operator composition for improved data imaging

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$$A : \mathcal{E}'(Y) \mapsto \mathcal{D}'(X)$$

$$A(u(y))(x) =$$

$$(2\pi)^{-n} \int\!\!\int a(x, y, \theta) e^{\mathrm{i}\phi(x,y,\theta)} u(y) \mathrm{d}y \, \mathrm{d}\theta$$

special case: $\theta = \omega$,

$$\phi(x, y, \omega) = \omega(T(x, y) - t)$$

$$(2\pi)^{-n}\iint a(x,y,\theta)e^{\mathrm{i}\phi(x,y,\theta)}u(y)\,\mathrm{d}y\,\mathrm{d}\theta$$

$$(2\pi)^{-n}\iint b(z,x,\sigma)e^{\mathrm{i}\psi(z,x,\sigma)}v(x)\,\mathrm{d}x\,\mathrm{d}\sigma$$

$$(2\pi)^{-2n}\iiiint\Big\{b(z,x,\sigma)a(x,y,\theta)\,e^{\mathrm{i}\psi(z,\underset{\uparrow}{x},\sigma)+\mathrm{i}\phi(\underset{\uparrow}{x},y,\theta)}u(y)\,\mathrm{d}y\,\mathrm{d}\theta\,\mathrm{d}x\,\mathrm{d}\sigma\Big\}$$

phase variable

$$(2\boldsymbol{\pi})^{-2n}\int\!\!\int \boldsymbol{c}(z,y,\theta')e^{\mathrm{i}\eta(z,y,\theta')}\boldsymbol{u}(y)\,\mathrm{d}y\,\mathrm{d}\theta'$$

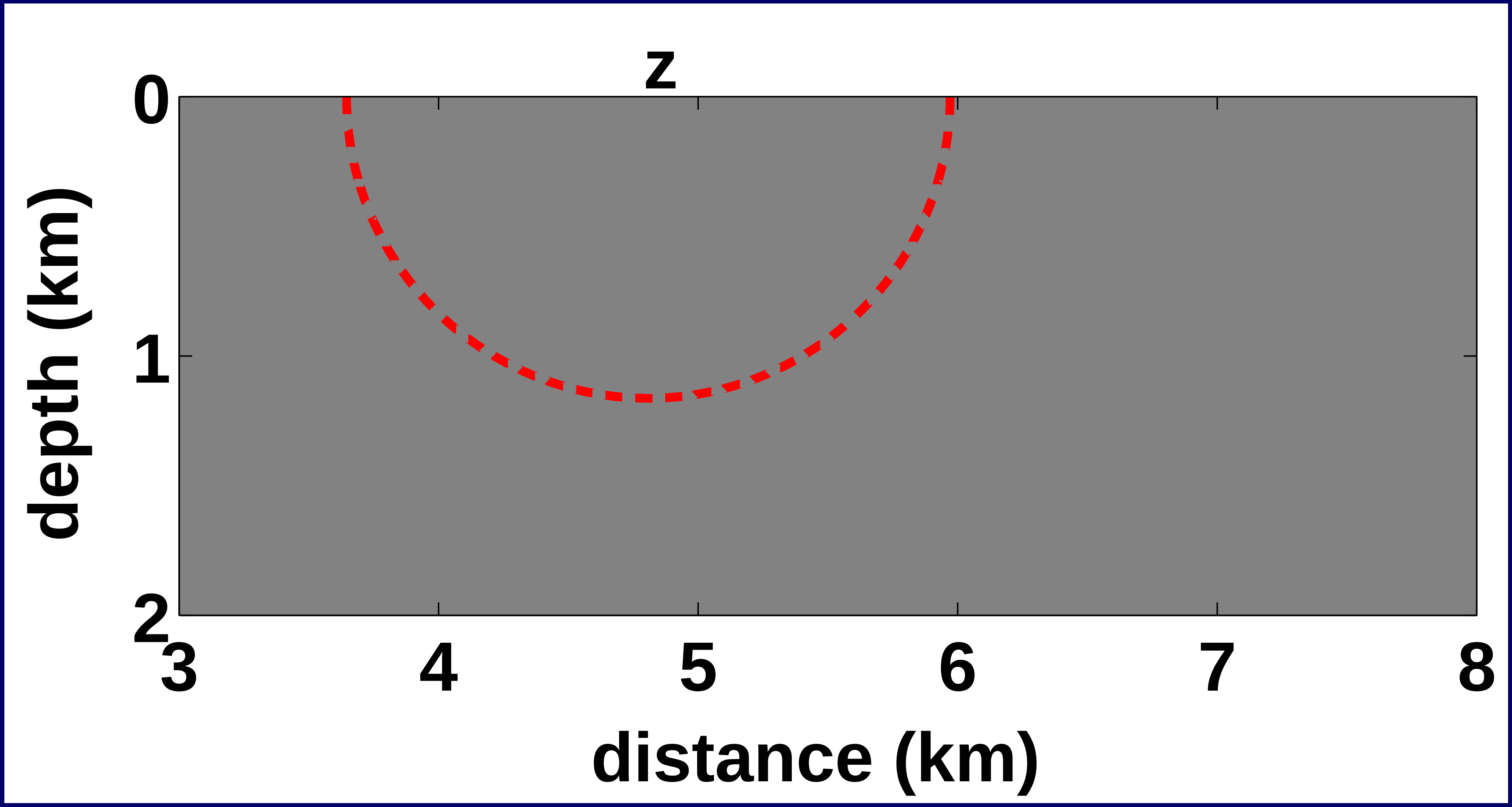
$$(2\pi)^{-2n}\int\!\!\int c(z,y,\theta')e^{\mathrm{i}\eta(z,y,\theta')}u(y)\,\mathrm{d}y\,\mathrm{d}\theta'$$

clean intersection calculus

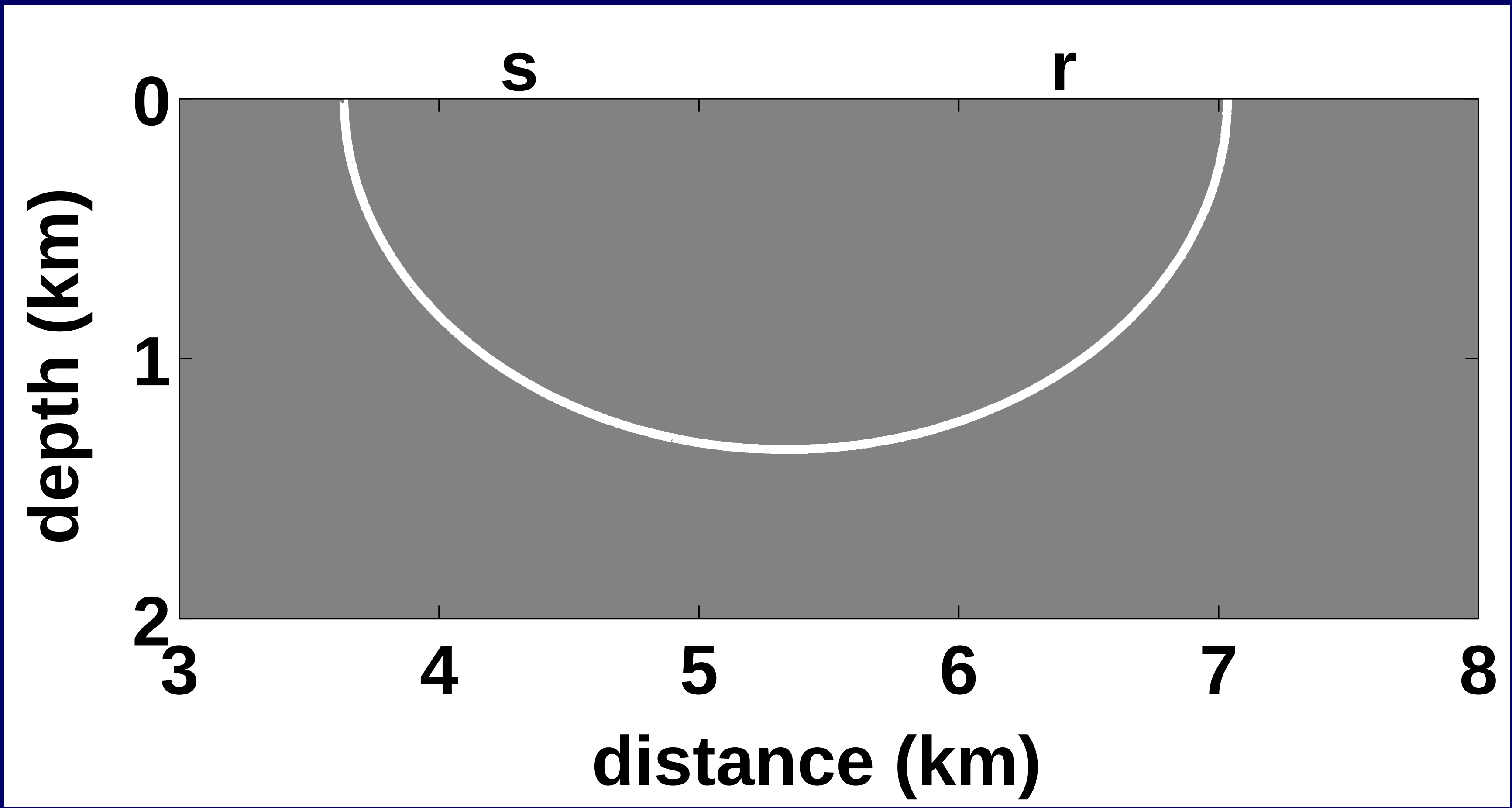
$$(2\pi)^{-2n}\int\!\!\int c(z,y,\theta')e^{\mathrm{i}\eta(z,y,\theta')}u(y)\,\mathrm{d}y\,\mathrm{d}\theta'$$

clean intersection calculus

min number of phase vars



z
depth (km)
0
1
2
3
4
5
6
7
8
distance (km)


s
r
0
1
2
depth (km)
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4
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7
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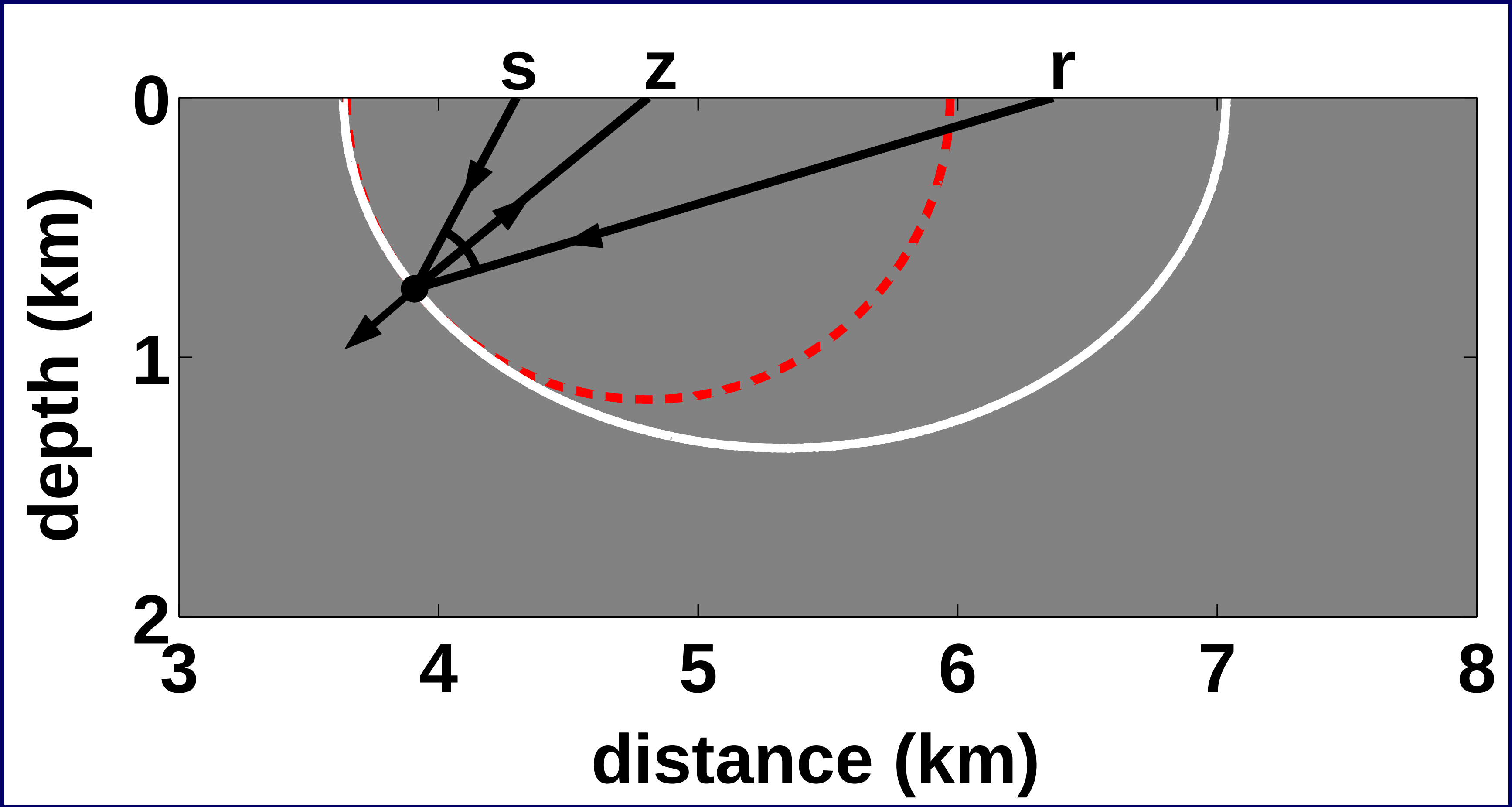
distance (km)



s
z
r
0
1
2
depth (km)
3
4
5
6
7
8
distance (km)

dip moveout

$$F_0 F^*$$

F_0: **exploding reflector modelling**

F^*: **imaging/migration**

3D DMO

$$\phi(y,t,x,\omega) = \omega\left(\frac{|x-y-h|}{c} + \frac{|x-y+h|}{c} - t\right)$$

$$\phi_0(z,t_0,x,\omega_0) = -\omega_0\left(\frac{2|x-z|}{c} - t_0\right)$$

phase variables: $(x/|\omega|, \omega_0, \omega)$

3D DMO

$$\phi_D(y,t,z,t_0,x,\omega,\omega_0) = \phi + \phi_0 =$$

$$\omega\left(\frac{|x-y-h|}{c} + \frac{|x-y+h|}{c} - t\right)$$

$$-\,\omega_0\left(\frac{2|x-z|}{c} - t_0\right)$$

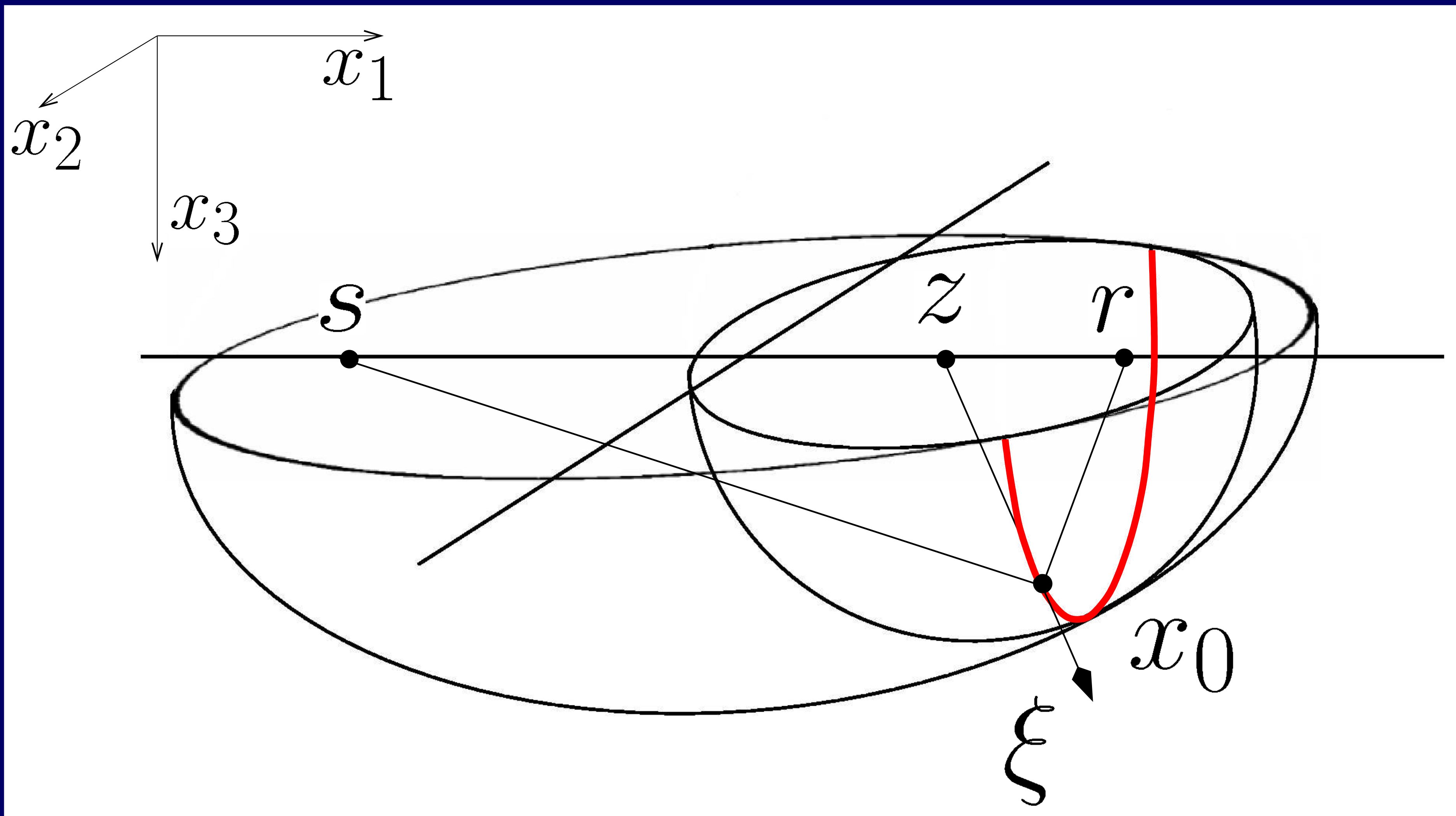
- $\mathrm{d}(\partial_{x,\omega,\omega_0}\phi_D)$ **full rank**

$$S_{\phi_D} = \left\{ (y, t, z, t_0, x, \omega, \omega_0) \mid \underbrace{\partial_x \phi_D = 0, \partial_\omega \phi_D = 0, \partial_{\omega_0} \phi_D = 0}_{f=0} \right\}$$

$$\pi : (y, t, z, t_0, x, \omega, \omega_0) \mapsto (y, t, z, t_0)$$

$$\operatorname{corank}(D\pi|_{S_{\phi_D}}) = \dim\{(x, \omega, \omega_0)\} - \operatorname{rank}(\partial_x f, \partial_\omega f, \partial_{\omega_0} f) = 2$$

- **phase variables:** (x_2, ω)



data regularization

$$N = (F^* F)^{-1}$$

$$\downarrow$$

$$(R) \qquad F \qquad \qquad F^*$$

$$\uparrow$$

AMO...

$N = F^*F$ is elliptic ΨDO

$$\phi_N = (x - x') \cdot \xi$$

in general FF^* not ΨDO

time processing $F_h F_h^*$ pseudo

no caustics

fixed offset

$$\phi = -\omega(t - T(x, h, y))$$
$$\phi' = \omega'(t' - T(x, h, y'))$$

$$\phi_r = \phi + \phi' =$$
$$-\omega(t - T(x, h, y)) +$$
$$\omega'(t' - T(x, h, y'))$$

phase variables: $(x/|\omega|, \omega', \omega)$

$$S_{\phi_r} = \{(y, h, t, y', h, t', x, \omega, \omega') \mid$$
$$\partial_x \phi_r = 0, \partial_\omega \phi_r = 0, \partial_{\omega'} \phi_r = 0\}$$

stationary phase in ω, x_3

$$\partial_\omega \phi_r = -t + T(x, y, h) = 0$$
$$\partial_{x_3} \phi_r = \omega \partial_{x_3} T(x, y, h)$$
$$-\omega' \partial_{x_3} T(x, y', h) = 0$$

$$\rightarrow (\omega, x_3) = (\omega^0, x_3^0)$$

at stationarity

$$\phi_r = \omega'(t' - T(x_1, x_2, x_3^0, h, y'))$$

$$= \omega'(t' - T(\cdot, y') \underbrace{-t + T(\cdot, y))}_{=0})$$

$$= \omega'(t' - t) + \omega'(T(\cdot, y) - T(\cdot, y'))$$

$$\approx \omega'(t' - t) + \underbrace{\omega' \partial_{y'} T|_{y'=y}}_{\eta'} \cdot (y - y')$$

$F_h F_h^*$ **pseudo**

$$G_h := F_h N^{1/2}$$

unitary

$$\downarrow$$

continuation: $G_{h'} G_h^*$

$$\uparrow$$

'true' amplitude

$$
\begin{tikzcd}
 & \Lambda_F \arrow[dl] \arrow[dr] & & \Lambda_{F^*} \arrow[dl] \arrow[dr] & \\
T^*Y & & T^*X(\times E) & & T^*Y
\end{tikzcd}
$$

submersion

- **composition is clean**

continuation still FIO