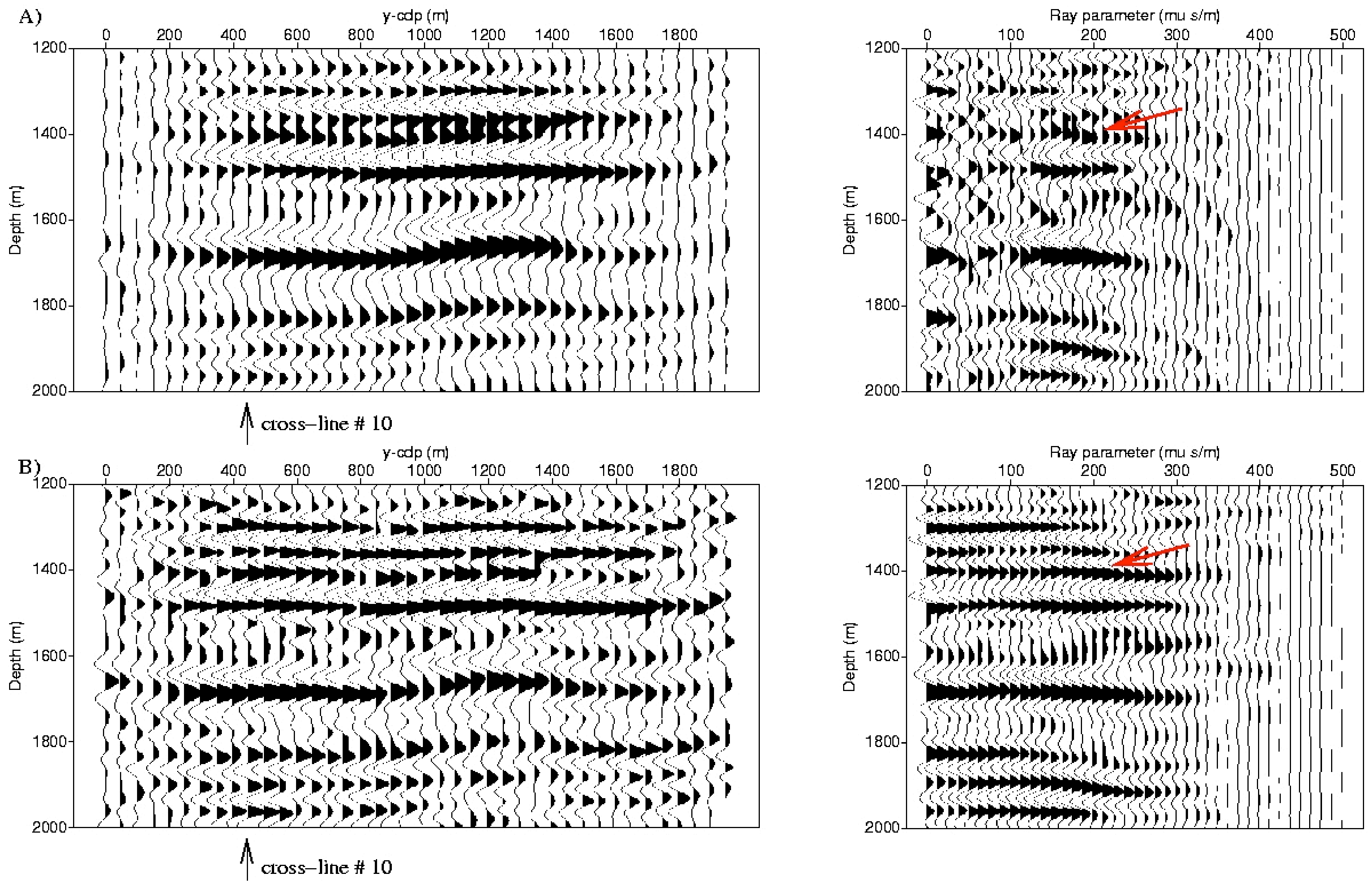
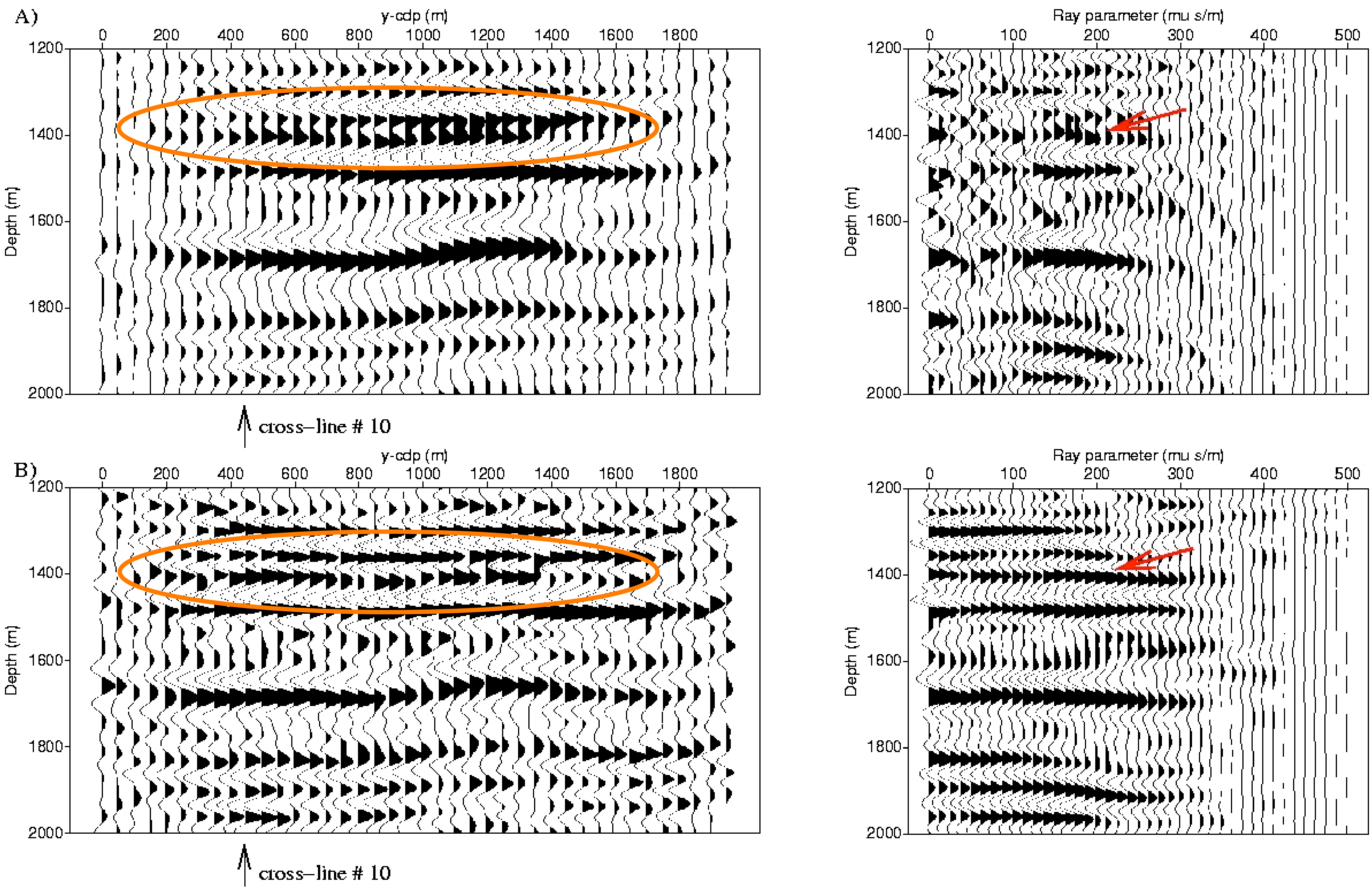
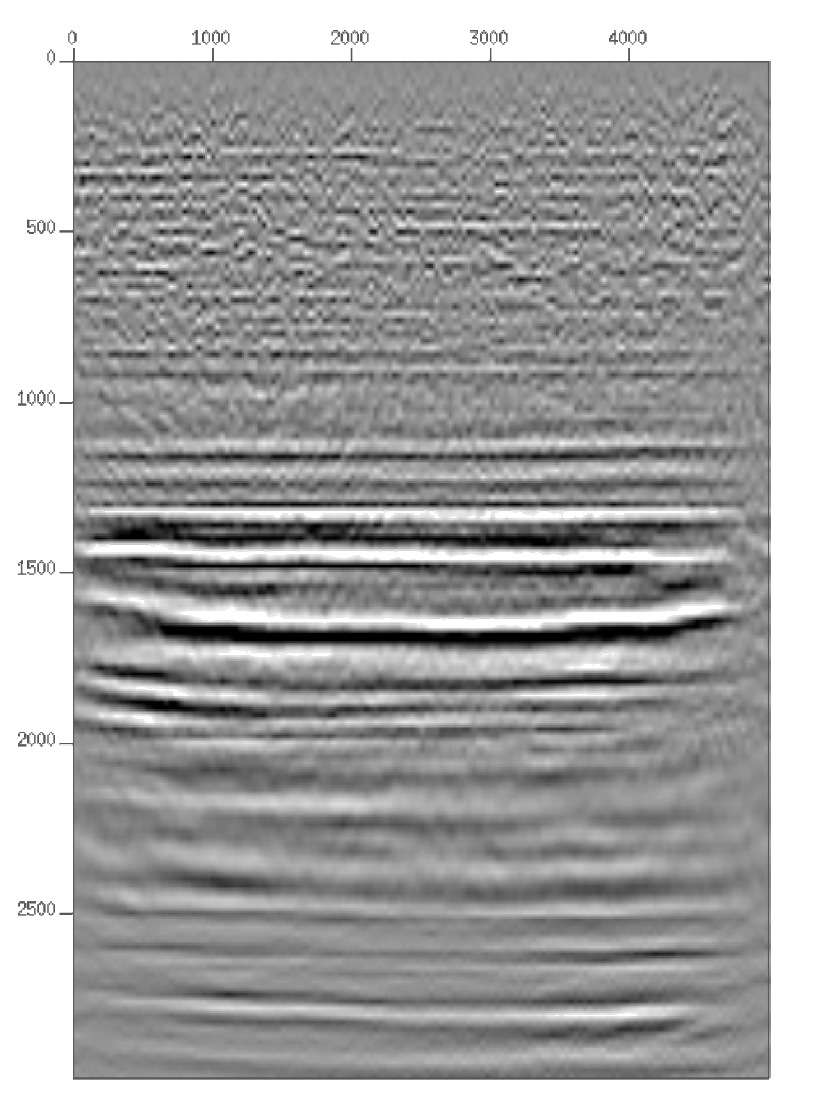


Image and Common Image Gather (detail)

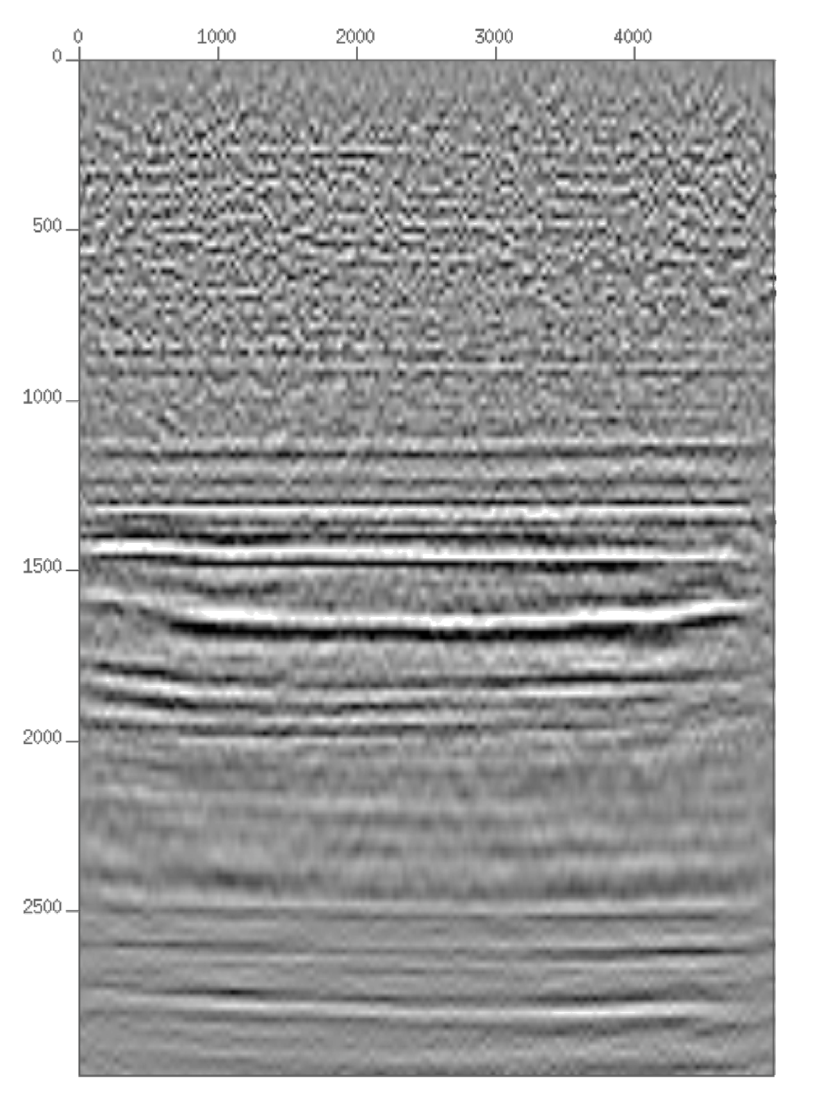


Stacked Image (x-line #10) comparison

Migration



LS Migration



Non-quadratic regularization applied to imaging

Non-quadratic regularization

But first, a little about smoothing:

$$d \square s + n$$

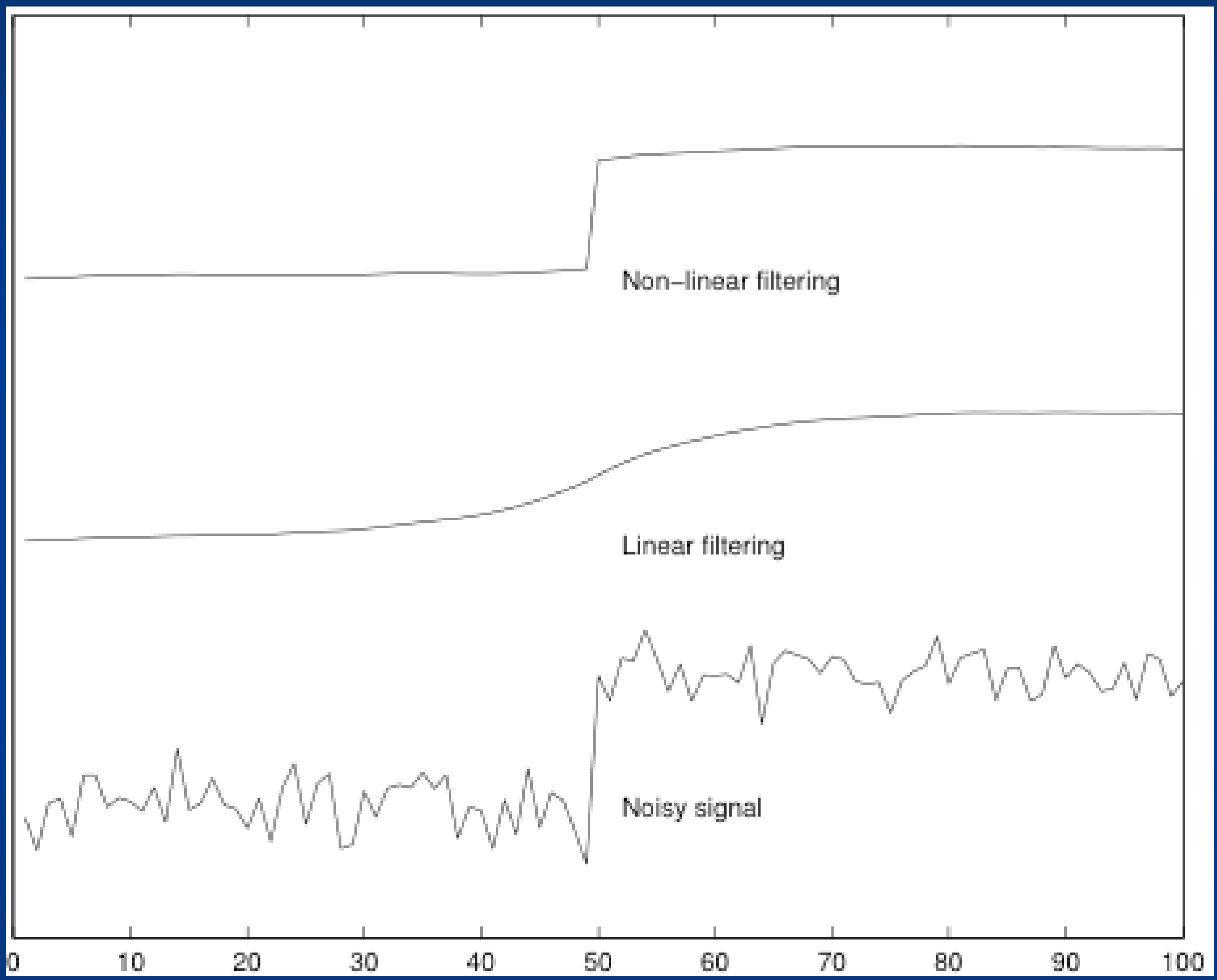
$$Rs \square 0$$

Quadratic Smoothing

$$J = \| d \square s \|^2 + \square \| Ds \|^2, \quad Ds = \partial_x s$$

Non-Quadratic Smoothing

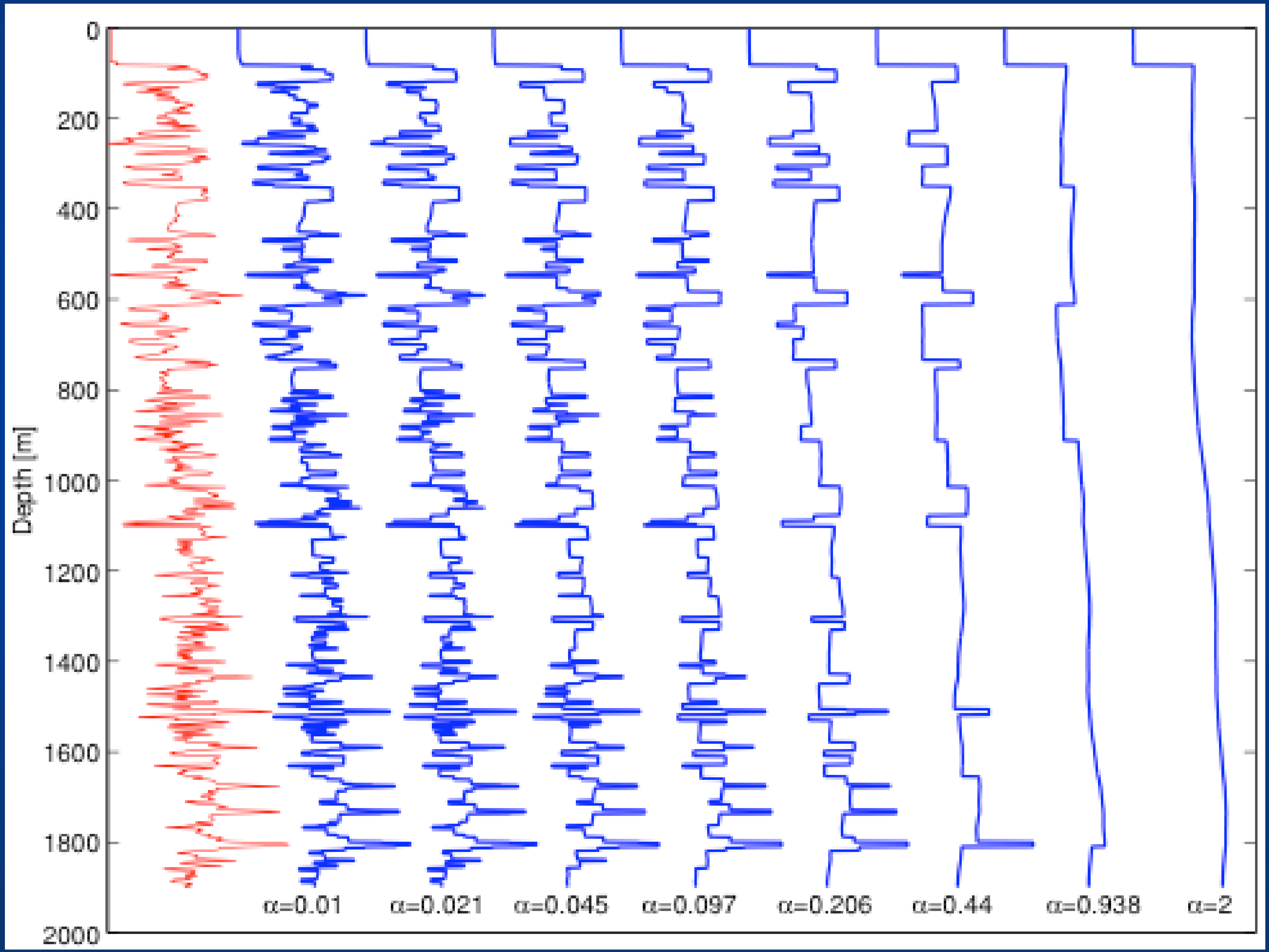
$$J = \| d \square s \|^2 + \square \square (Ds)$$



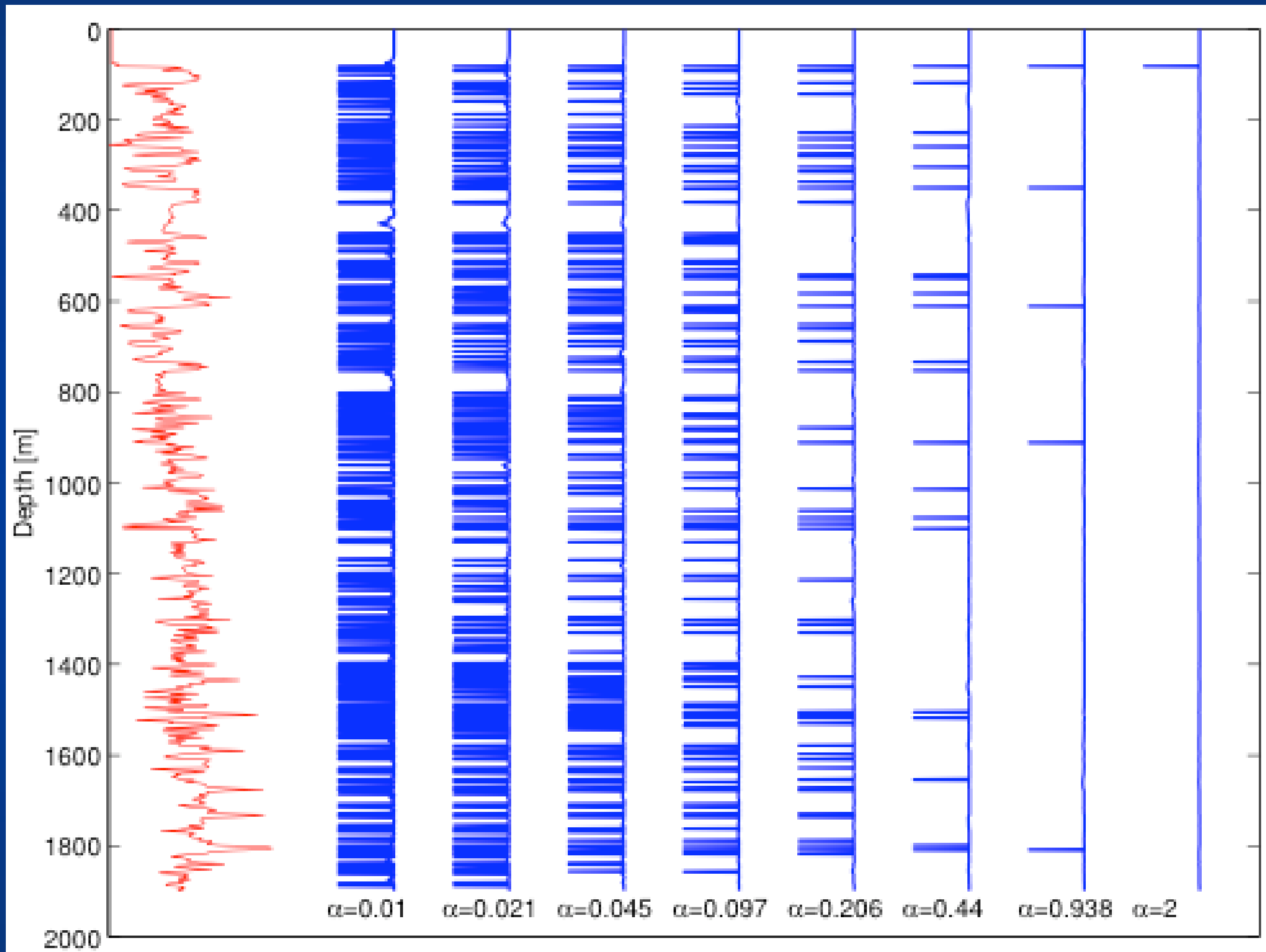
Quadratic regularization ->Linear filters

Non-quadratic -> Non-linear filters

Segmentation/Non-linear Smoothing

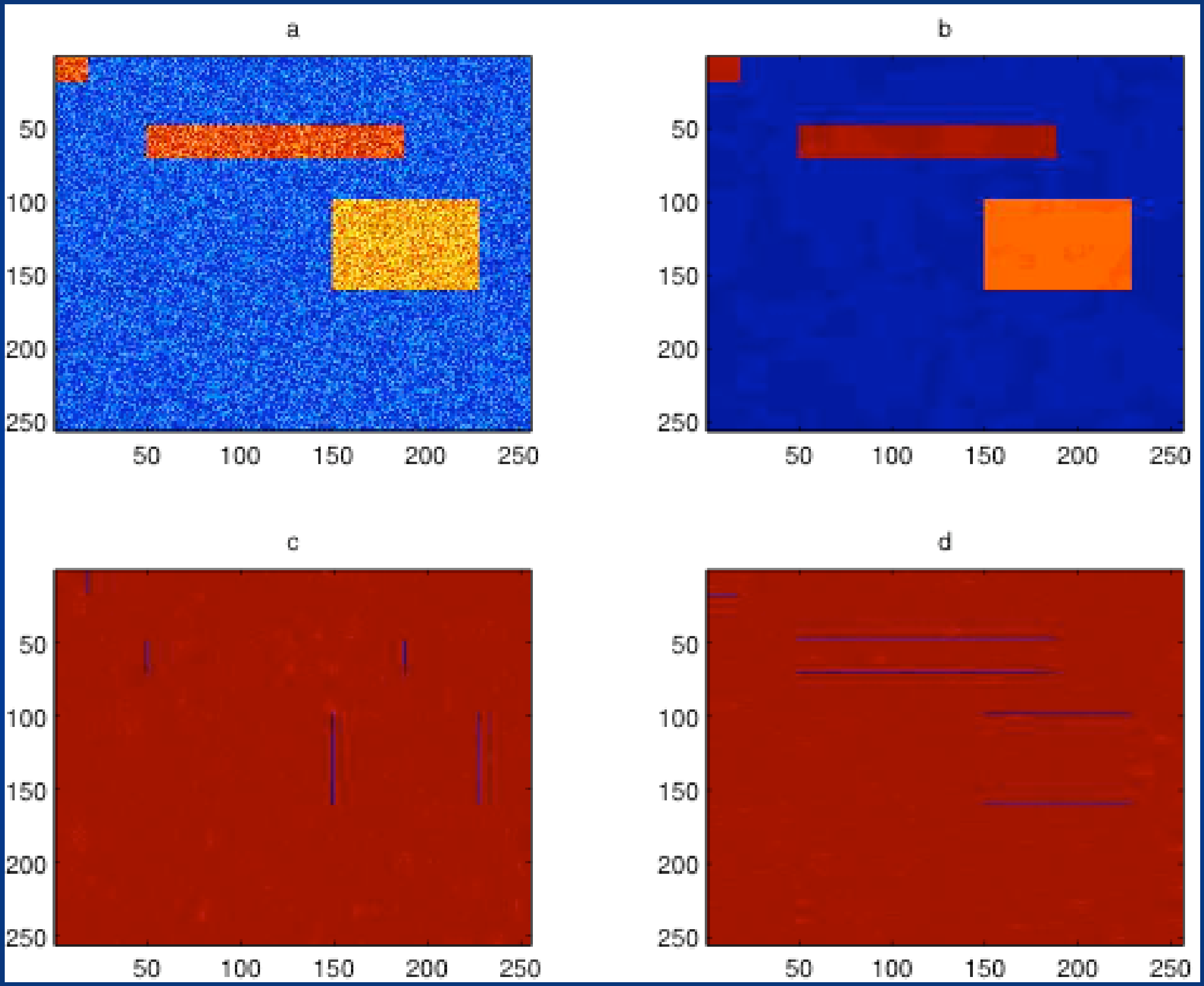


Segmentation/Non-linear Smoothing

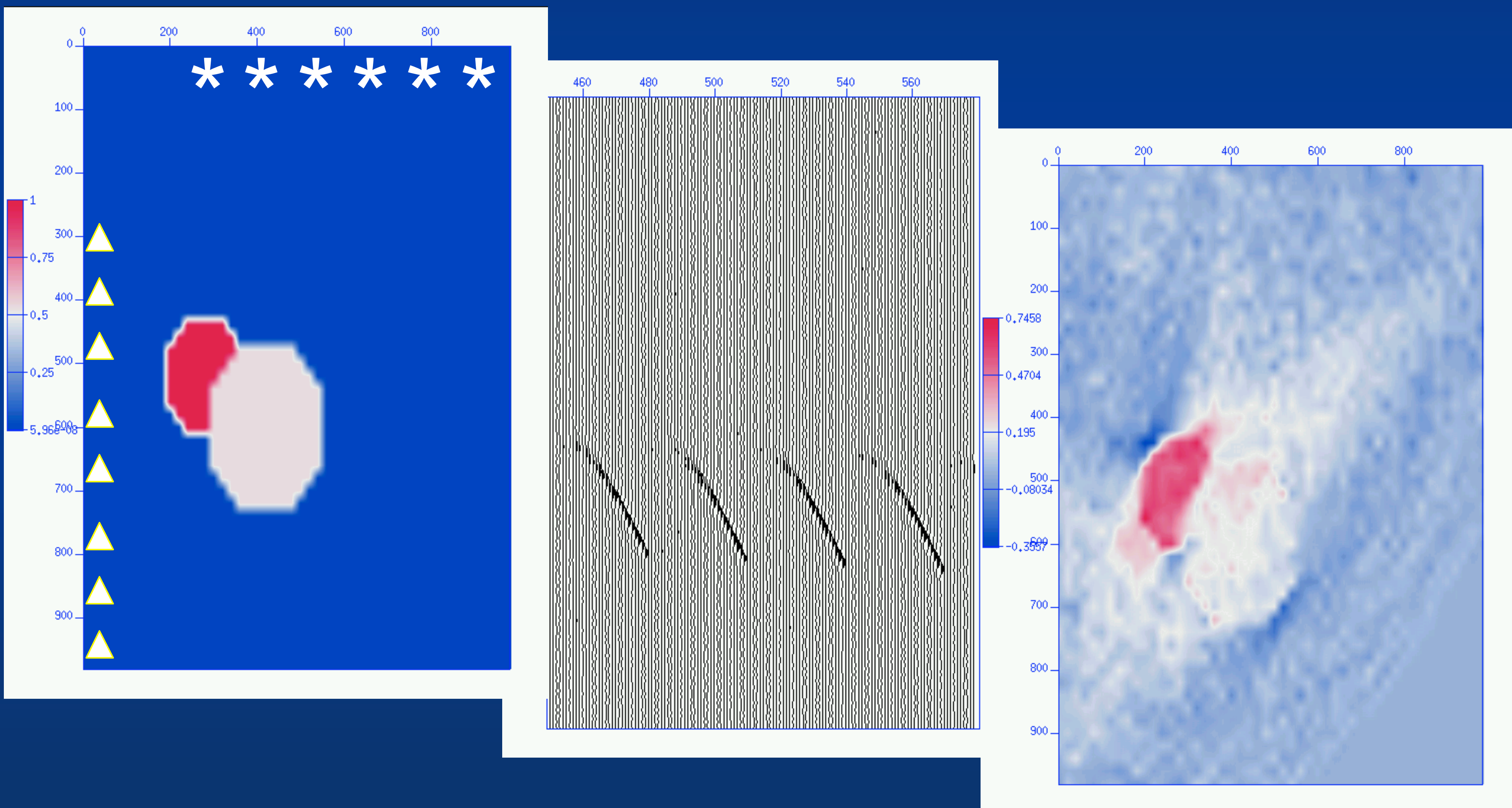


1/Variance at edge position

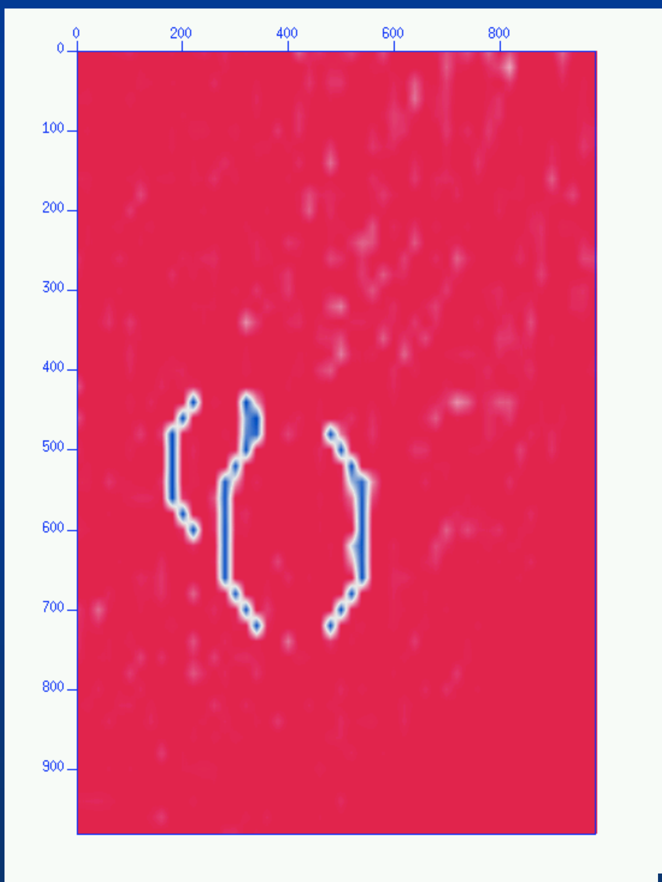
2D Segmentation/Non-linear Smoothing



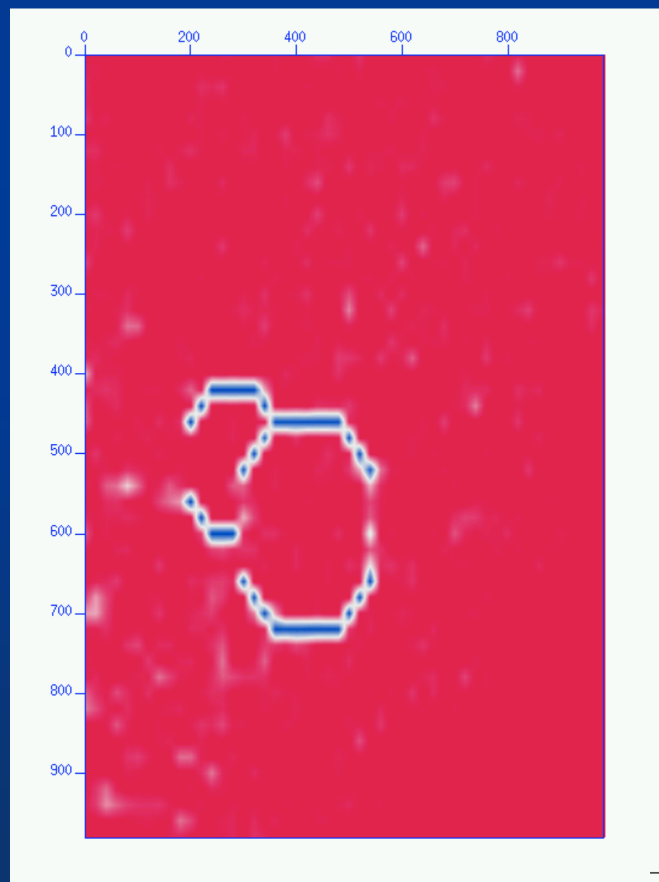
GRT Inversion - VSP
(Carrie Youzwishen, MSc 2001)



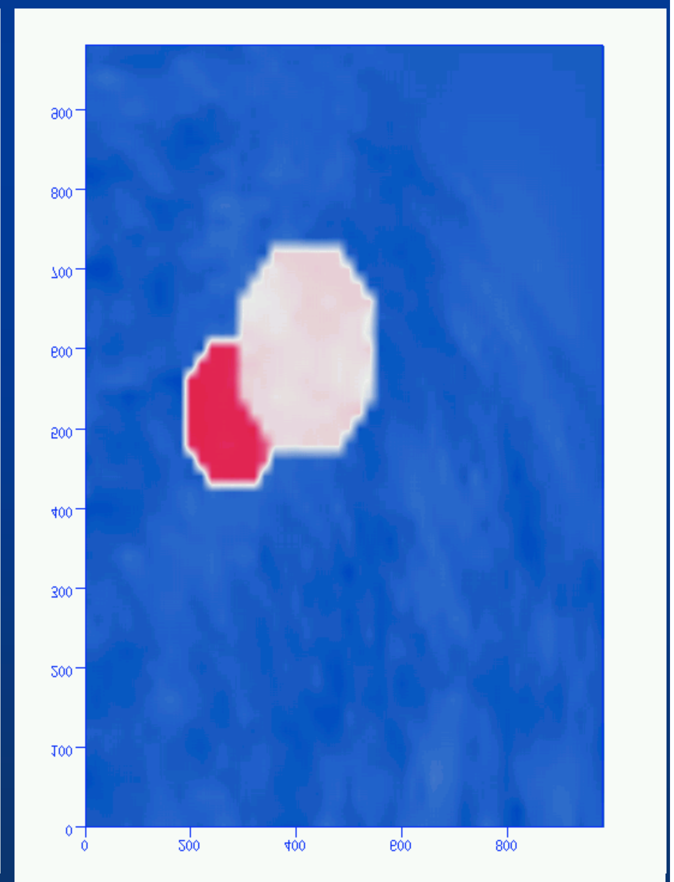
D_x


0
200
400
600
800
0
100
200
300
400
500
600
700
800
900

D_z


0
200
400
600
800
0
100
200
300
400
500
600
700
800
900

m



HR Migration

(Current Direction)

- Quadratic constraint (Dp) to smooth along p (or h)
- Non-quadratic constraint to force vertical sparseness

$$J = \| W\, L\, m - d \|^2 + \square \| D_h\, m(x,z,h) \|^2 + \square\, \square(m)$$

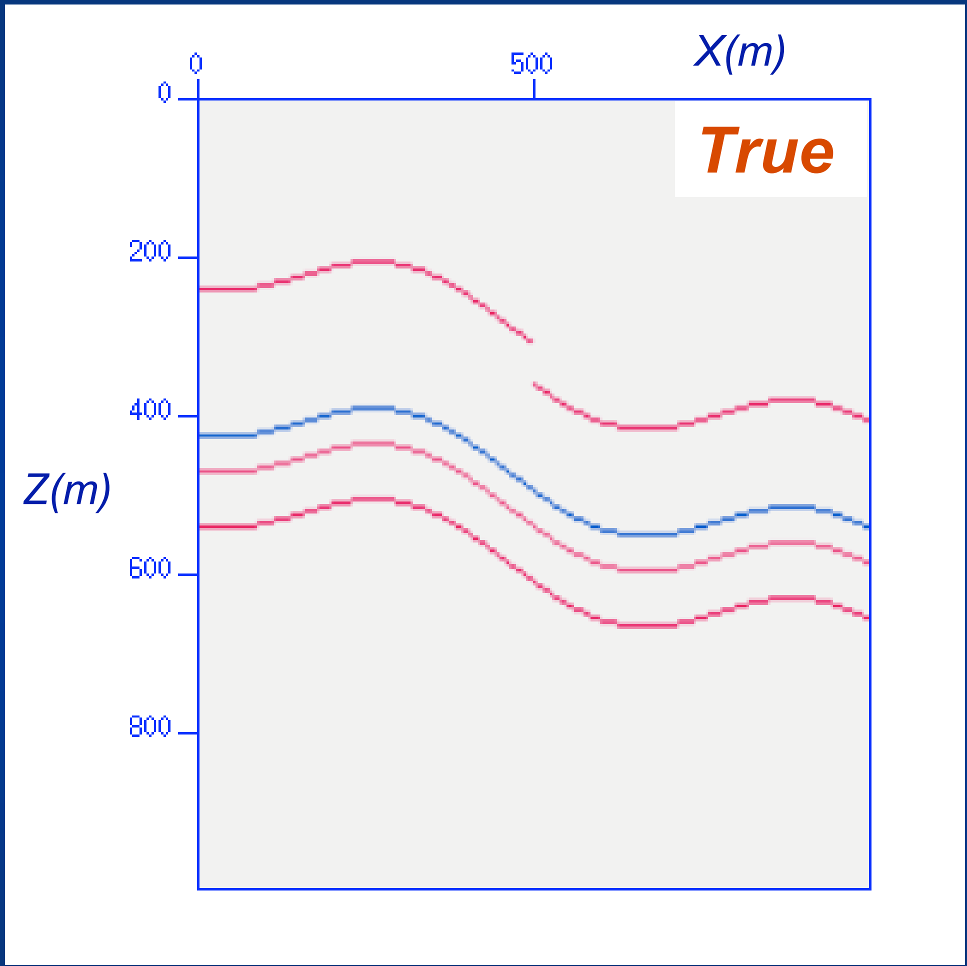
Example: we compare migrated images m(x,z,h) for the following 3 imaging methods:

Adj $\tilde{m} = L'W'd$

LS $\min\{\, J = \| W\, L\, m \,\square\, d \|^2 + \square \| m(x,z,h) \|^2 \,\}$

HR $\min\{\, J = \| W\, L\, m \,\square\, d \|^2 + \square \| D_h\, m(x,z,h) \|^2 + \square\, \square(m) \,\}$

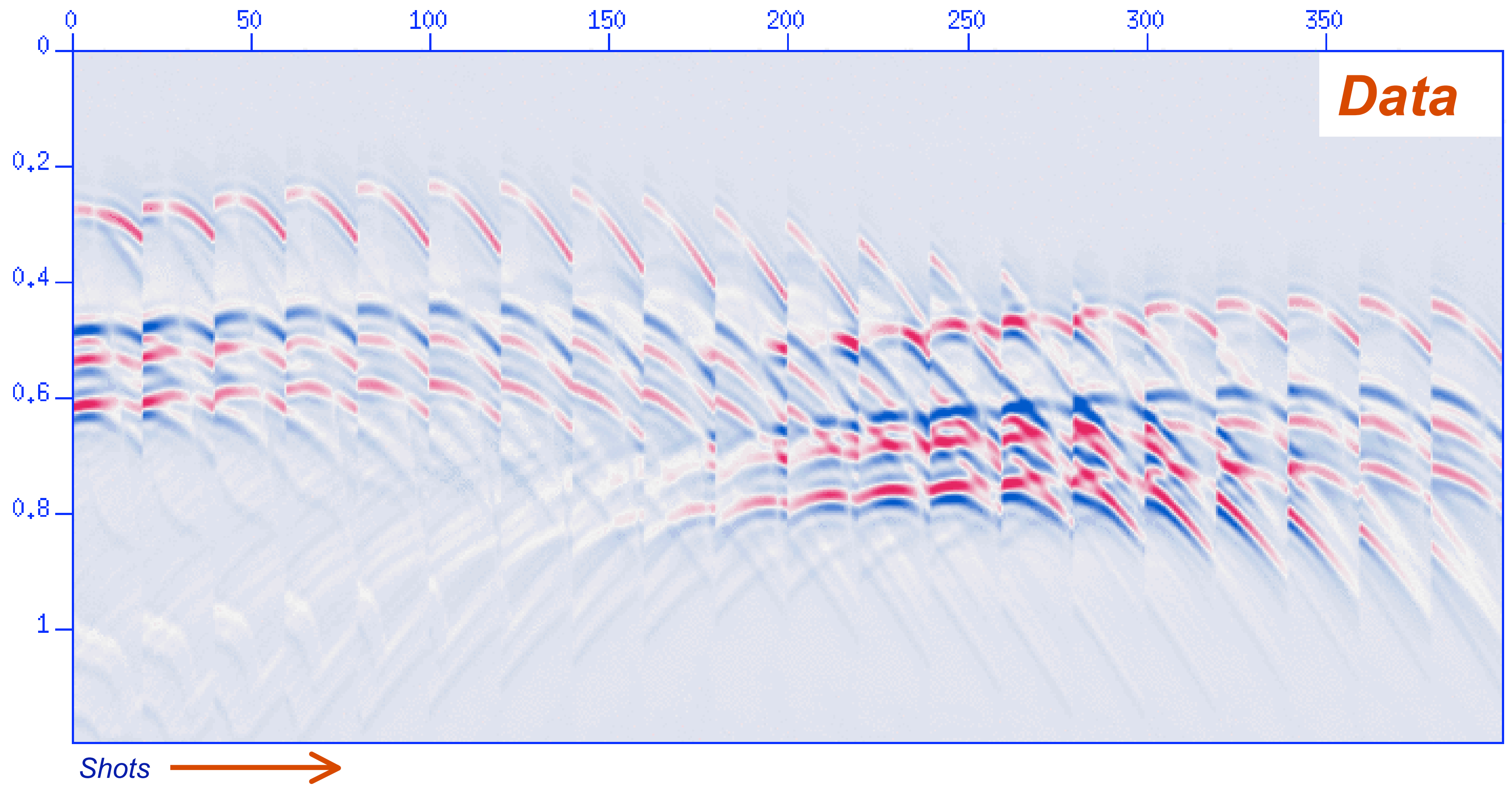
Synthetic Model



m(x,z)

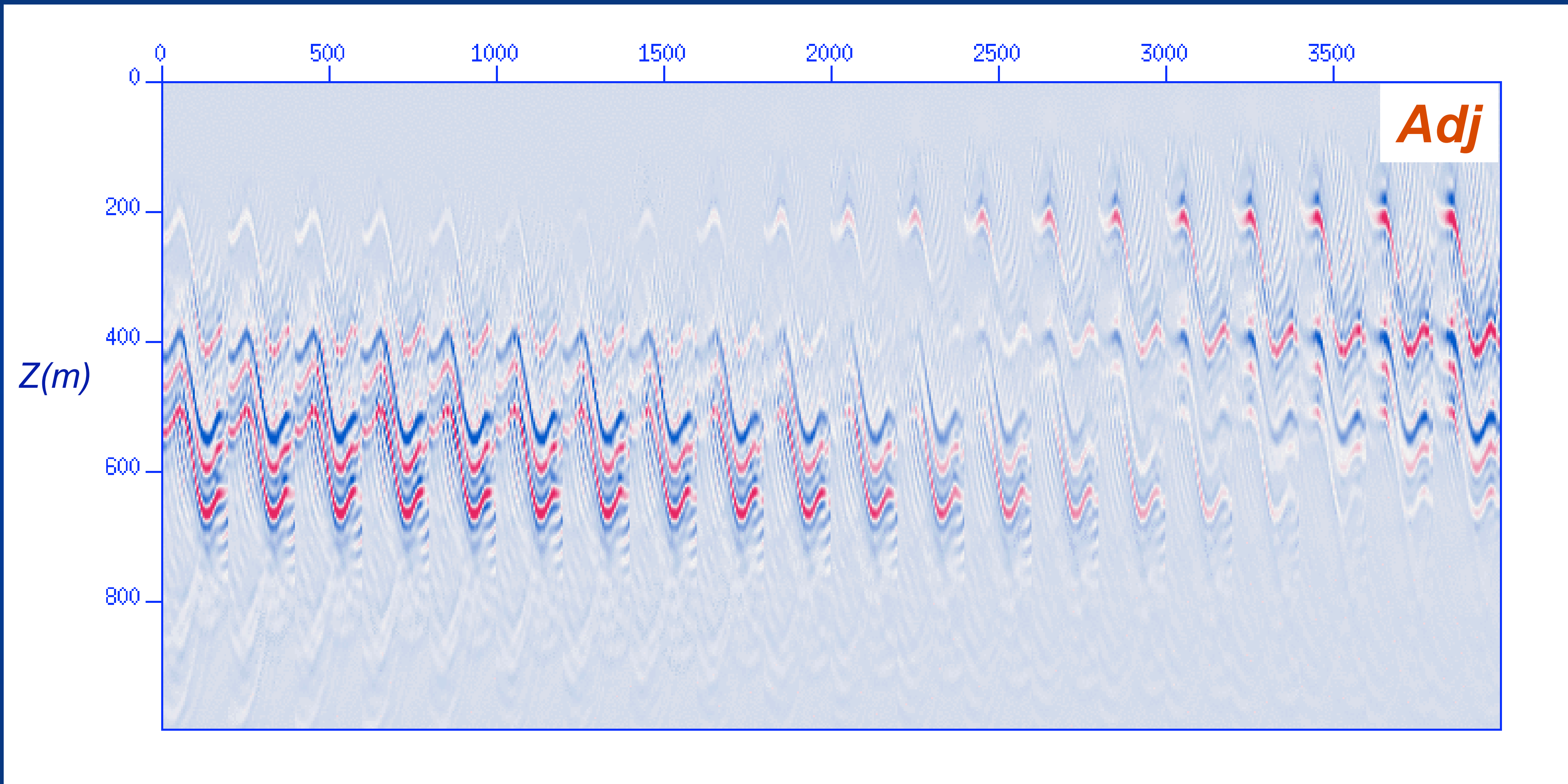
Acoustic, Linearized, Constant V, Variable Density

Pre-stack data



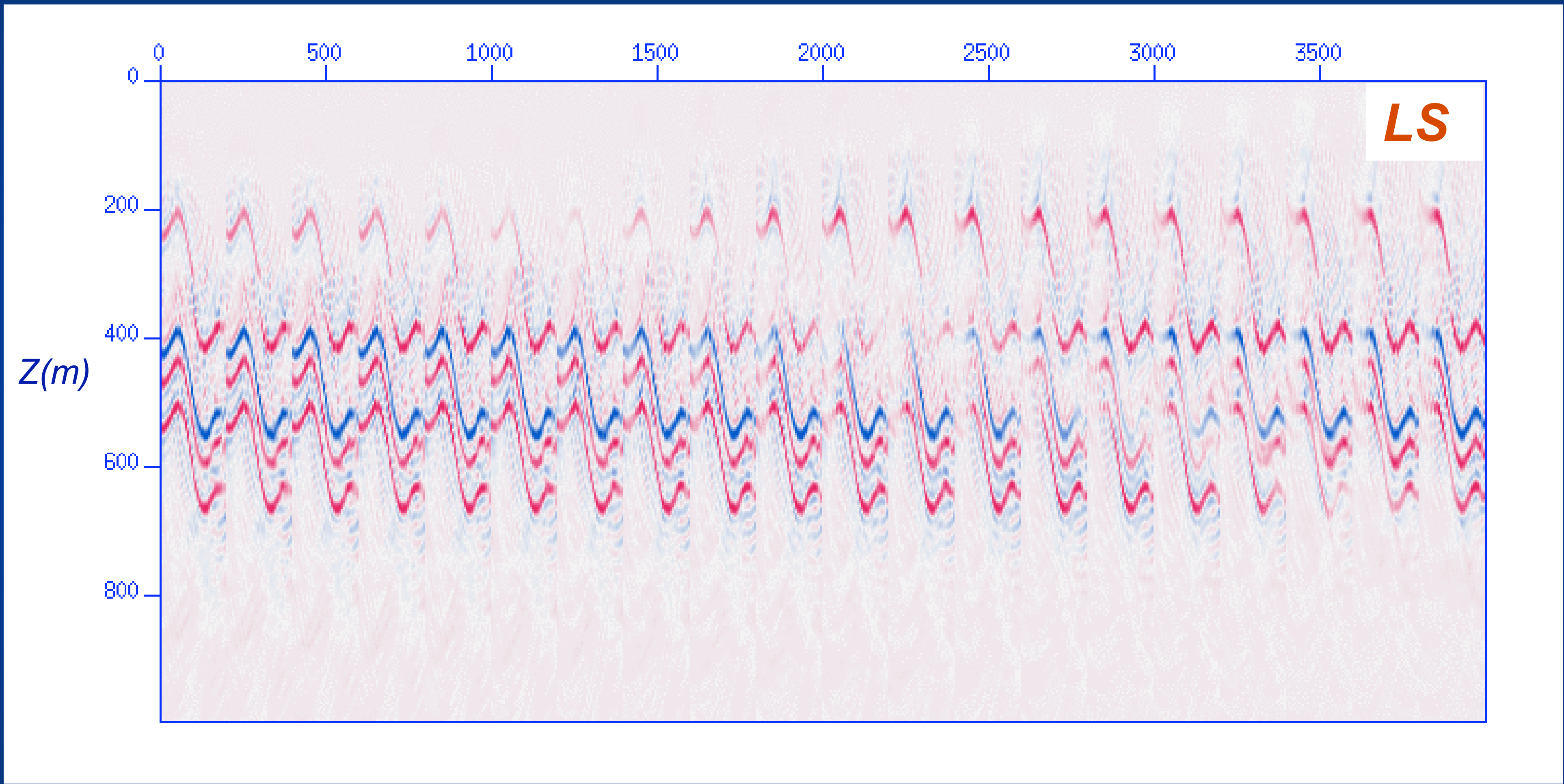
d(s,g,t)

Common offset images



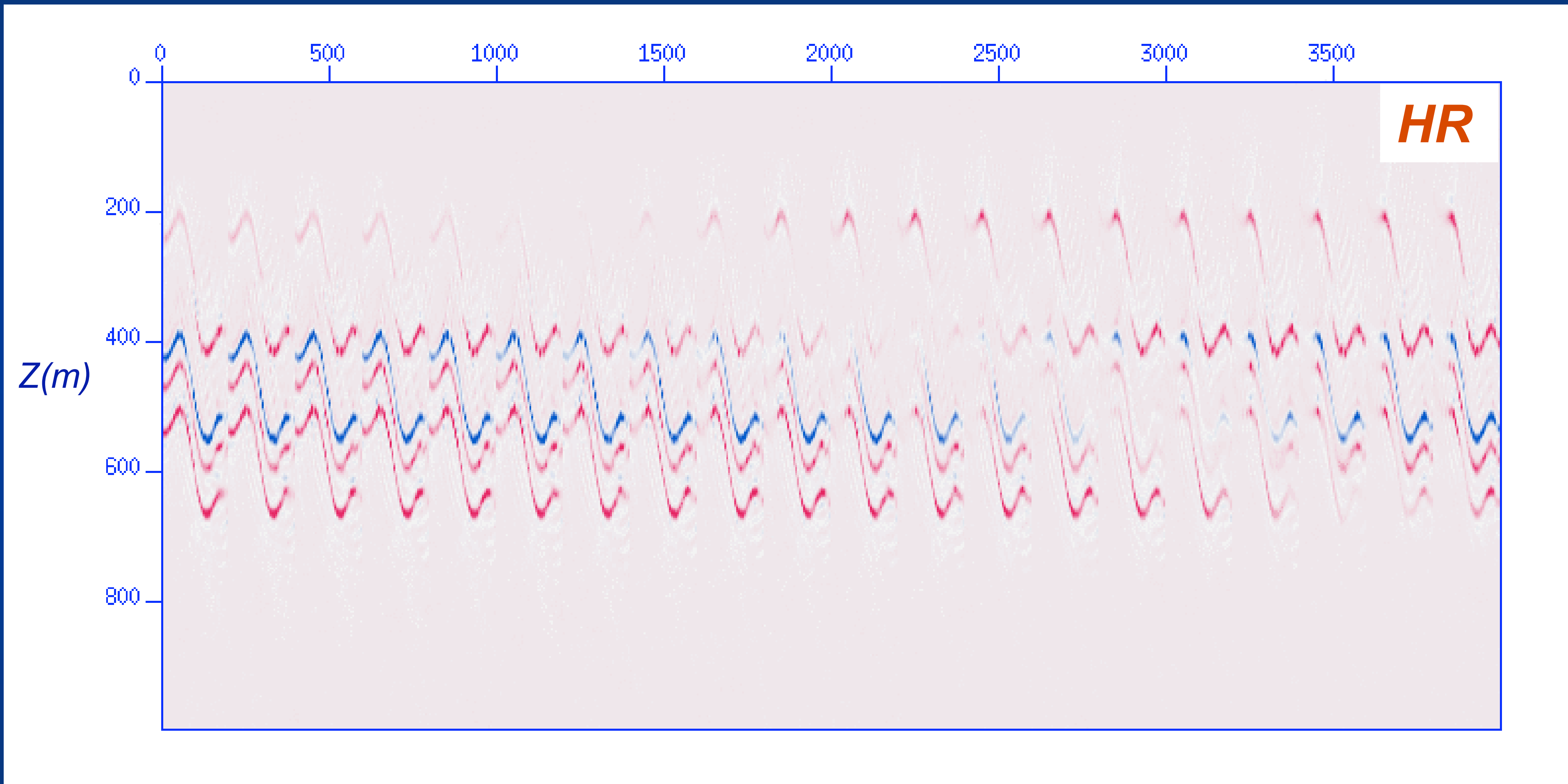
m(x,z,h)

Common offset images



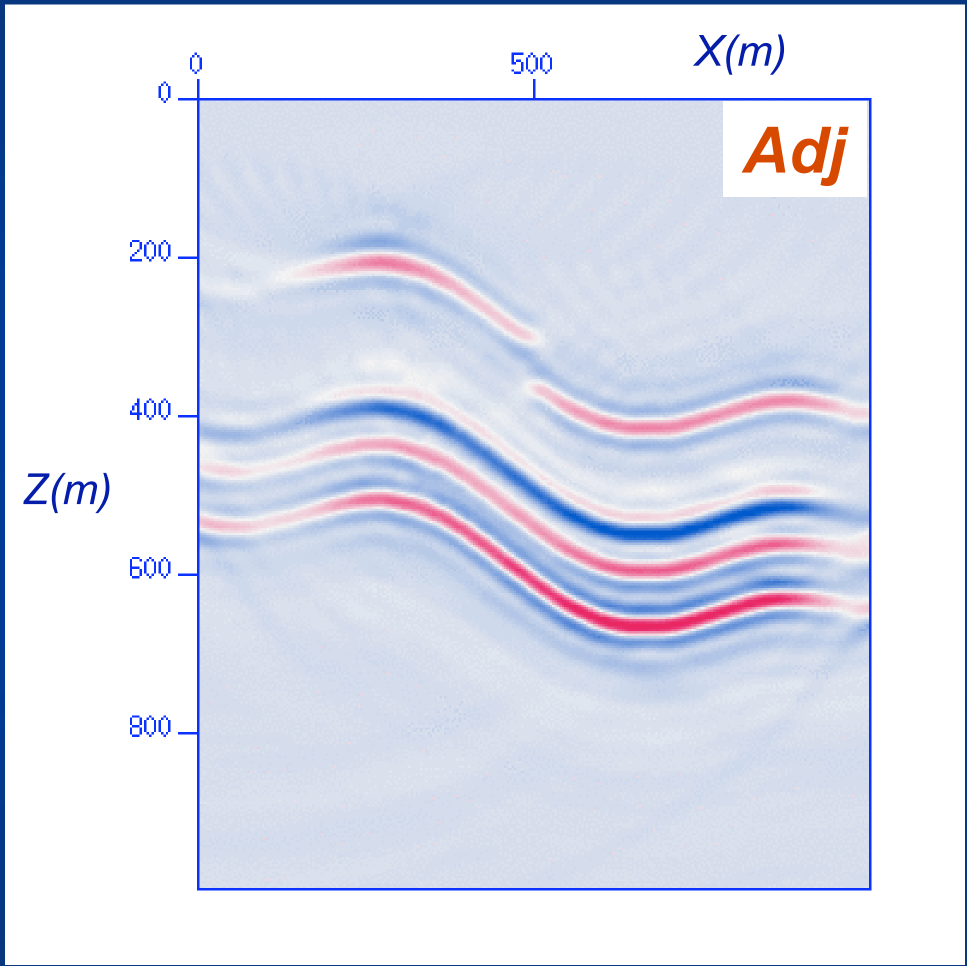
m(x,z,h)

Common offset images



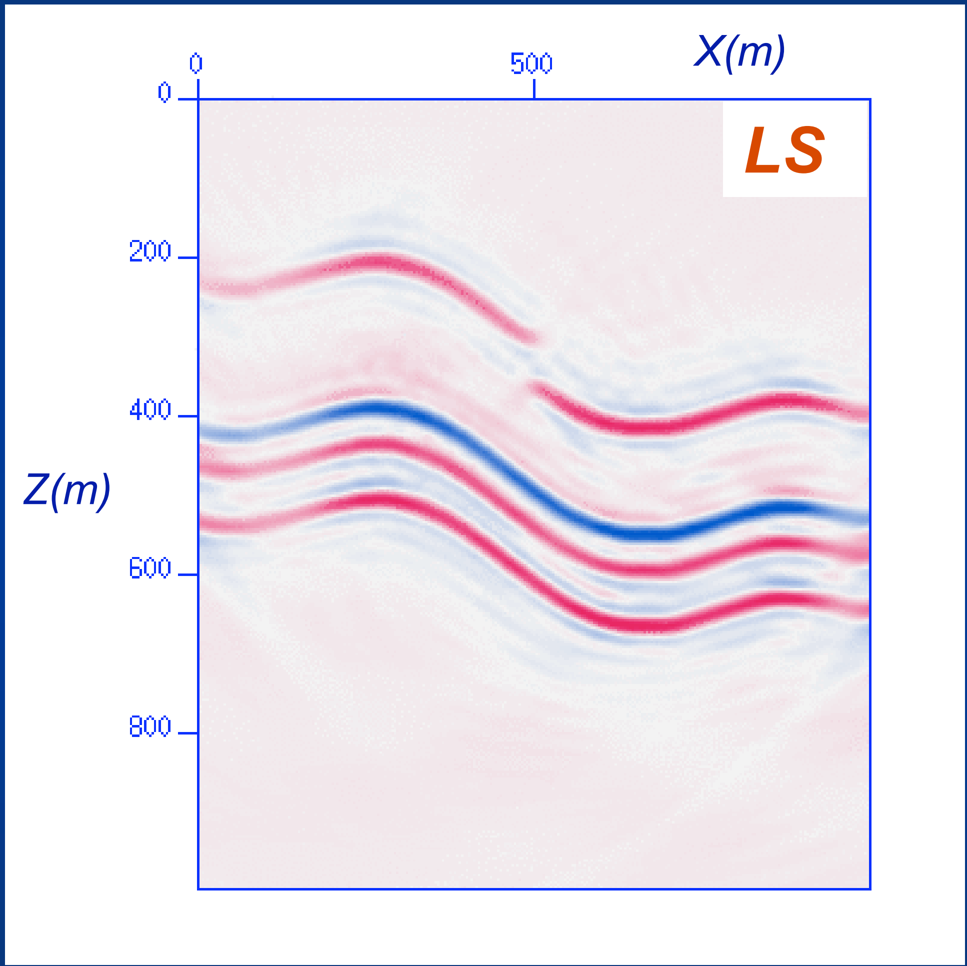
m(x,z,h)

Stacked CIGs



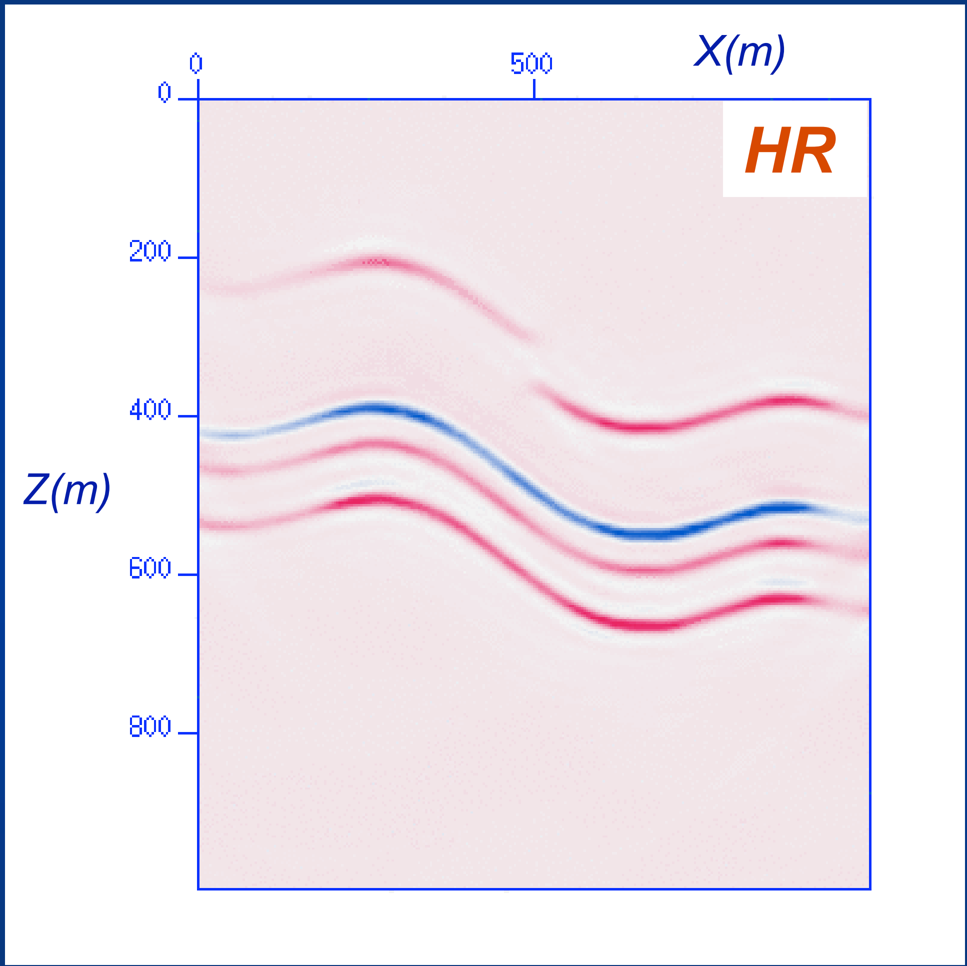
m(x,z)

Stacked CIGs



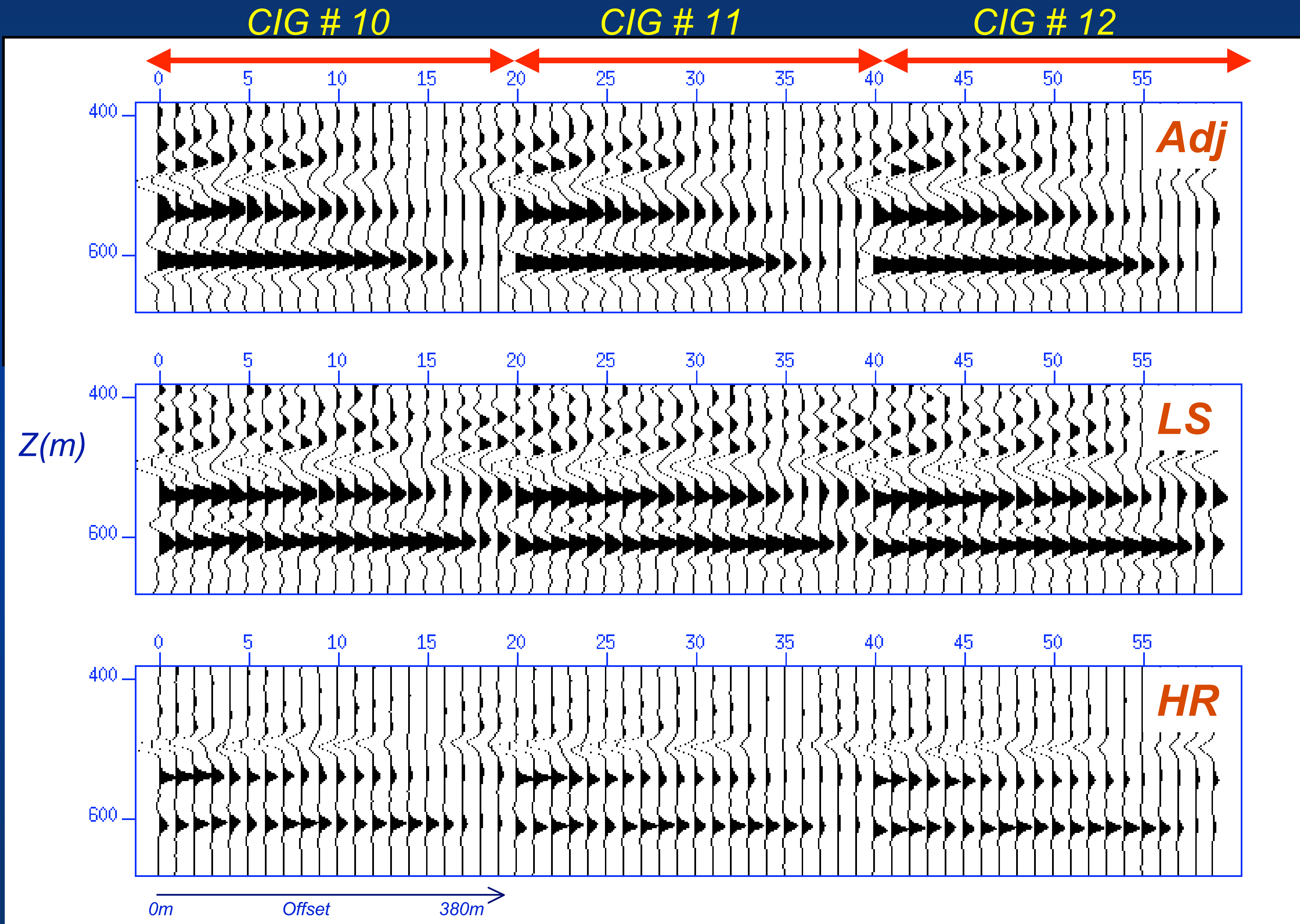
m(x,z)

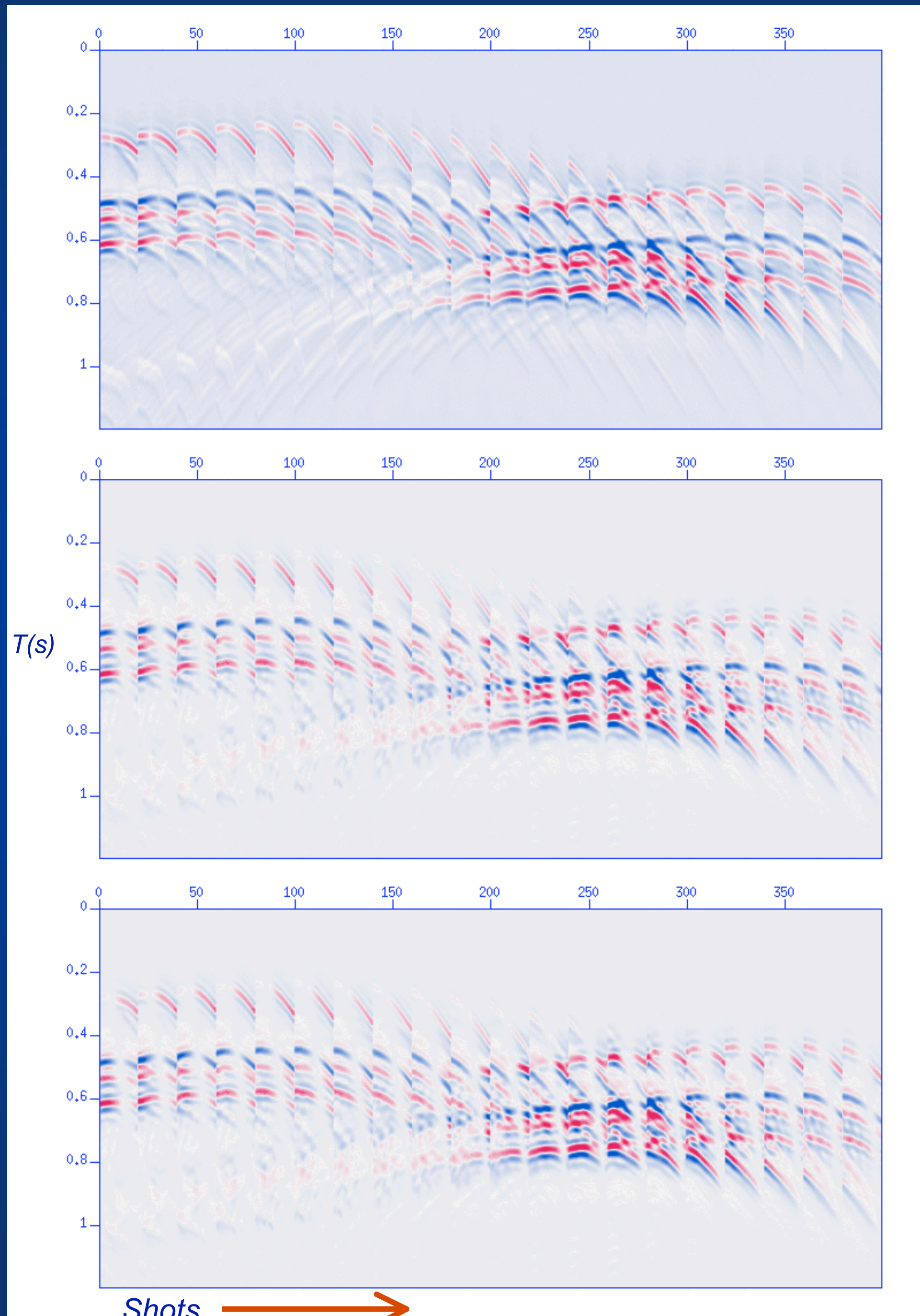
Stacked CIGs



m(x,z)

CIGs


CIG # 10
CIG # 11
CIG # 12
0 5 10 15 20 25 30 35 40 45 50 55
400
600
Adj
LS
HR
Z(m)
0m
Offset
380m



Data

LS Prediction

HR Prediction

d(s,g,t)

Conclusions

- *Imaging/Inversion with the addition of quadratic and non-quadratic constraints could lead to a new class of imaging algorithms where the resolution of the inverted image can be enhanced beyond the limits imposed by the data (aperture and band-width).*
- *This is not a completely new idea. Exploration geophysicists have been using similar concepts to invert post-stack data (sparse spike inversion) and to design Radon operators.*
- *Finally, it is important to stress that any regularization strategy capable of enhancing the resolution of seismic images must be applied in the CIG domain. Continuity along the CIG horizontal variable (offset, angle, ray parameter) in conjunction with sparseness in depth, appears to be reasonable choice.*

Acknowledgments

- EnCana
- Geo-X
- Veritas
- Schlumberger foundation
- NSERC
- AERI

- 3D Real data set was provided by Dr Cheadle