RESERVOIR CHARACTERIZATION:

i –waveform elastic inversion

ii – petrophysical inference

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Core Lab

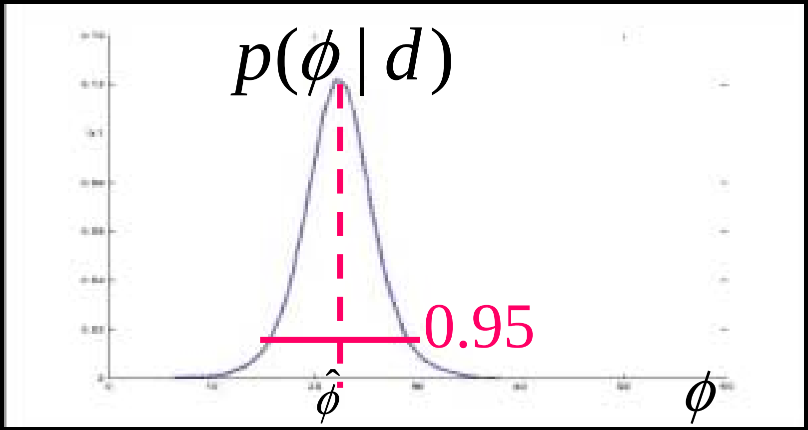
Bayesian Methodology of inference

- Integration of independent sources of information
  - seismic elastic attributes: pre-stack elastic inversion
  - rock physics models: empirical models
  - petrophysical observations: well log data
    core laboratory experiments
- Uncertainty analysis associated with

Bayes Theorem

$$p(\phi \mid d) \propto l(d \mid \phi) I(\phi)$$

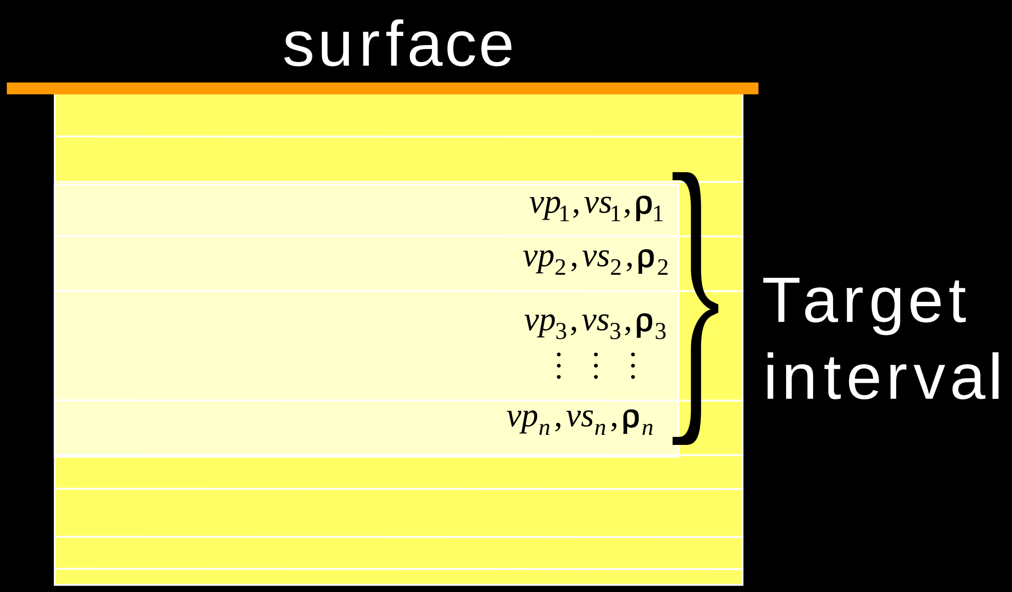
posterior probability	likelihood function	prior probability



FULL-WAVEFORM ELASTIC INVERSION

Gouveia & Scales (1998), Bayesian Seismic waveform inversion: estimation and uncertainty analysis

$$p(\mathbf{m}\,|\,\mathbf{d_s}) \propto l(\mathbf{d_s}\,|\,\mathbf{m})\, I(\mathbf{m})$$



$$\mathbf{m} = \begin{bmatrix} vp_1 \\ vp_2 \\ \vdots \\ vp_m \\ vs_1 \\ vs_2 \\ \vdots \\ vs_m \\ \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix}$$

Formulation

$$I(m) \propto \exp\left\{-\frac{1}{2}\left[m_{prior}-m\right]^{T} C_{prior}^{-1}\left[m_{prior}-m\right]\right\}$$

$$l(d_s|m) \propto \exp\left\{-\frac{1}{2}\left[d_s-g(m_s)\right]^{T} C_{d}^{-1}\left[d_s-g(m_s)\right]\right\}$$

$$p(m|d_s) \propto \exp\left\{-\frac{1}{2}\left[m_{map}-m\right]^{T} C_{map}^{-1}\left[m_{map}-m\right]\right\}$$

Prior distribution

experimental variogram data **v** → spatial variability information

$$\mathbf{v} = \gamma(\mathrm{h}) = \frac{1}{2\mathrm{NP}} \sum_{\mathrm{i}=1}^{\mathrm{NP}} \left[\mathrm{m}(\mathrm{r_i}) - \mathrm{m}(\mathrm{r_i} + \mathrm{h})\right]^2$$

$$I(\mathrm{m_i}, \boldsymbol{\sigma} \,|\, \mathbf{v}) \propto exp\left\{-\frac{1}{2\pi\sigma^2}\left[\mathbf{v} - \gamma(\mathbf{h})\right]^{\mathrm{T}}\left[\mathbf{v} - \gamma(\mathbf{h})\right]\right\}$$

Marginal distribution

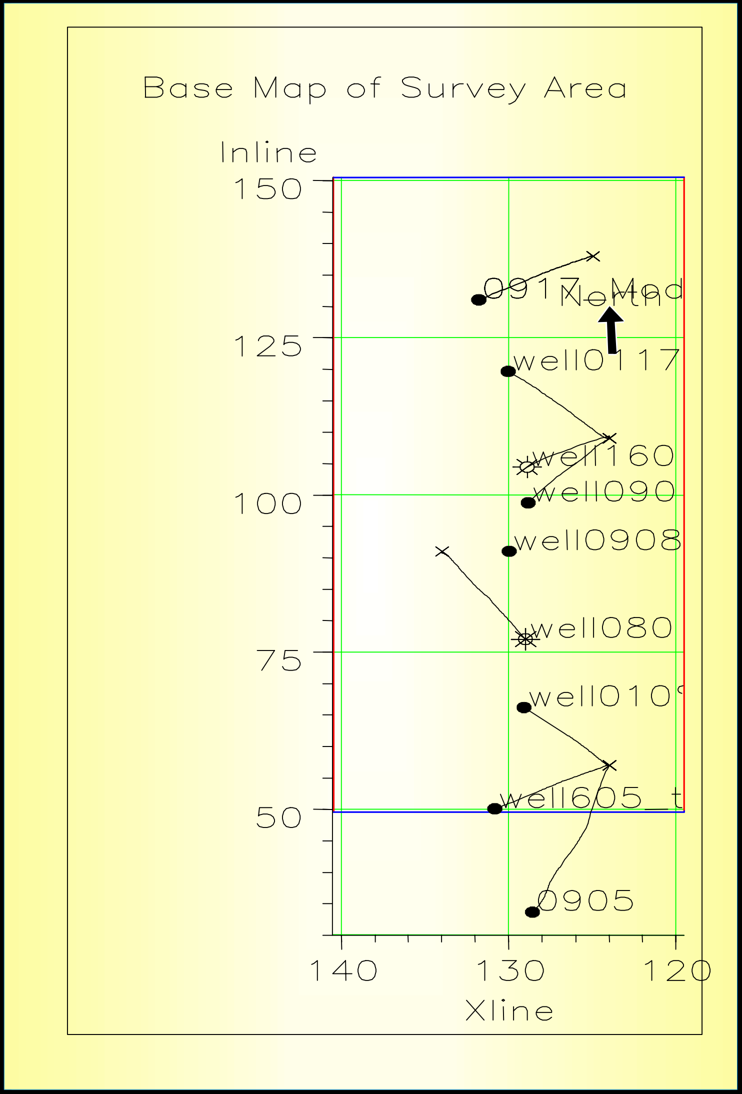
$$I(\mathrm{m_i} \mid \mathbf{v}) \propto \int I(\mathrm{m_i}, \sigma \mid \mathbf{v})\, \mathrm{d}\sigma$$

$$\left\{ [\mathbf{v} - \chi(\mathbf{h})]^{\mathrm{T}} [\mathbf{v} - \chi(\mathbf{h})] \right\}^{-\frac{\mathrm{NP}}{2}}$$

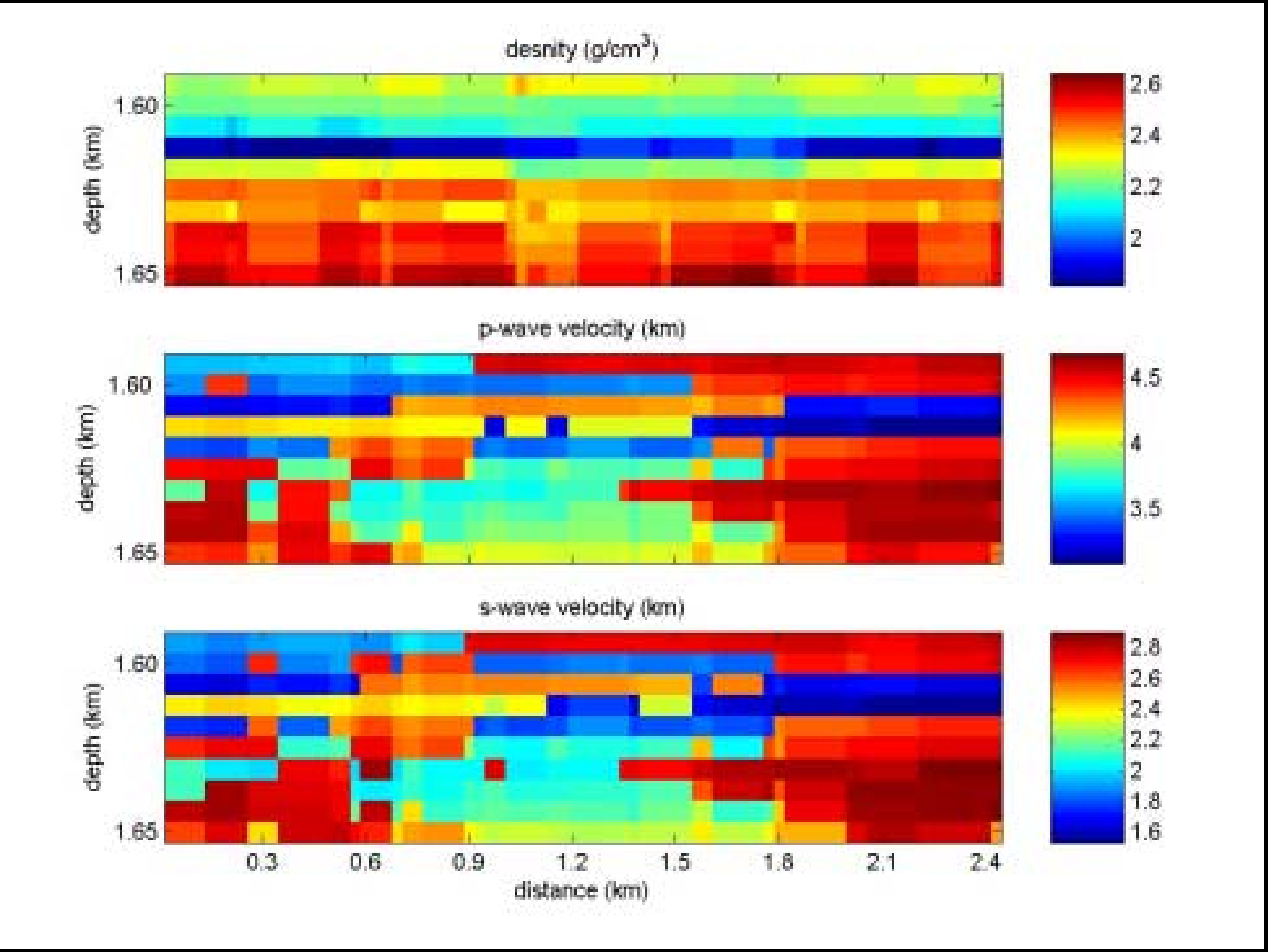
$$I(\sigma \mid \mathbf{v}) \propto \int I(\mathrm{m_i}, \sigma \mid \mathbf{v})\, \mathrm{dm_i}$$

$$\propto \sigma^{-\mathrm{NP}}\, exp\left\{ -\frac{\mathrm{NP}-1}{2\sigma^{-2}} \right\}, 0 < \sigma < \infty$$

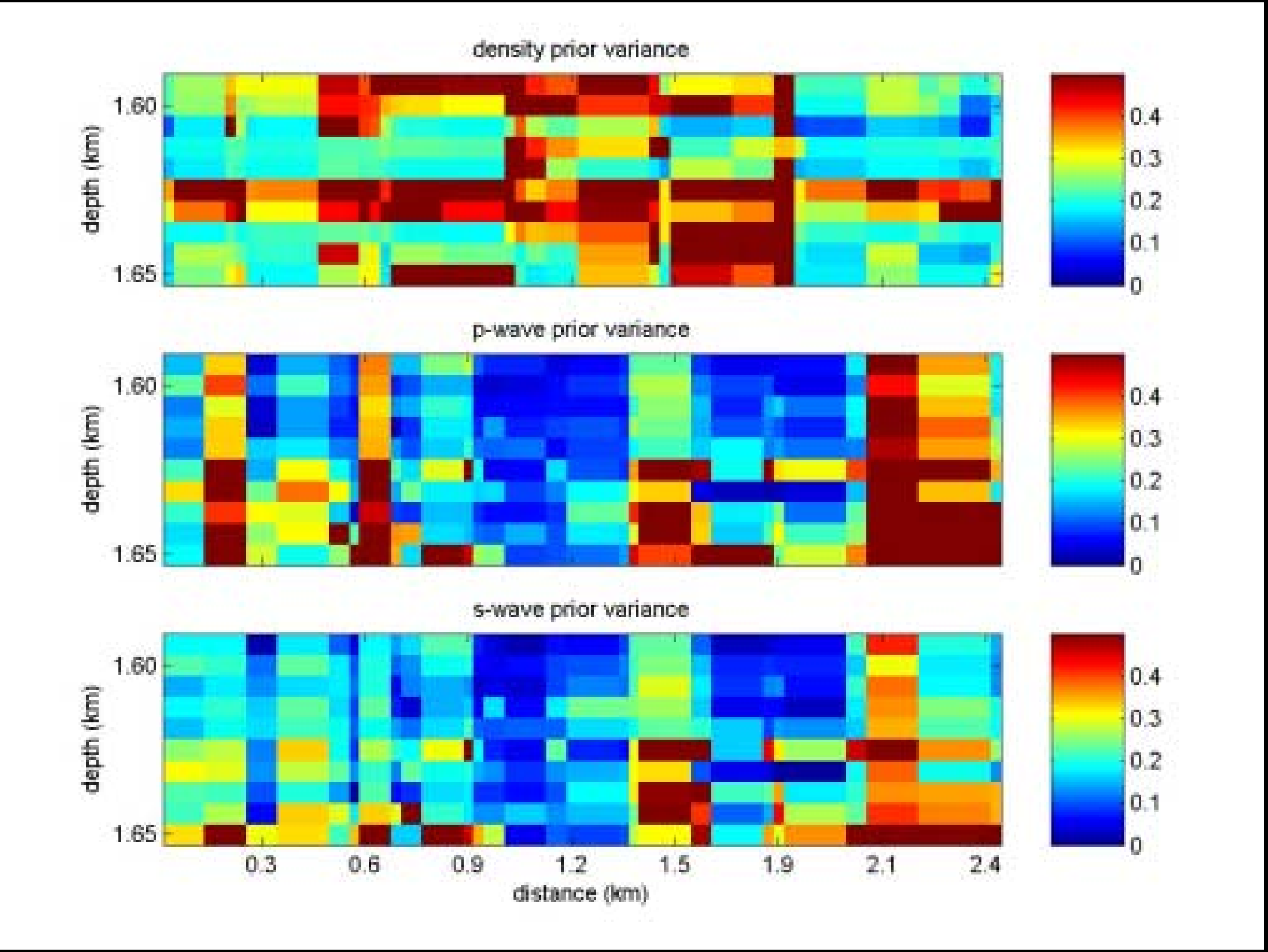
Example


Base Map of Survey Area
Inline
150
125
100
75
50
0917_Mod
North
well0117
well160
well090
well0908
well080
well010
well605_t
0905
140
130
120
Xline

Example: Prior elastic model



Example: Prior variance



optimization

$$p(\mathbf{m}\,|\,\mathbf{d_s}) \propto l(\mathbf{d_s}\,|\,\mathbf{m})\,I(\mathbf{m})$$

$$\Theta(\mathbf{m}) = \frac{1}{2}\left\{[\mathbf{d}_\mathrm{s} - g(\mathbf{m})]^\mathrm{T}\mathbf{C}_\mathrm{d}^{\mathbf{-1}}[\mathbf{d}_\mathrm{s} - g(\mathbf{m})] + [\mathbf{m}_{prior} - \mathbf{m}]^\mathrm{T}\mathbf{C}_{prior}^{\mathbf{-1}}[\mathbf{m}_{prior} - \mathbf{m}]\right\};$$

$$\mathbf{m}_{n+1} = \mathbf{m}_n + \alpha\kappa_\mathrm{n};$$

$$\nabla g(\mathbf{m}) = \Gamma(\mathbf{m})\mathrm{C}_\mathrm{d}^{\mathbf{-1}}[\mathbf{d}_\mathrm{s} - g(\mathbf{m})] + \mathbf{C}_{prior}^{\mathbf{-1}}[\mathbf{m}_{prior} - \mathbf{m}];$$

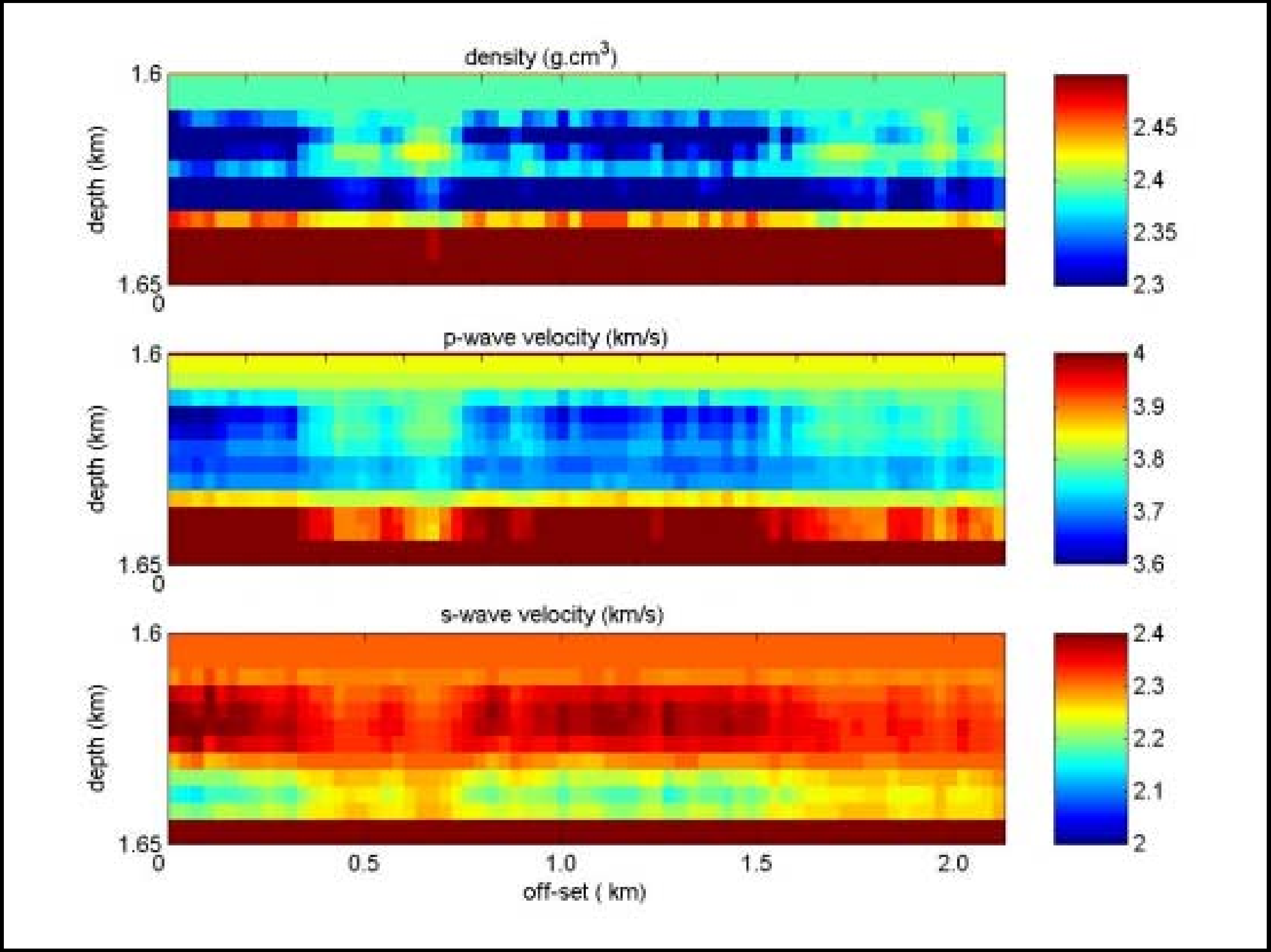
$$\Gamma(\mathbf{m}) \rightarrow \frac{\partial g(\mathbf{m})}{\partial \mathrm{m}_\mathrm{i}}$$

Posterior distribution

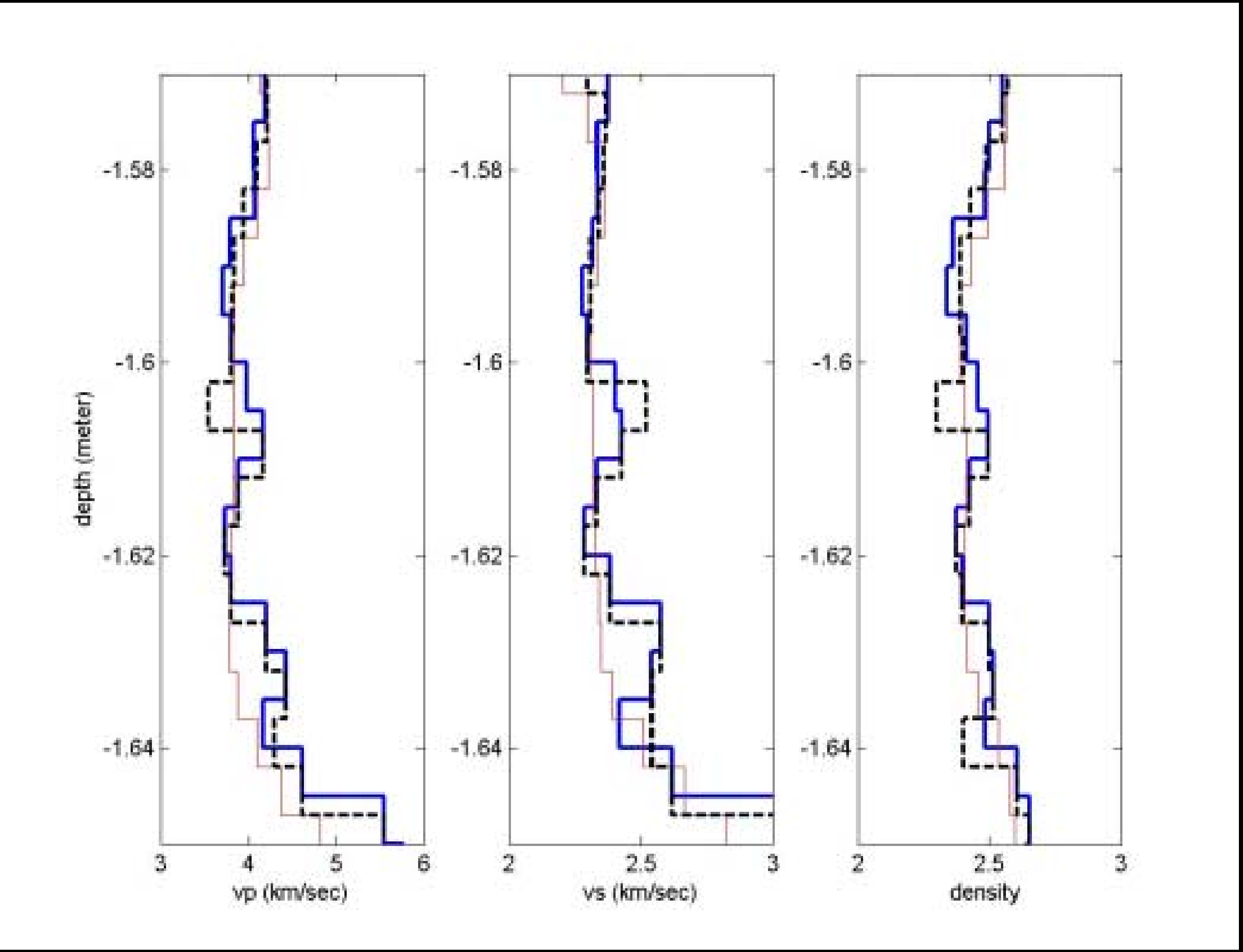
$$p(\mathbf{m}|\mathbf{d}_s) \propto exp\left\{-\frac{1}{2}\left[\mathbf{m}_{\mathrm{map}} - \mathbf{m}\right]^{\mathrm{T}} \mathbf{C}^{\mathbf{-1}}_{\mathrm{map}} \left[\mathbf{m}_{\mathrm{map}} - \mathbf{m}\right]\right\};$$

$$\mathbf{C}^{\mathbf{-1}}_{\mathrm{map}} = \left[\mathbf{G}^{\mathrm{T}} \mathbf{C}^{\mathbf{-1}}_{\mathrm{d}} \mathbf{G} + \mathbf{C}^{\mathbf{-1}}_{prior}\right]^{-1}$$

Example: MAP model



MAP model


-1.58
-1.6
-1.62
-1.64
depth (meter)
3
4
5
6
vp (km/sec)
2
2.5
3
vs (km/sec)
density

PETROPHYSICAL INFERENCE

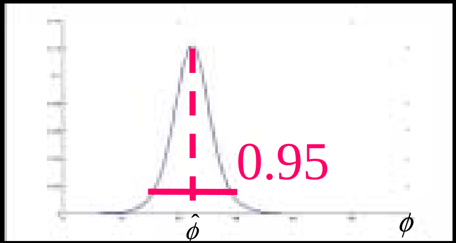
$$p(\phi\,|\,d) \propto l(\mathrm{d}\,|\,\phi)\;\; r(\phi)$$

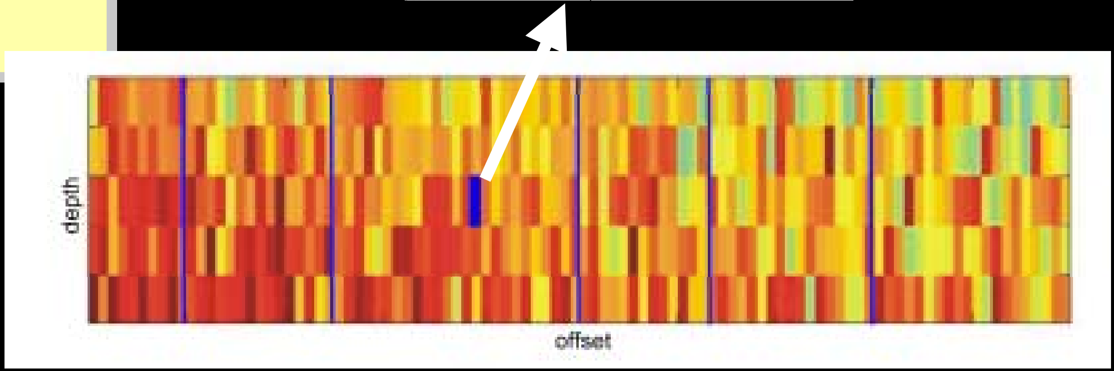
posterior pdf | likelihood pdf | prior pdf

Consider a **reservoir model** composed by a set of *N* block cells with average porosity ⌋.

Our problem is to find one local posterior pdf for porosity to each cell.

Local posterior pdf





prior distribution

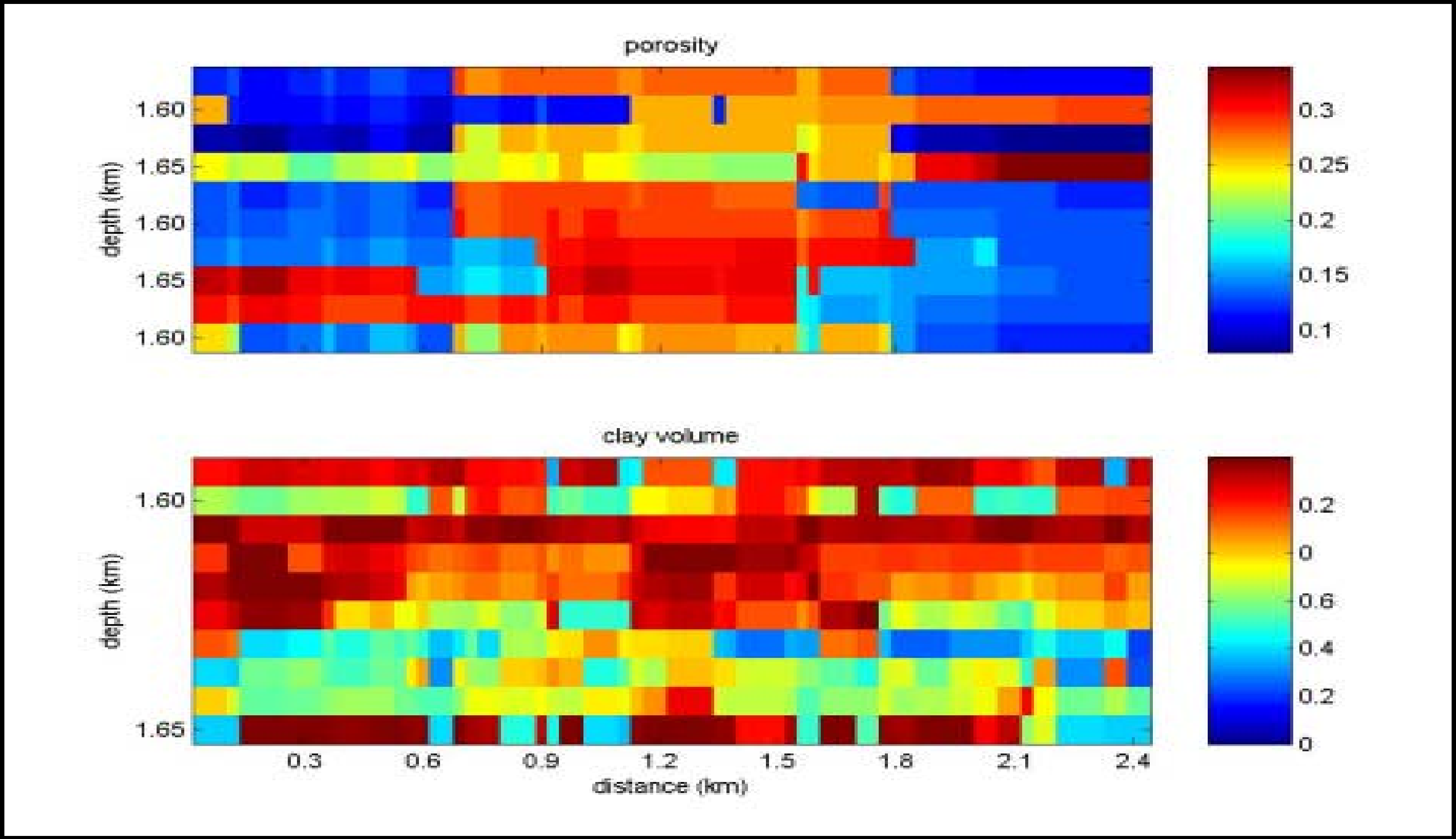
$I(\phi)$

- experimental variogram data from well to well data **v**: **information of the espacial variability**

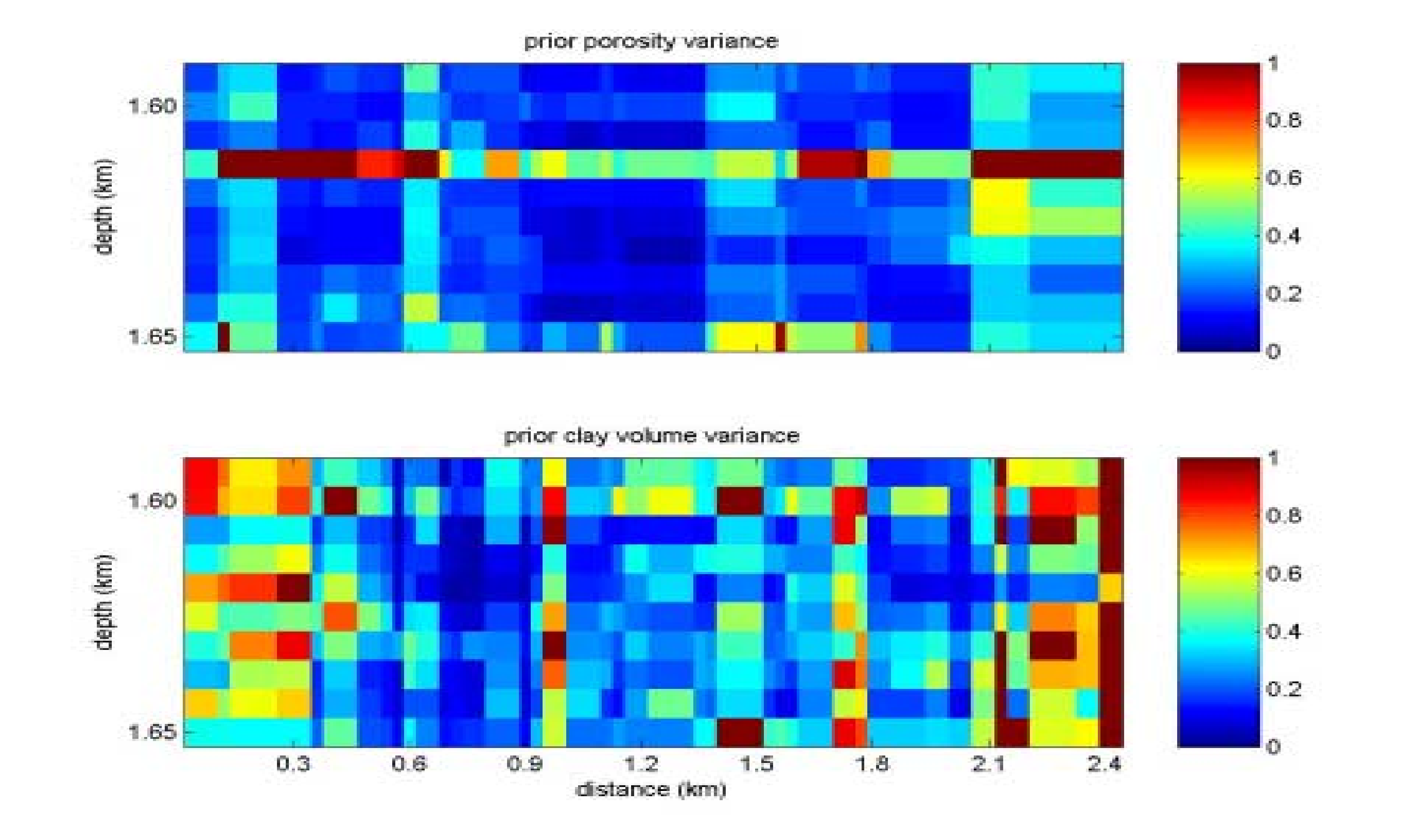
$$I(\phi,\sigma^2 \mid \mathbf{v}) \propto \mathbf{exp}\left\{ -\frac{1}{2\pi\sigma^2} \left[\mathbf{v}_i - f(\phi)\right]^T \left[\mathbf{v}_i - f(\phi)\right] \right\}$$

$$\mathbf{v}(h) = f(\phi) = \frac{1}{2NP} \sum_{i=1}^{NP} \left[\hat{\phi}(r_i)_j - \phi(r_i + h)_j\right]^2$$

Example: prior model



Example: prior variance



Likelihood function

$$l(\mathbf{d}\,|\,\phi), \qquad \mathbf{d}=\begin{bmatrix}\mathbf{vp}\\ \mathbf{vs}\\ \boldsymbol{\rho}\end{bmatrix}, \qquad \mathbf{C}_{map}$$

$$\mathrm{vp}=\mathrm{a}_1-\mathrm{b}_1\phi-\mathrm{c}_1\gamma,$$

$$\mathrm{vs}=\mathrm{a}_2-\mathrm{b}_2\phi-\mathrm{c}_2\gamma,$$

$$\rho=(1-\phi)\rho_{\mathrm{m}}+\phi\rho_{\mathrm{f}},$$

$$l(\mathbf{d},\mathrm{a}_{\mathrm{i}},\mathrm{b}_{\mathrm{i}},\mathrm{c}_{\mathrm{i}},\rho_{\mathrm{m}},\rho_{\mathrm{f}}\,|\,\mathbf{d}^*,\phi,\delta,\phi^*,\delta^*),$$

$$i=1,2$$

BAYESIAN CRITERION TO PREDICT A PDF
OF NEW OBSERVATIONS WITH THE KNOWLEDGE OF OLD OBSERVATIONS:
the predictive PDF

Predictive pdf

i- the cells at the well positions with

-a vector of seismic attributes s*

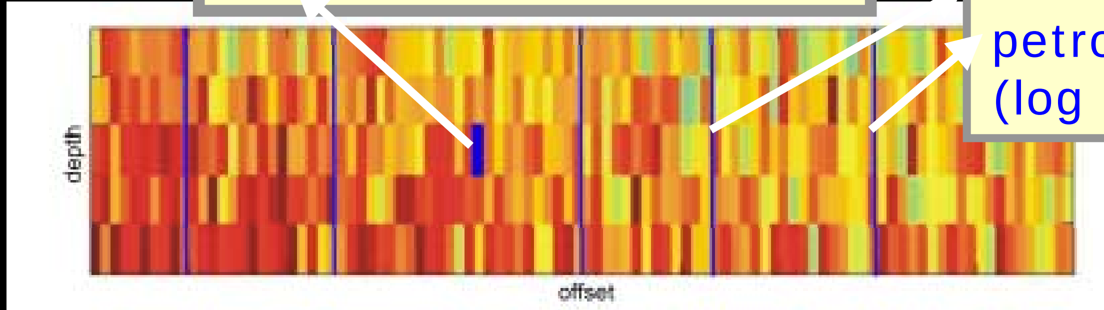
-petrophysical observations of porosity and clay content nside the well.

ii- one cell at a inter-well position with

-estimated seismic attributes s

-unknown porosity and clay content.

cell at inter-well position



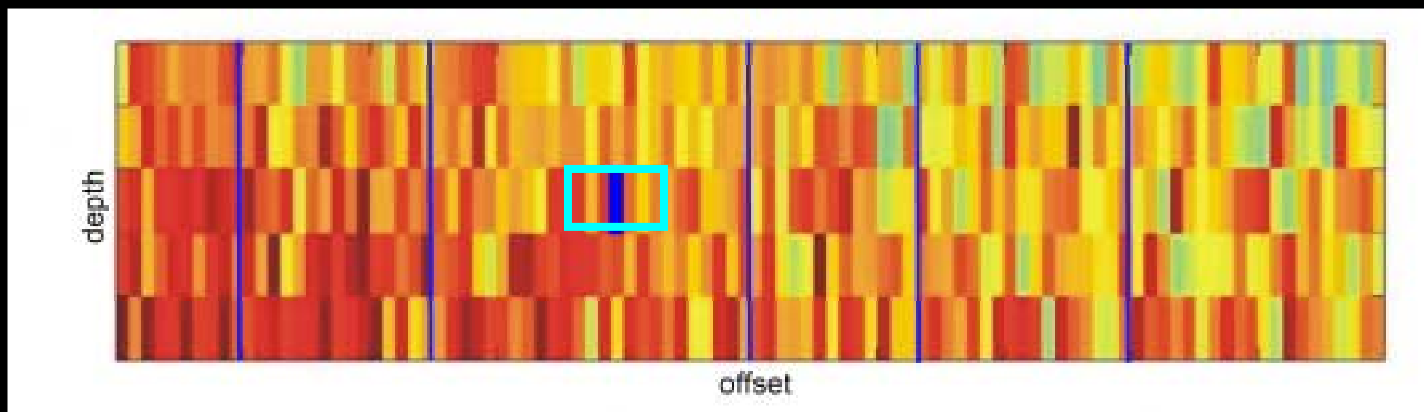
well positions with petrophysical observations (log and/or core)

$$l(\mathrm{d}, a_i, b_i, c_i, \rho_m, \rho_f \mid \mathrm{d}^*, \phi, \delta, \phi^*, \delta^*) = l^+(\mathrm{d} \mid a_i, b_i, c_i, \rho_m, \rho_f, \mathrm{d}^*, \phi, \delta)\, l^*(a_i, b_i, c_i, \rho_m, \rho_f \mid \mathrm{d}^*, \phi^*, \delta^*),\quad i = 1,2$$

PRACTICAL IMPLEMENTATION:

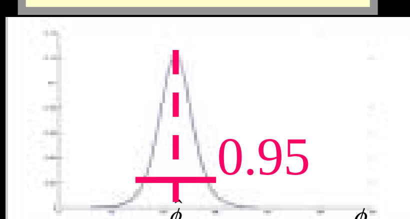
MOVING WINDOW

$$p(\phi \mid d) \propto l(\mathrm{d} \mid \phi)\ r(\phi)$$



Local posterior pdf

$$p(\phi \mid d)$$



$$p(\phi \mid d, \delta = \delta_o)$$

$$\int_{\hat{\phi}+\frac{l}{2}}^{\hat{\phi}-\frac{l}{2}} \mathrm{d}\phi\ p(\phi \mid d, \delta = \delta_o) = 0.95$$

Posterior pdf

$$p(\phi,\delta\,|\,\mathbf{d},\mathbf{d}^*,\phi^*,\delta^*,\mathrm{a}_i,\mathrm{b}_i,\mathrm{c}_i,\rho_m,\rho_f) = l(\mathbf{d},\mathrm{a}_i,\mathrm{b}_i,\mathrm{c}_i,\rho_m,\rho_f\,|\,\mathbf{d}^*,\phi,\delta,\phi^*,\delta^*)\,I(\phi,\delta)$$

$$i = 1,2$$

MARGINALIZATION OF THE NUISANCE PARAMETER

$$p(\phi,\delta\,|\,\mathbf{d},\mathbf{d}^*,\phi^*,\delta^*) = \int p(\phi,\delta\,|\,\mathbf{d},\mathbf{d}^*,\phi^*,\delta^*,\mathrm{a}_i,\mathrm{b}_i,\mathrm{c}_i,\rho_m,\rho_f)\,d\mathrm{a}_i\,d\mathrm{b}_i\,d\mathrm{c}_i\,d\rho_m\,d\rho_f$$

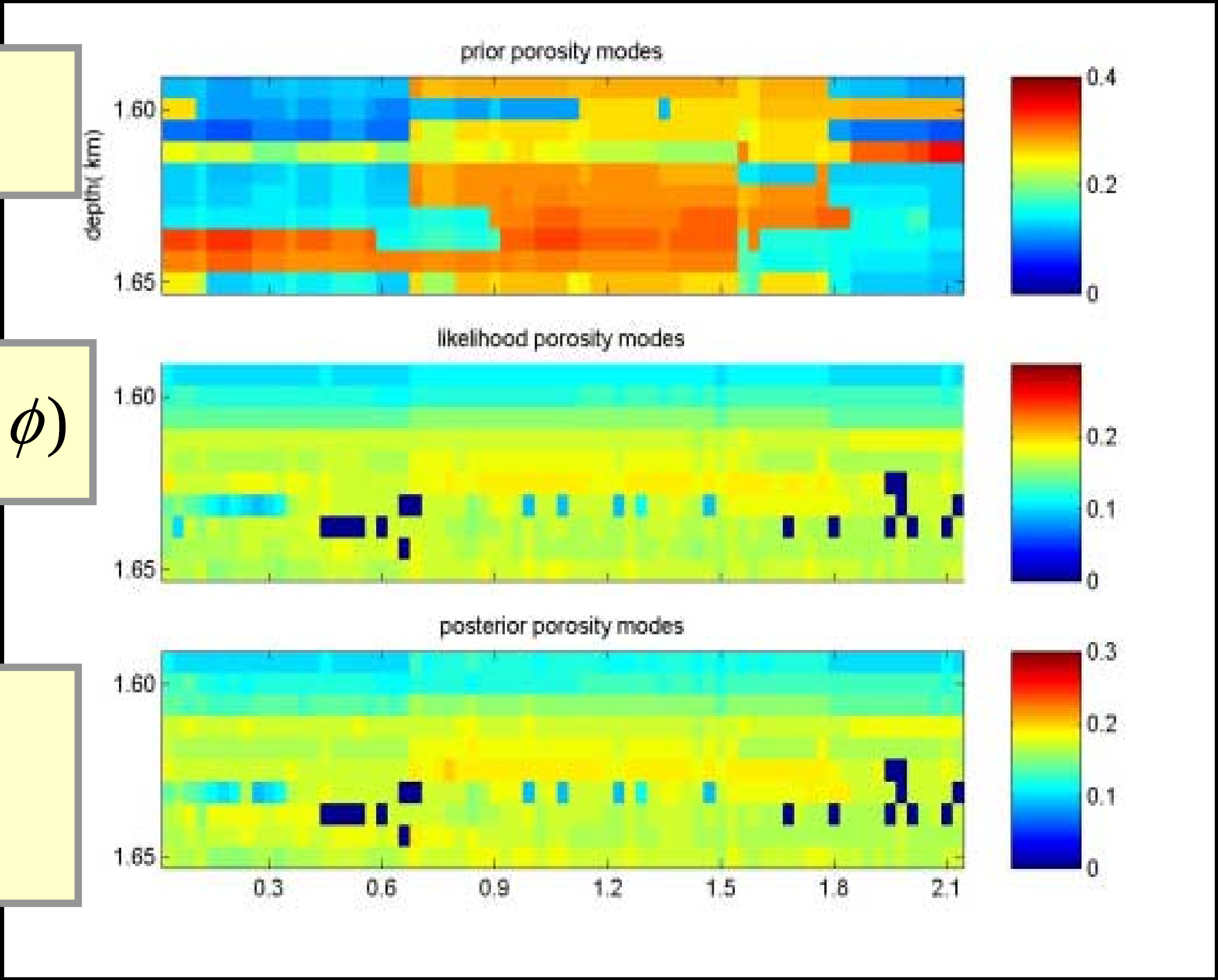
$$i = 1,2$$

The estimated porosity models

$$p(\phi \mid d) \propto r(\phi)$$

$$p(\phi \mid d) \propto l(\mathrm{d} \mid \phi)$$

$$p(\phi \mid d) \propto l(\mathrm{d} \mid \phi)\ r(\phi)$$

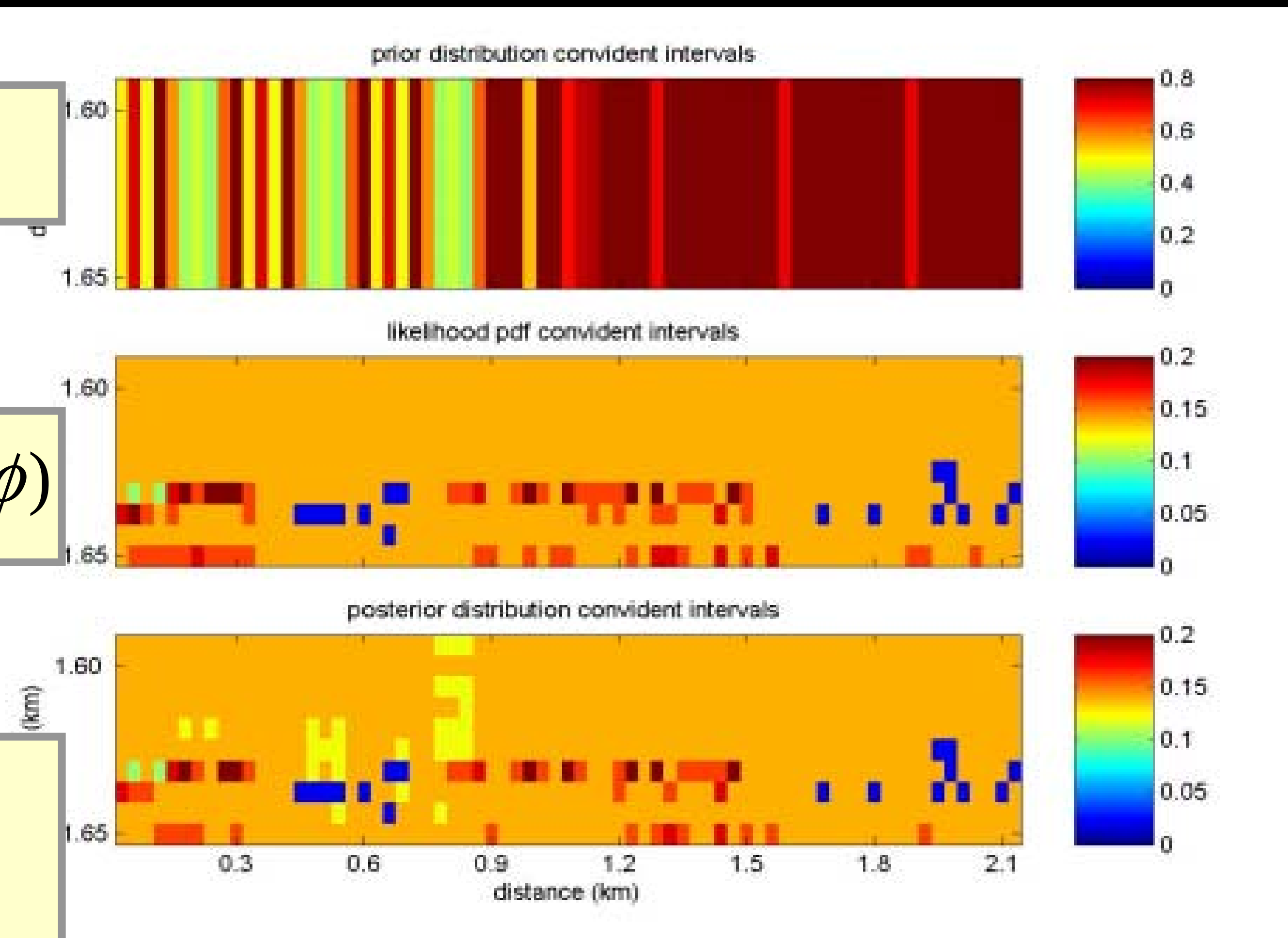


0.95 CONFIDENT INTERVALS

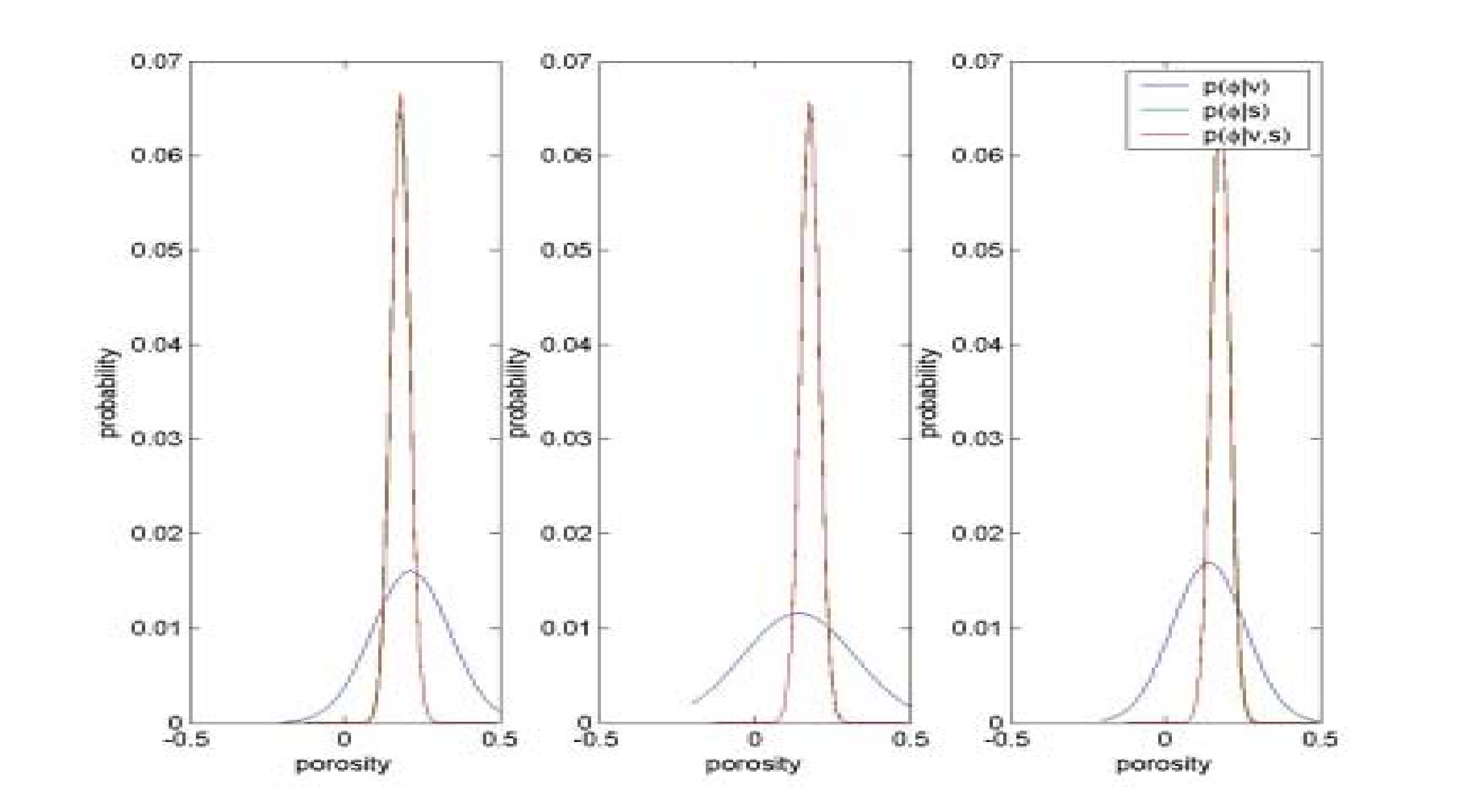
$$p(\phi|d) \propto r(\phi)$$

$$p(\phi|d) \propto l(\mathrm{d}|\phi)$$

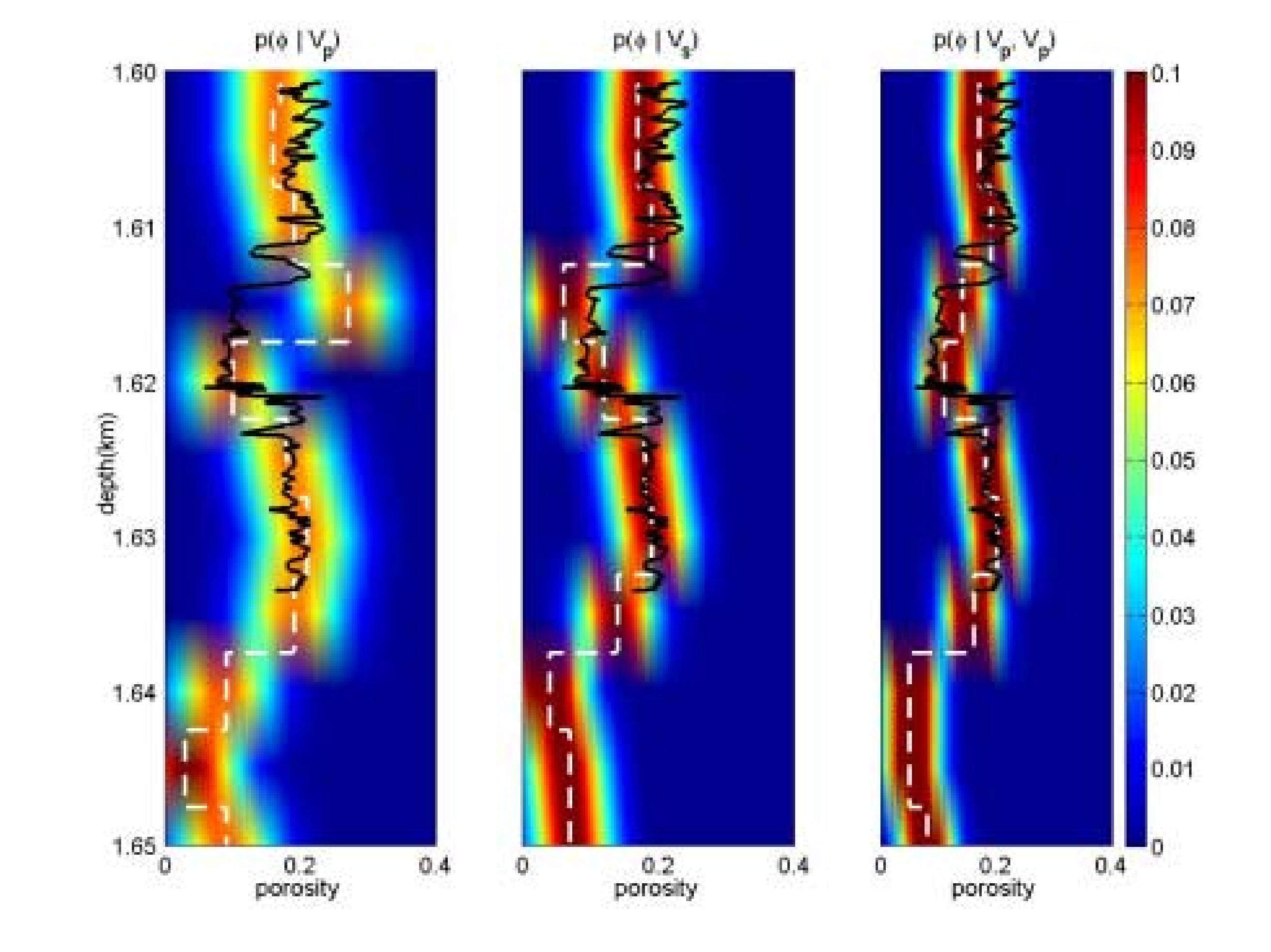
$$p(\phi|d) \propto l(\mathrm{d}|\phi)\ r(\phi)$$



Porosity pdfs'



POROSITY PDFS'



CONLUSION

- The results show reasonable porosity models obtained from the mode of the posterior pdfs,

- the associated uncertainty, represented by the length of 0.95 probability intervals, consistently varies depending on the amount of information available,

- the variogram fitting procedure allowed describing the information from the wells at inter-well locations.

- when combining variogram and attribute data, we observe that the main source of information is the seismic data.

Next development

- Lithological discrimination
- Fluid property inference

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