Fast wavefield extrapolation by phase-shift in the nonuniform Gabor domain

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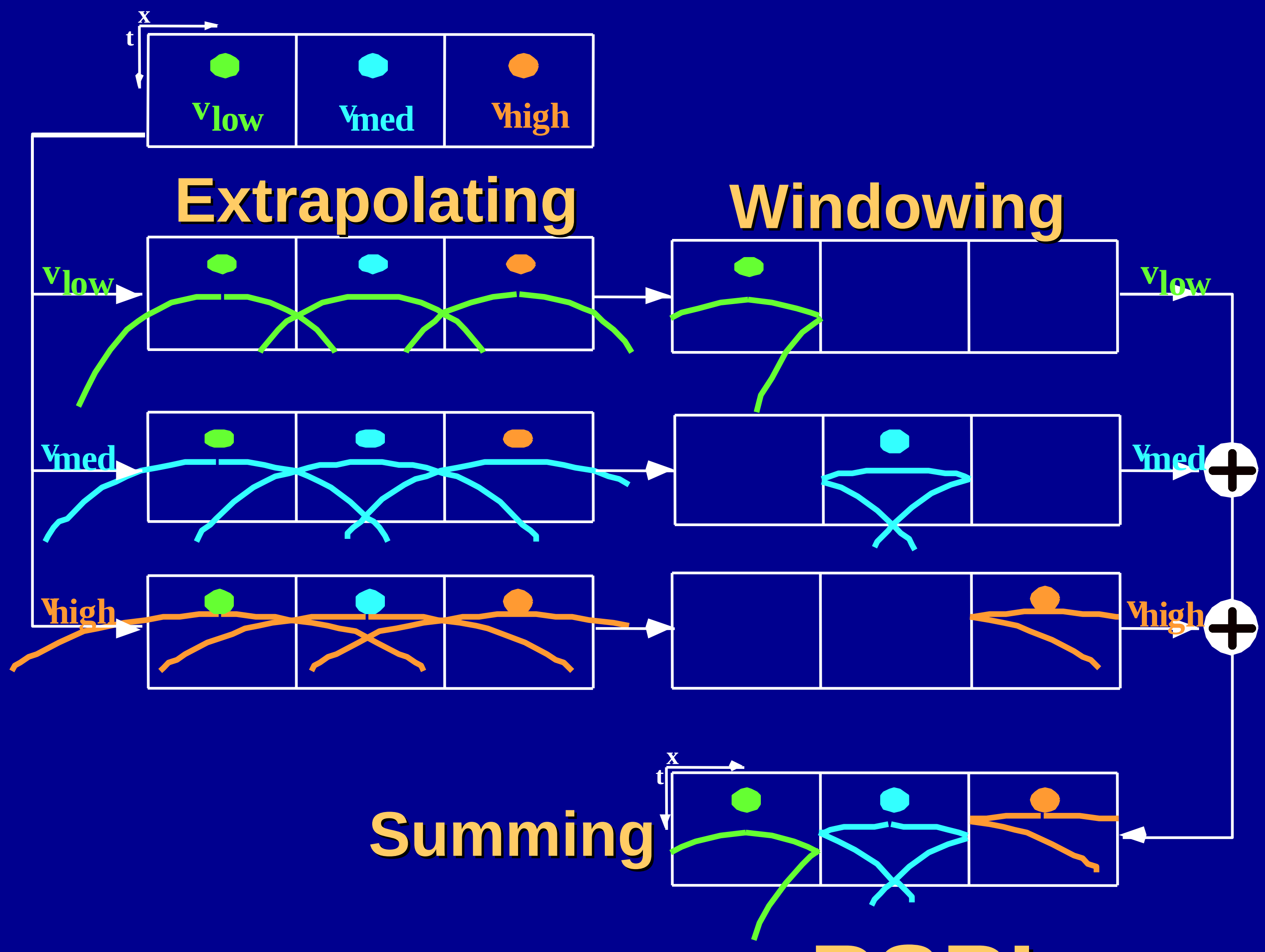
CREWES

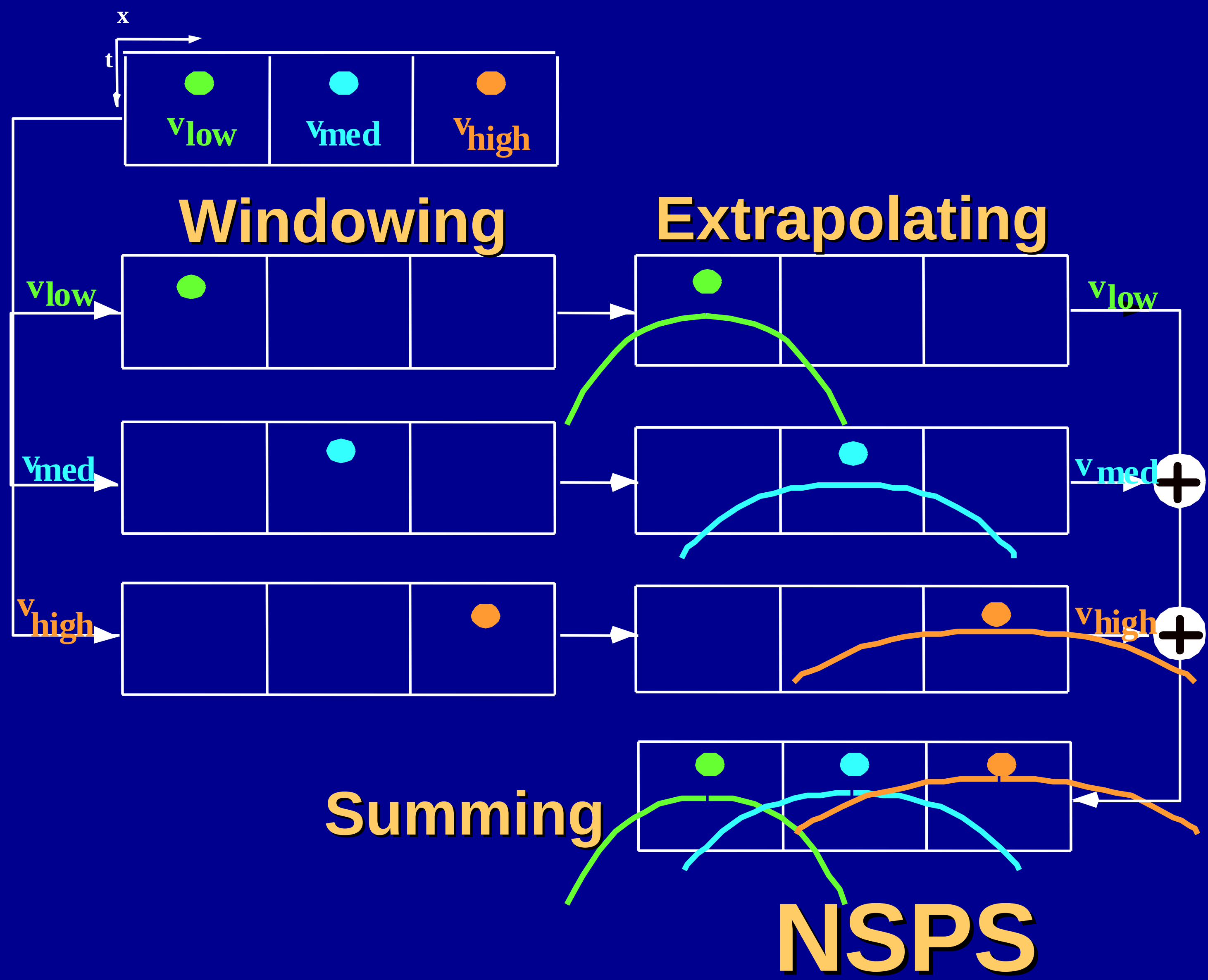
mitacs

Introduction

- PSPI and NSPS
- The Gabor transform
- Uniform POU Gabor frames
- Adaptive POU Gabor frames
- Gabor phase-shift extrapolation
- Synthetic examples
- Summary



Courtesy G. F. Margrave



Courtesy G. F. Margrave

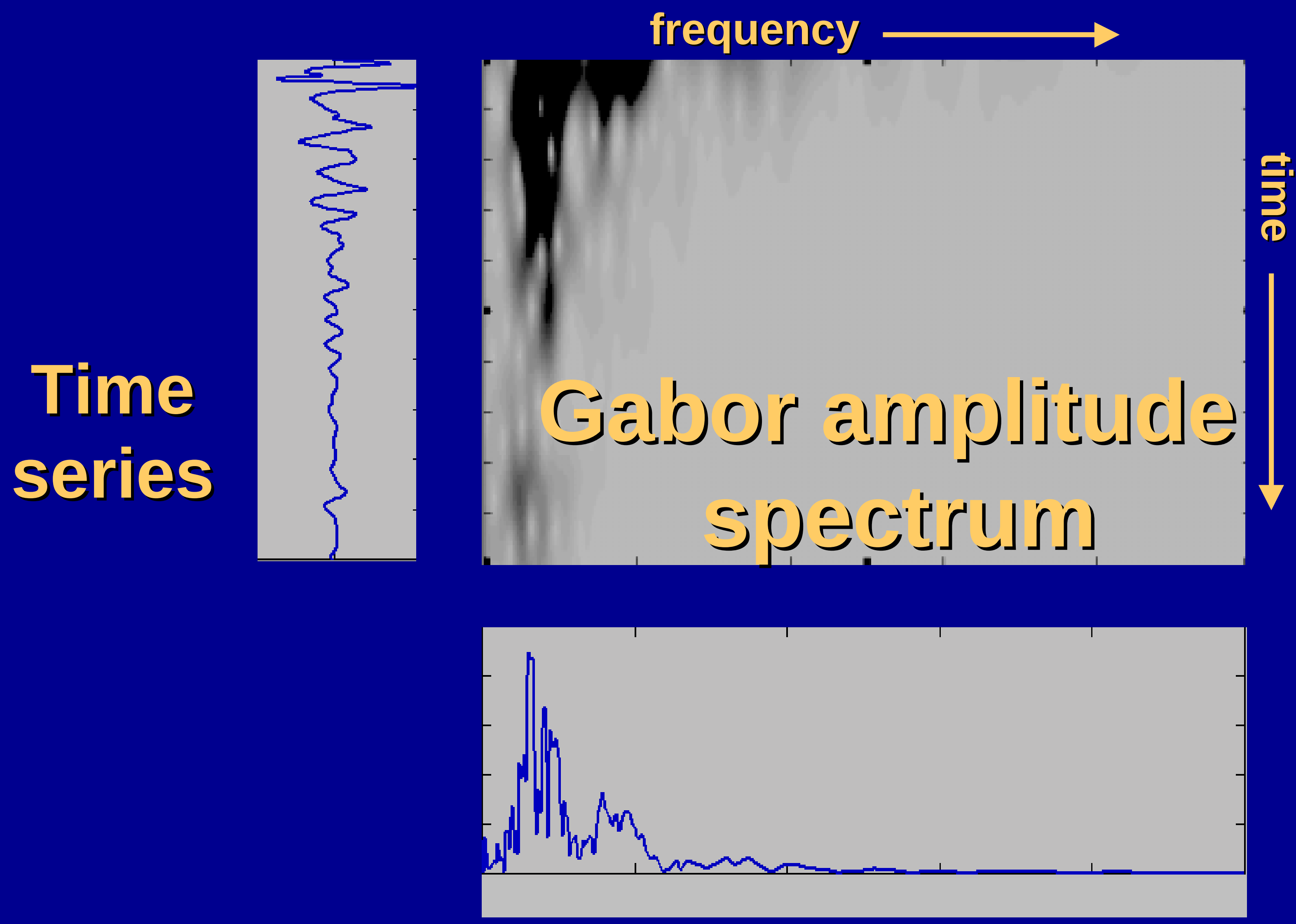
The Gabor transform

Gabor transform of $\psi(x)$

$$V_g\psi\left(\hat{x},k_x\right)=\int_{-\infty}^{\infty}\psi(x)\,g\left(x-\hat{x}\right)e^{-2\pi i\,x k_x}dx$$

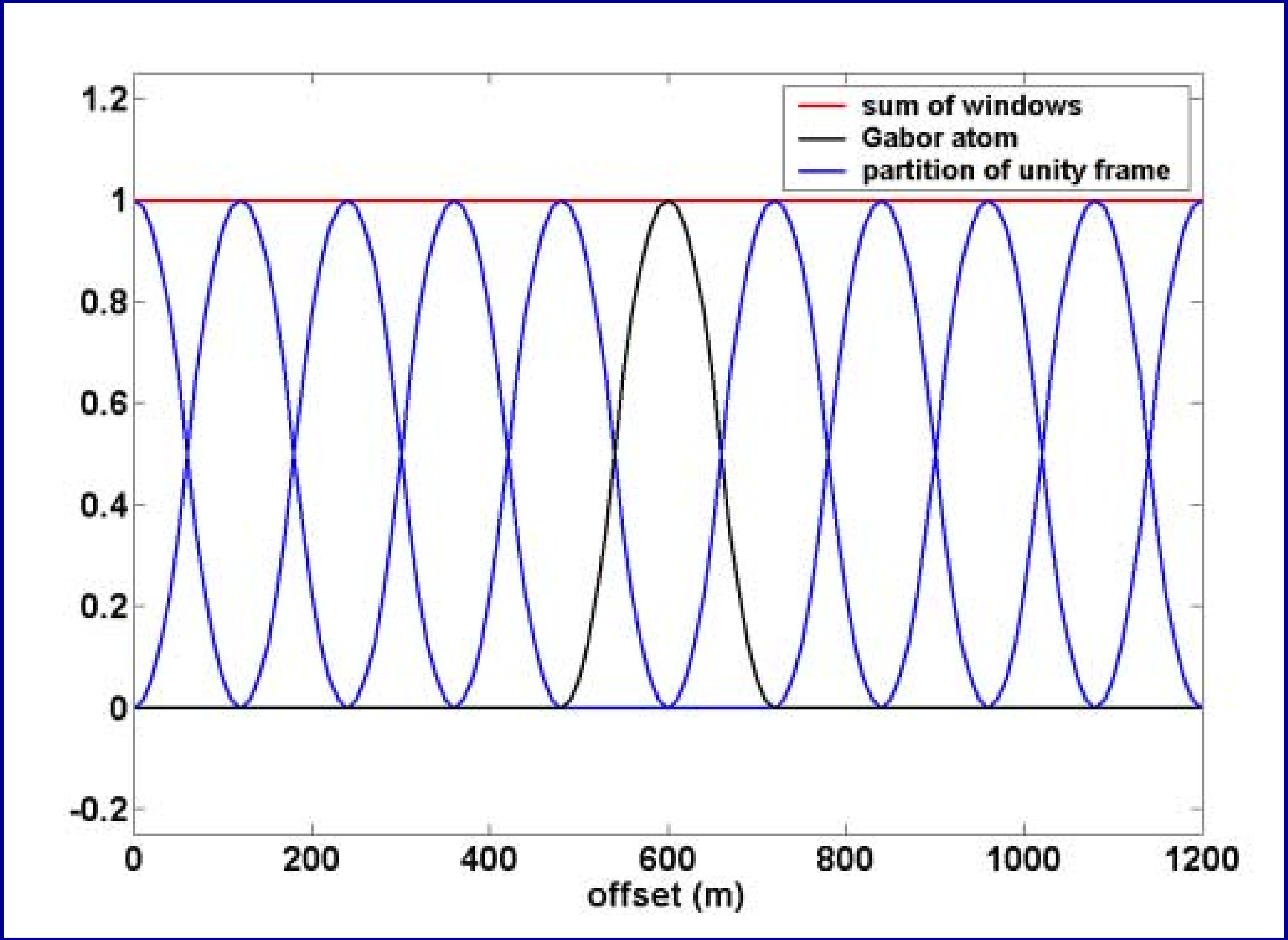
$g = g(x)$ **is a window, or atom**

$\hat{x}$ **indexes window position**

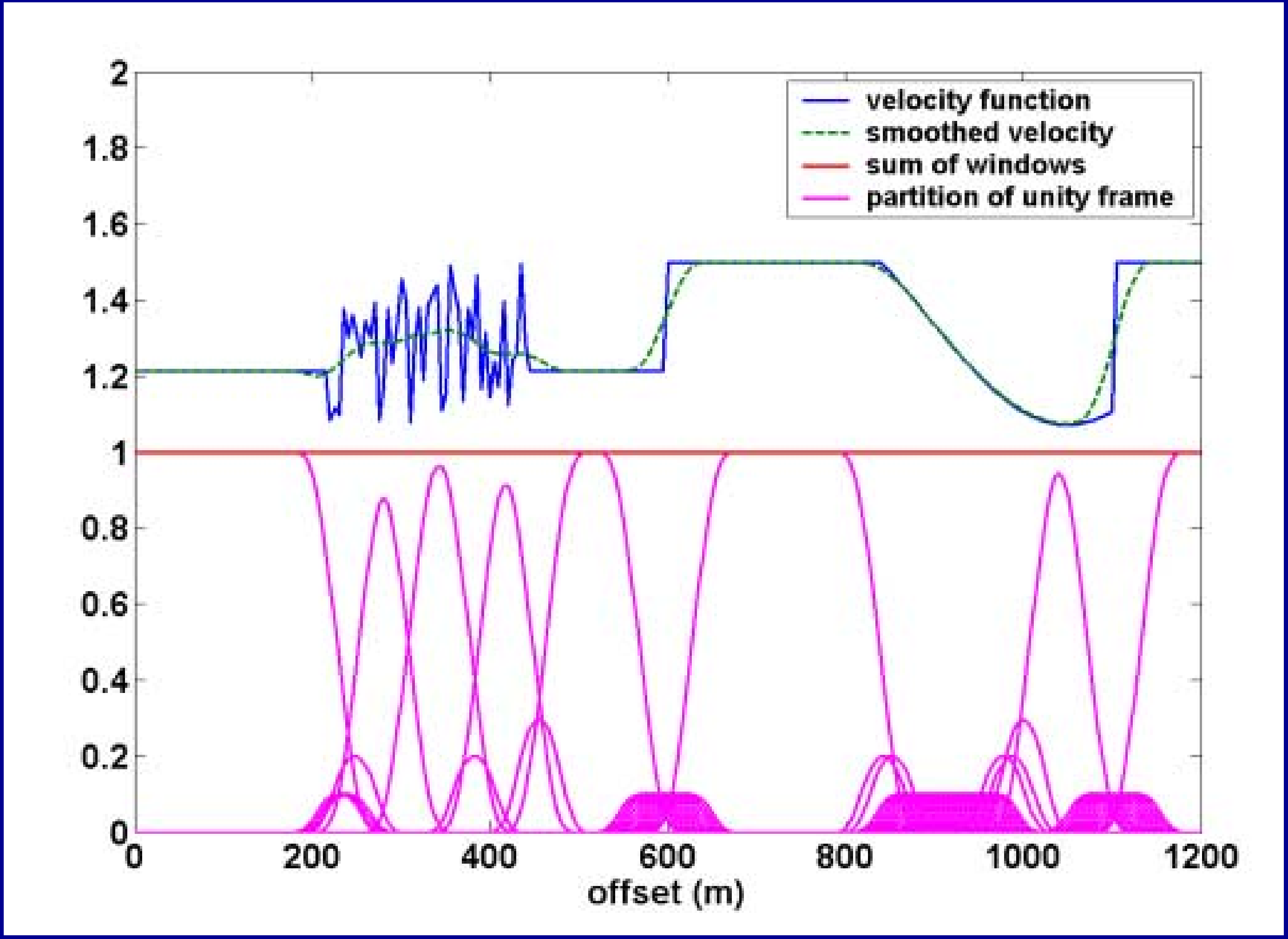


Fourier amplitude spectrum

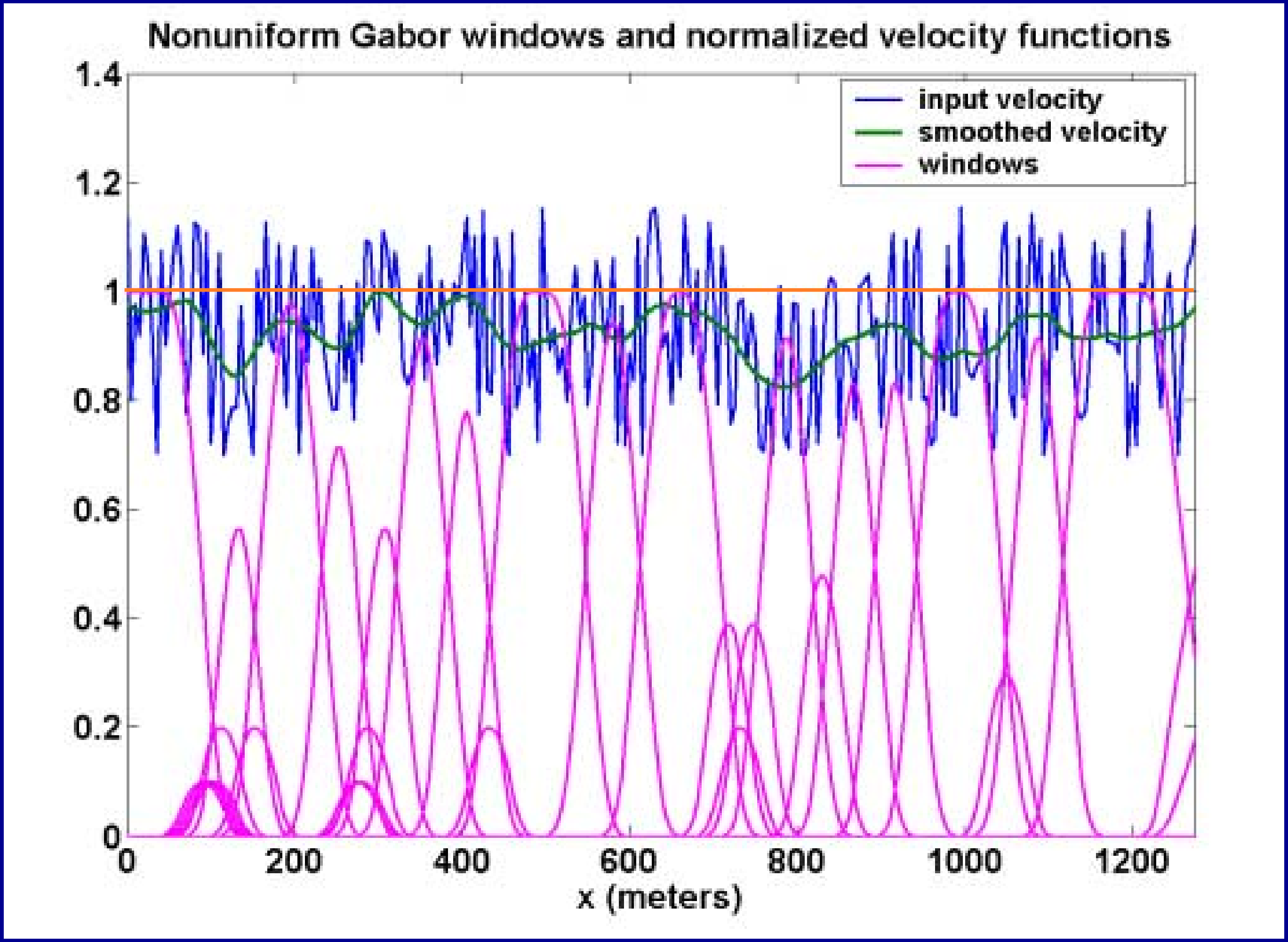
A uniform POU Gabor frame



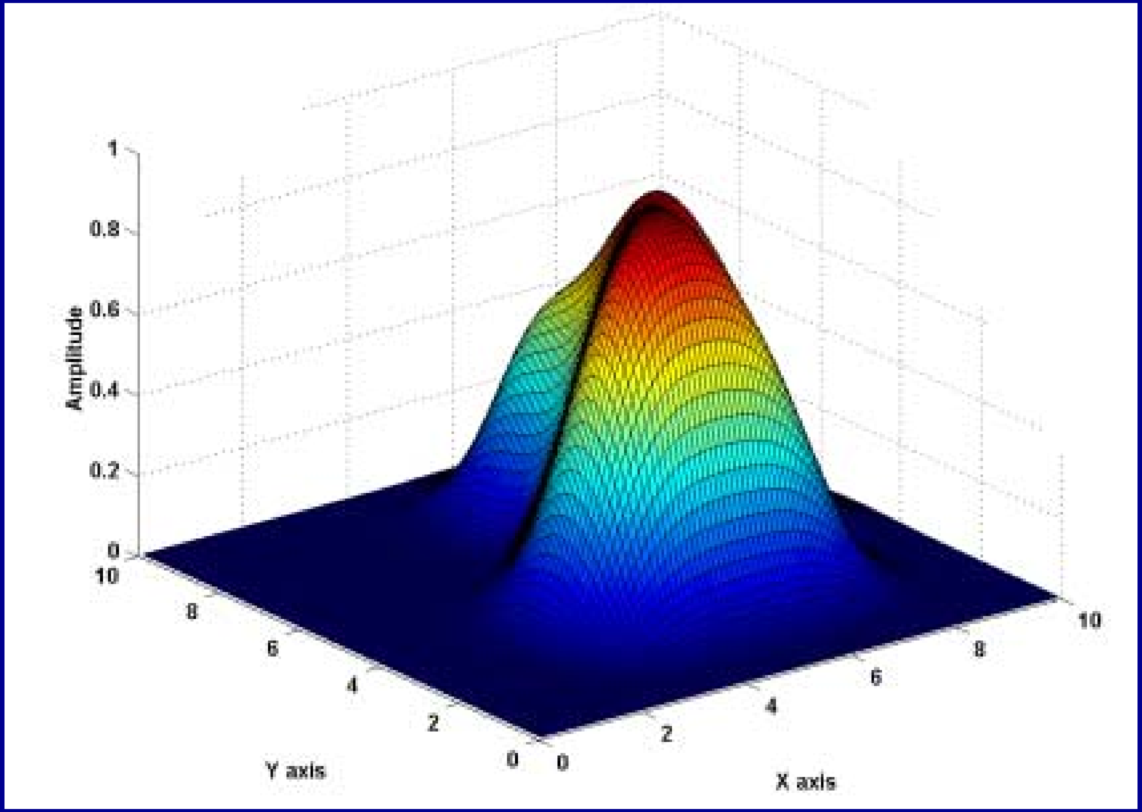
An adaptive POU frame



Another adaptive frame

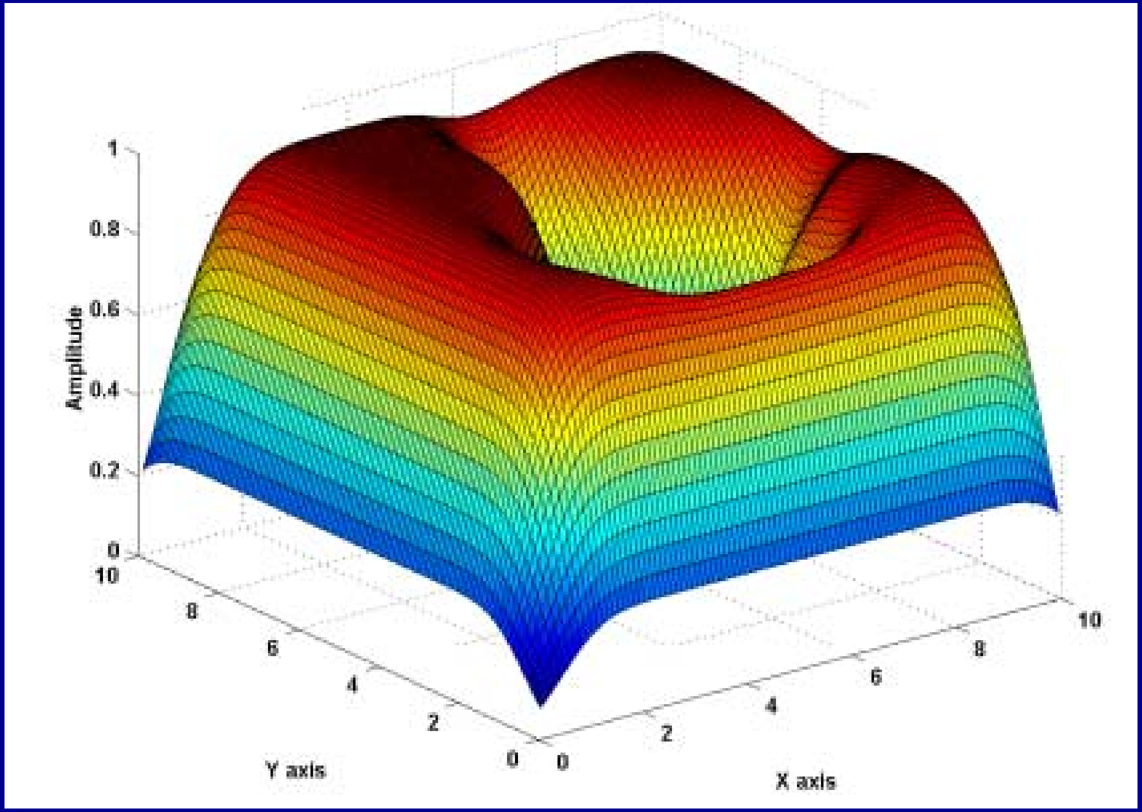


A 2-D window



Courtesy R. Bale

Its complement



Courtesy R. Bale

Extrapolation algorithm

$$\Psi\left(x, z = z_0, t\right) \longrightarrow \Psi\left(x, z_0, f\right)$$

$$k_z\left(\hat{x}, k_x, f\right) = \sqrt{\frac{f^2}{\left[\bar{V}\left(\hat{x}\right)\right]^2} - k_x^2}$$

$$\mathbf{GT}\left[\Psi\right]\left(\hat{x}, k_x, z_0, f\right)$$

$$\mathbf{GT}\left[\Psi\right]\left(\hat{x}, k_x, z_0 + \Delta z, f\right) = \exp\left(-2\pi i k_z \Delta z\right) \cdot \mathbf{GT}\left[\Psi\right]\left(\hat{x}, k_x, z_0, f\right)$$

$$\Psi\left(\tilde{x}, z_0 + \Delta z, f\right) \longrightarrow \Psi\left(\tilde{x}, z_0 + \Delta z, t\right)$$

Synthetic examples

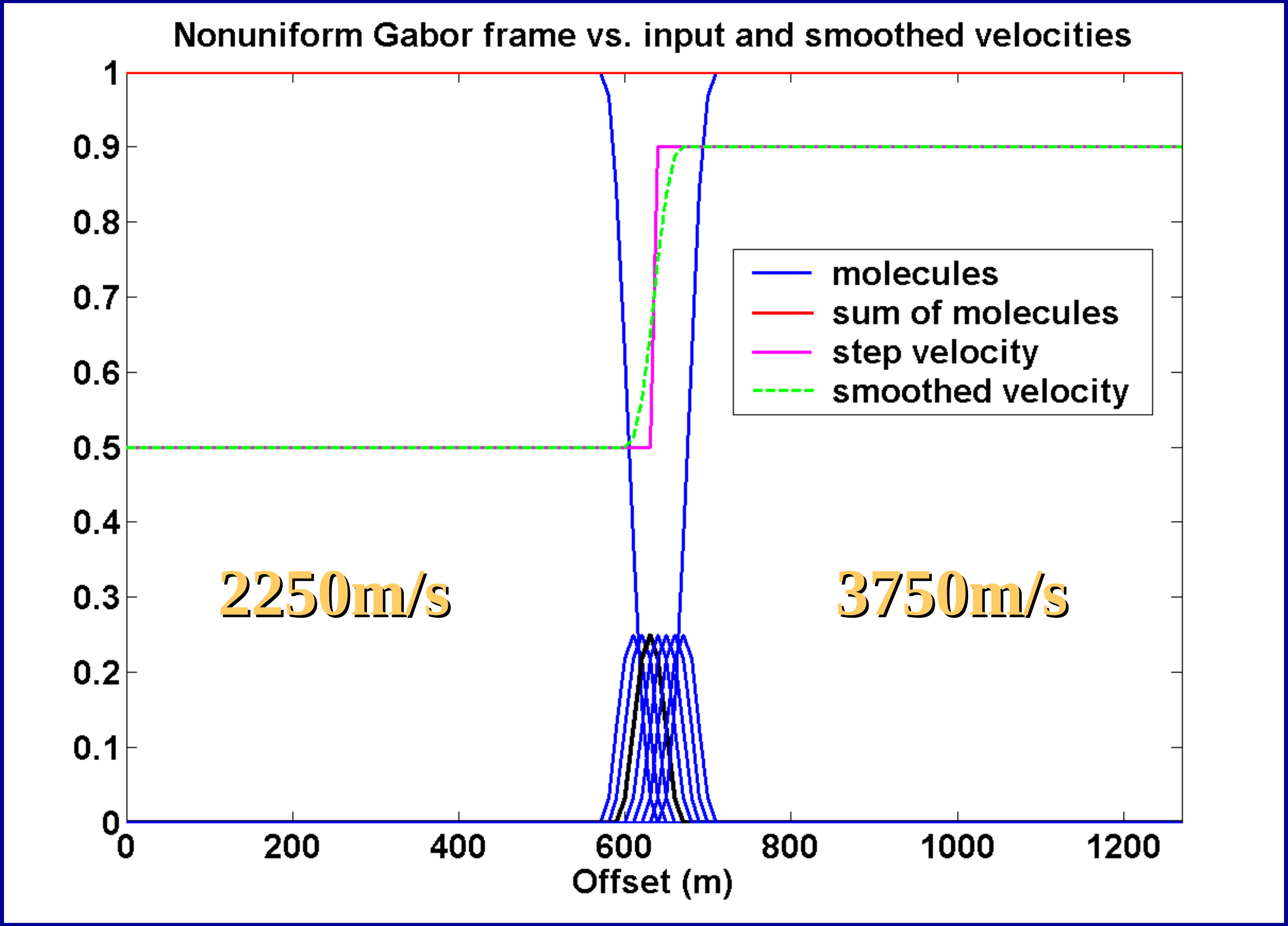
- Step velocity model

  2250 m/s to 3750 m/s

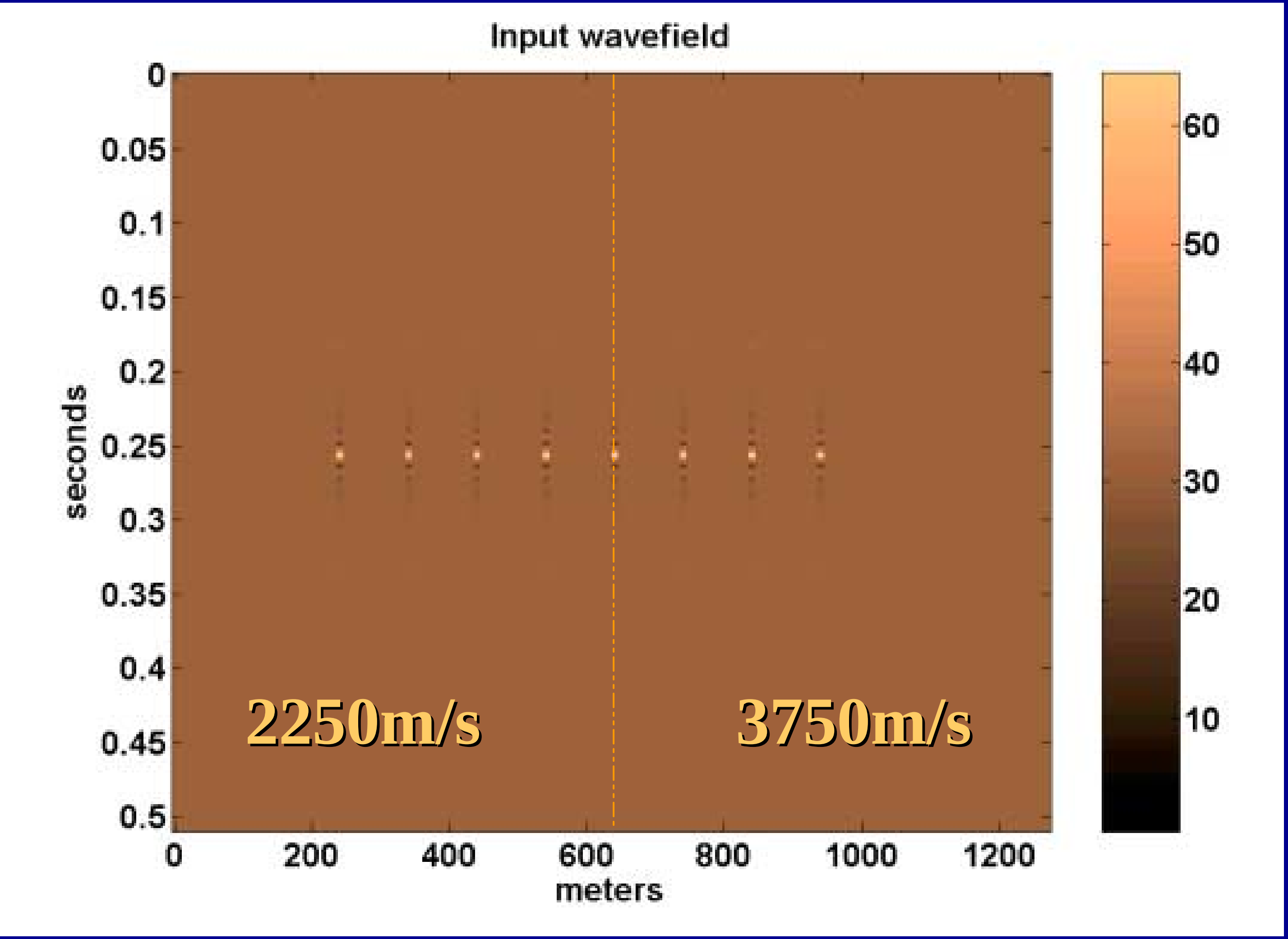
- Random velocity model

  1500 m/s and 2500 m/s

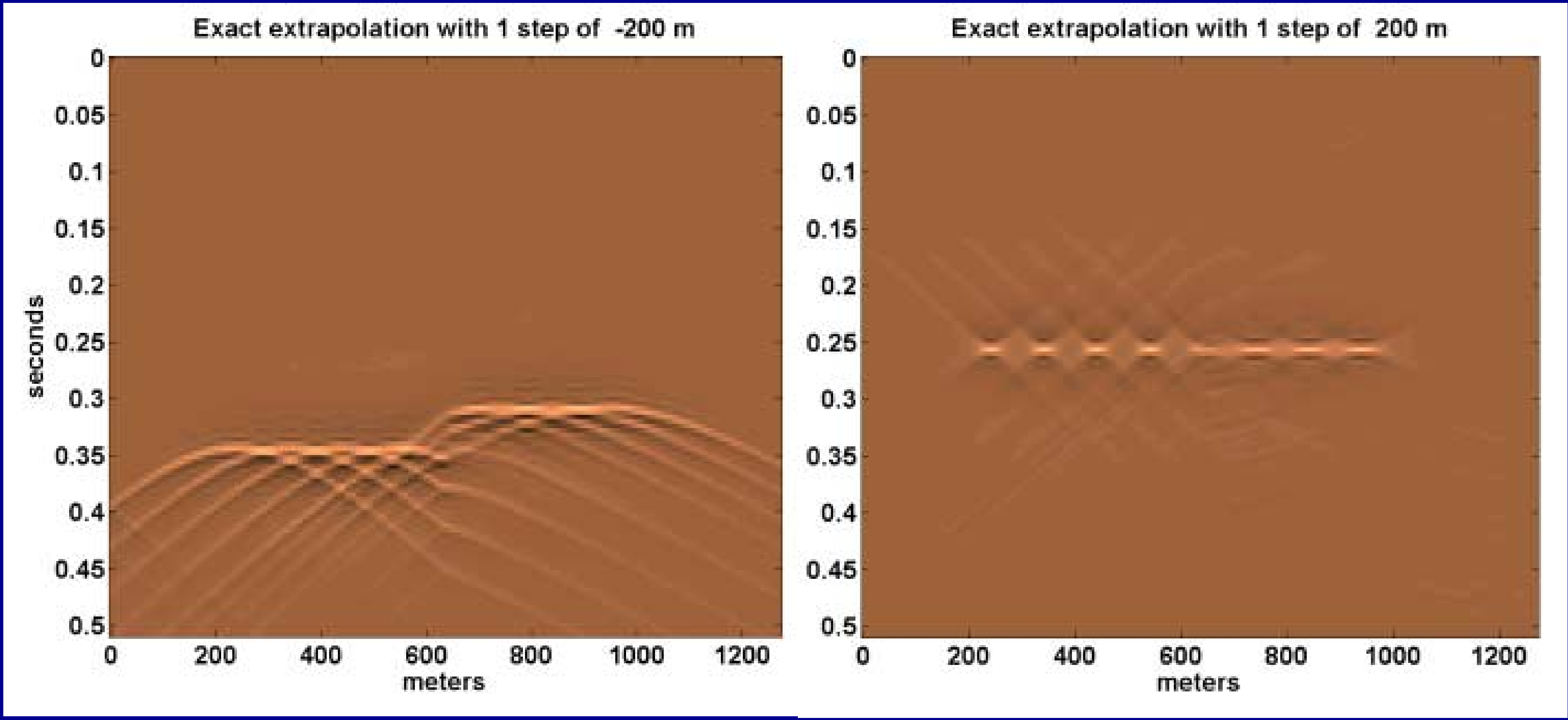
Step velocity



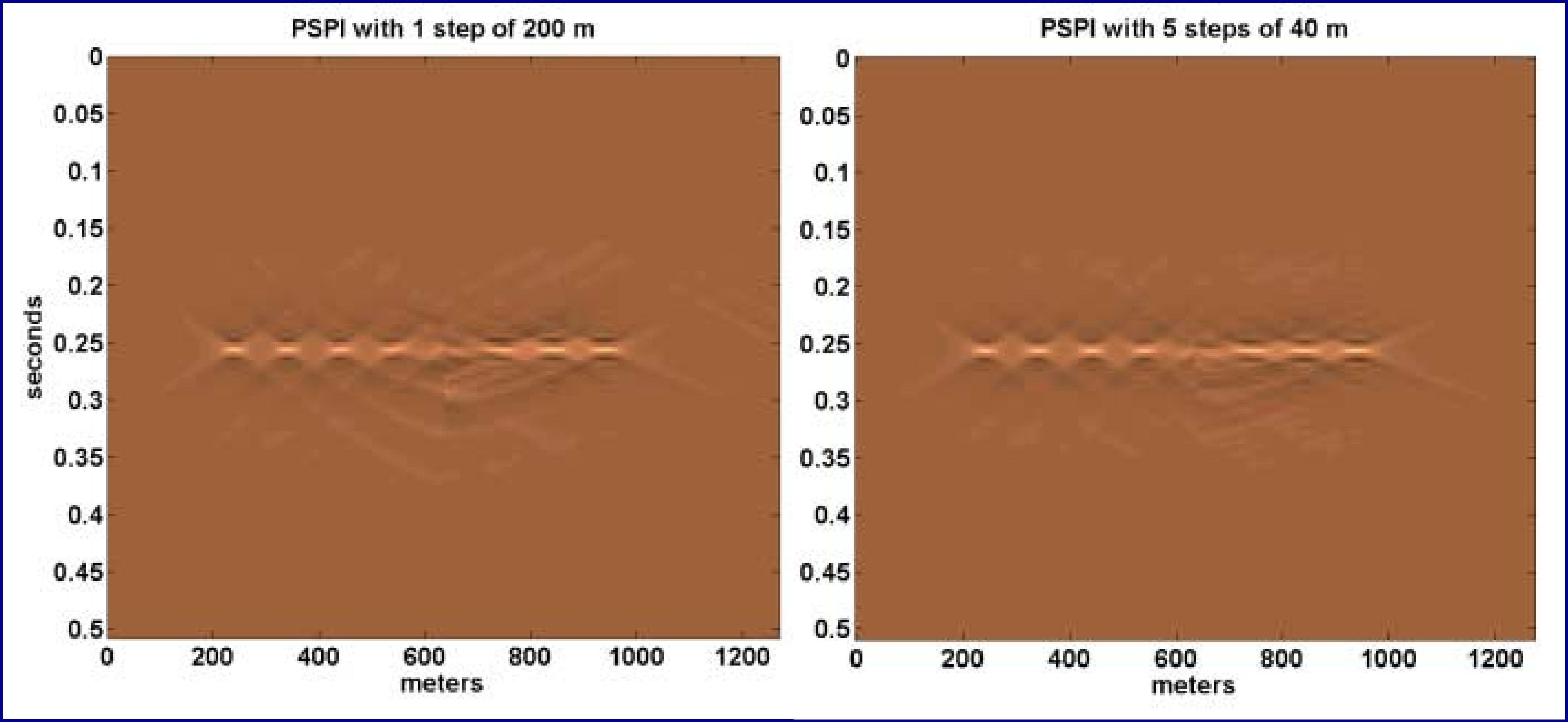
Bandlimited impulses



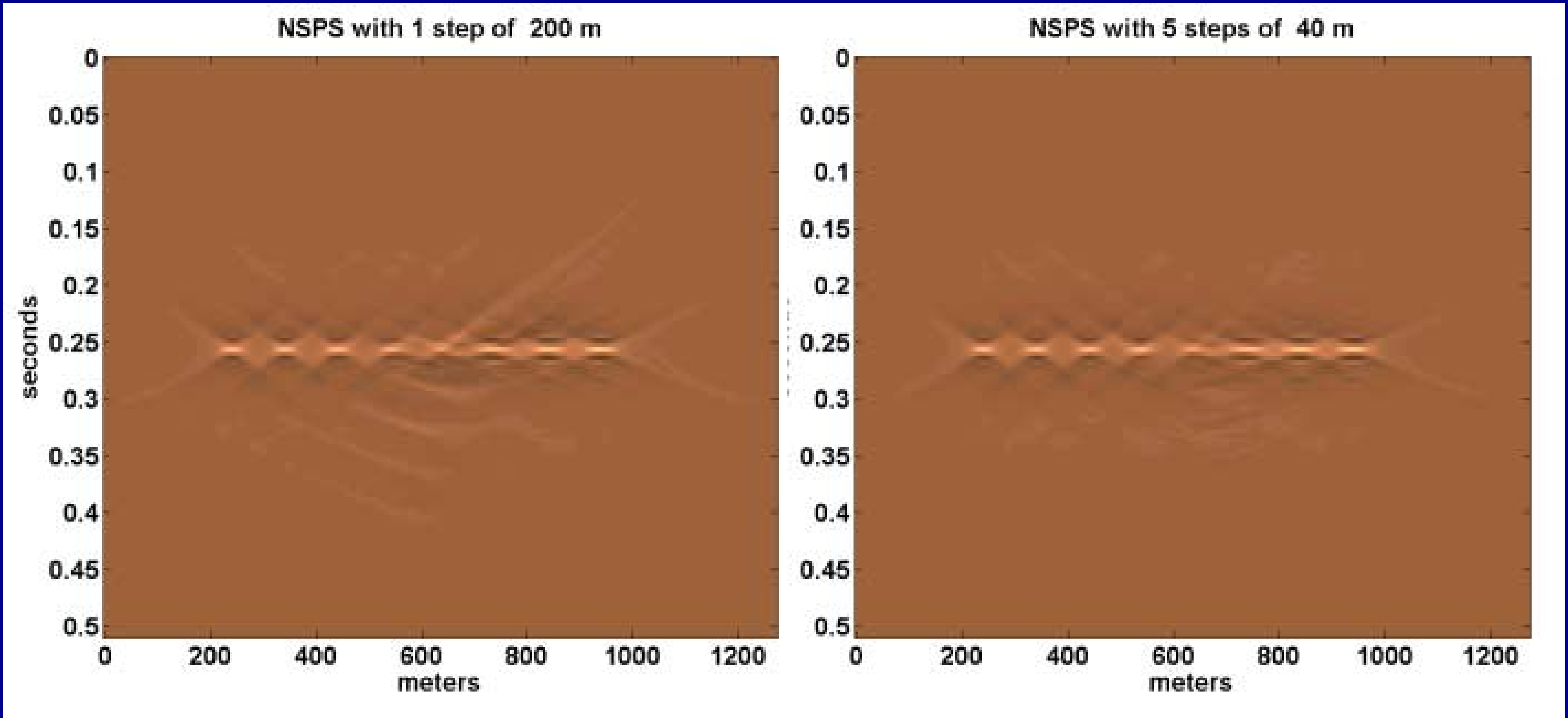
Exact extrapolator



PSPI

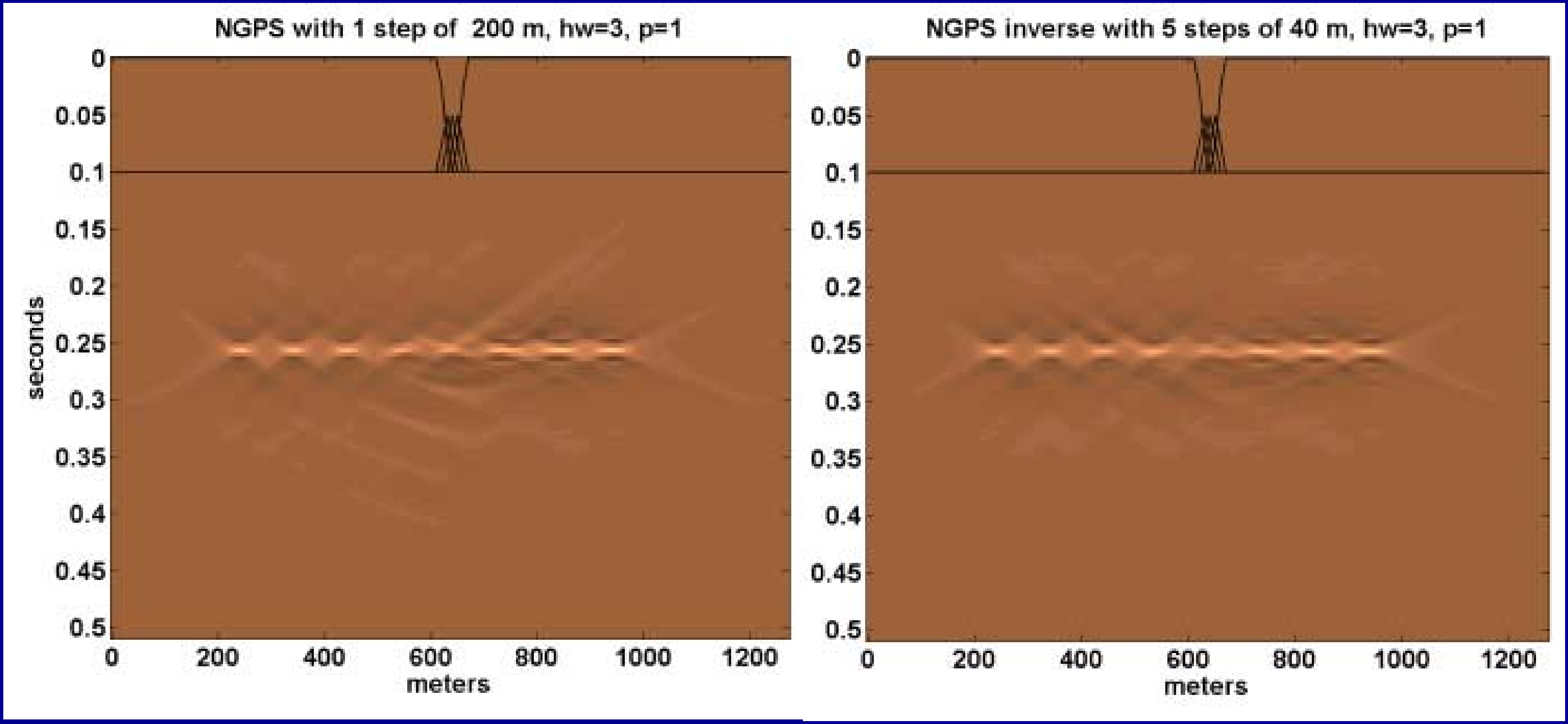

PSPI with 1 step of 200 m
PSPI with 5 steps of 40 m
seconds
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200
meters

NSPS



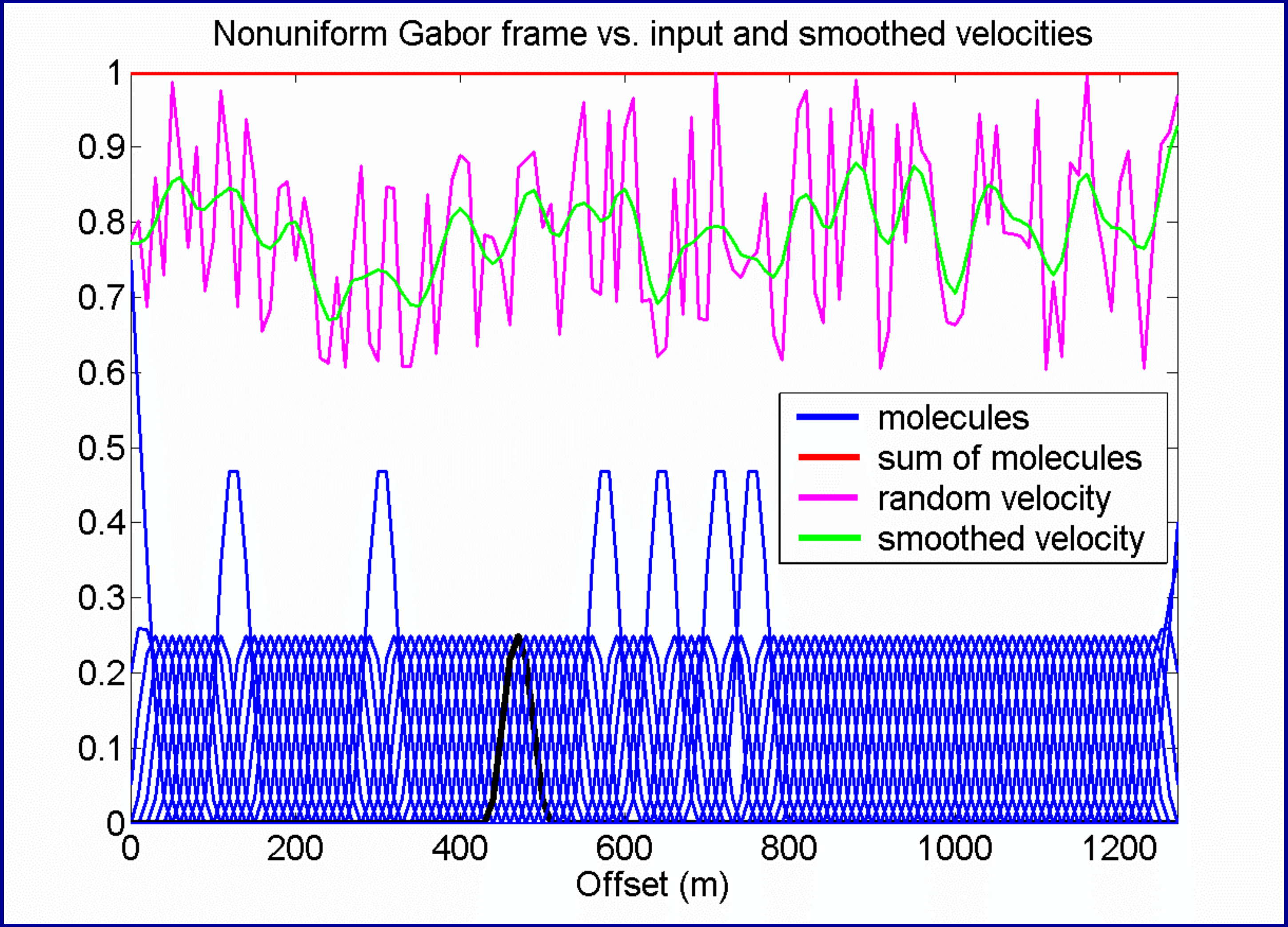
NSPS with 1 step of 200 m
NSPS with 5 steps of 40 m
seconds
meters
meters
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200

AGPS

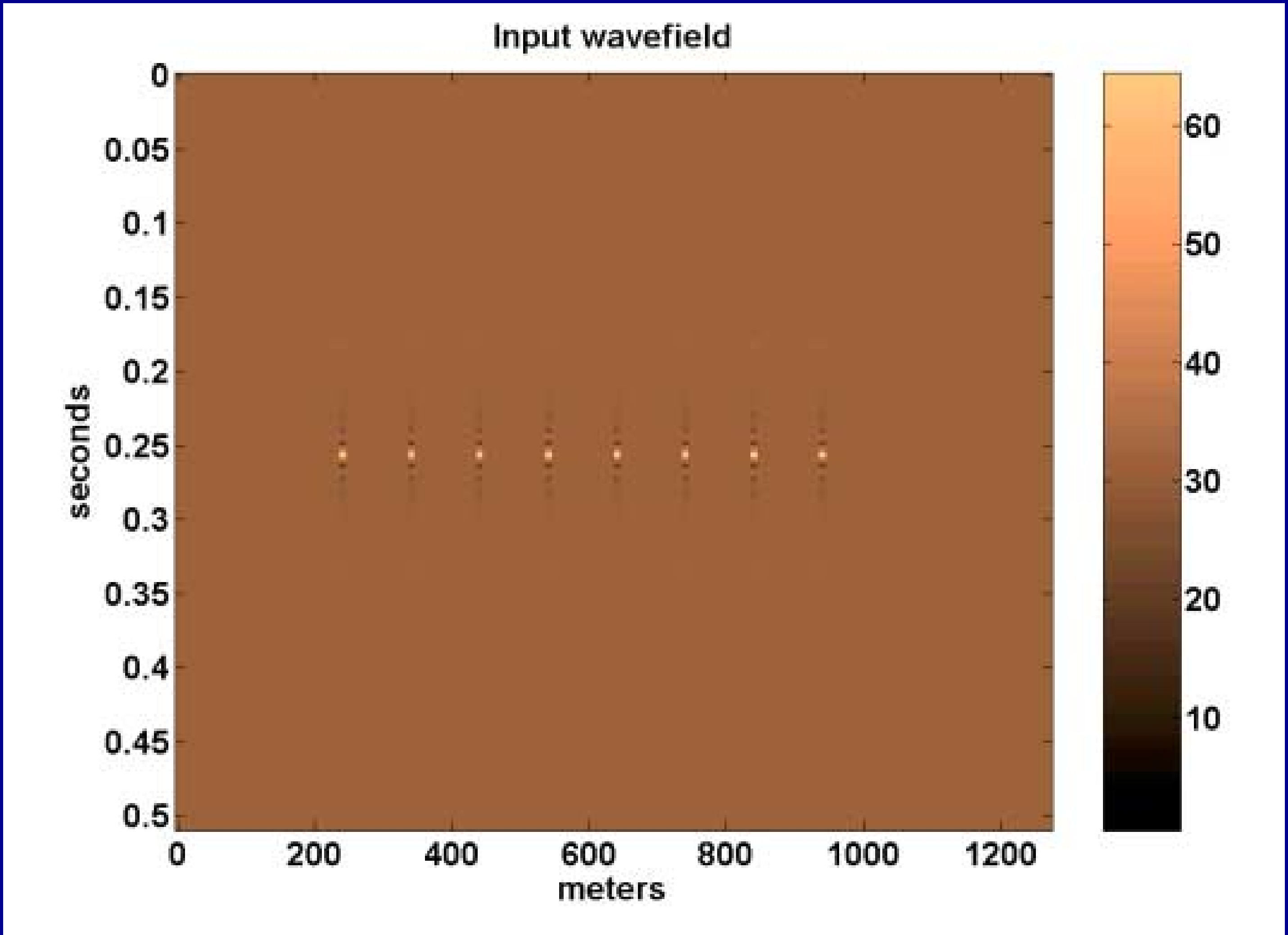


NGPS with 1 step of 200 m, hw=3, p=1
NGPS inverse with 5 steps of 40 m, hw=3, p=1
seconds
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200
meters

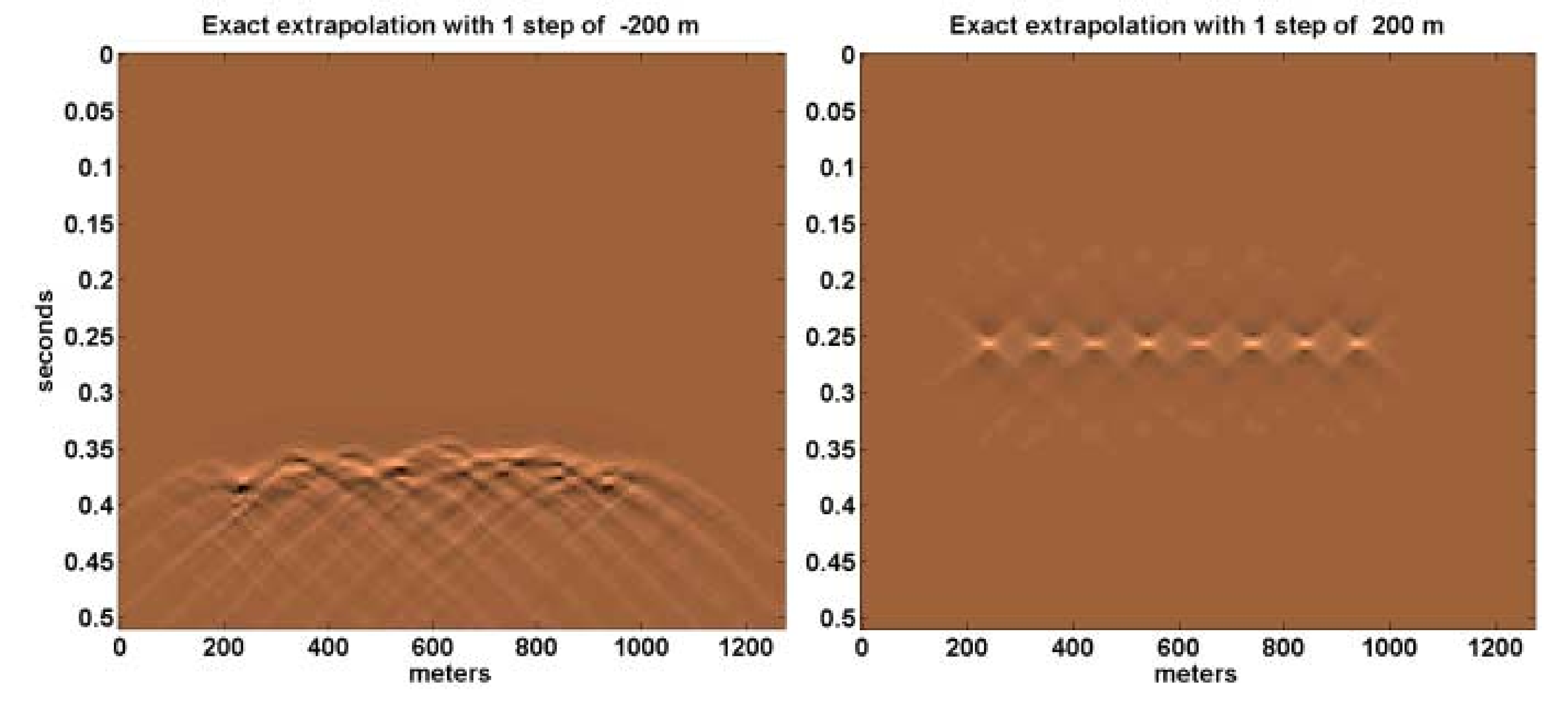
Random velocity



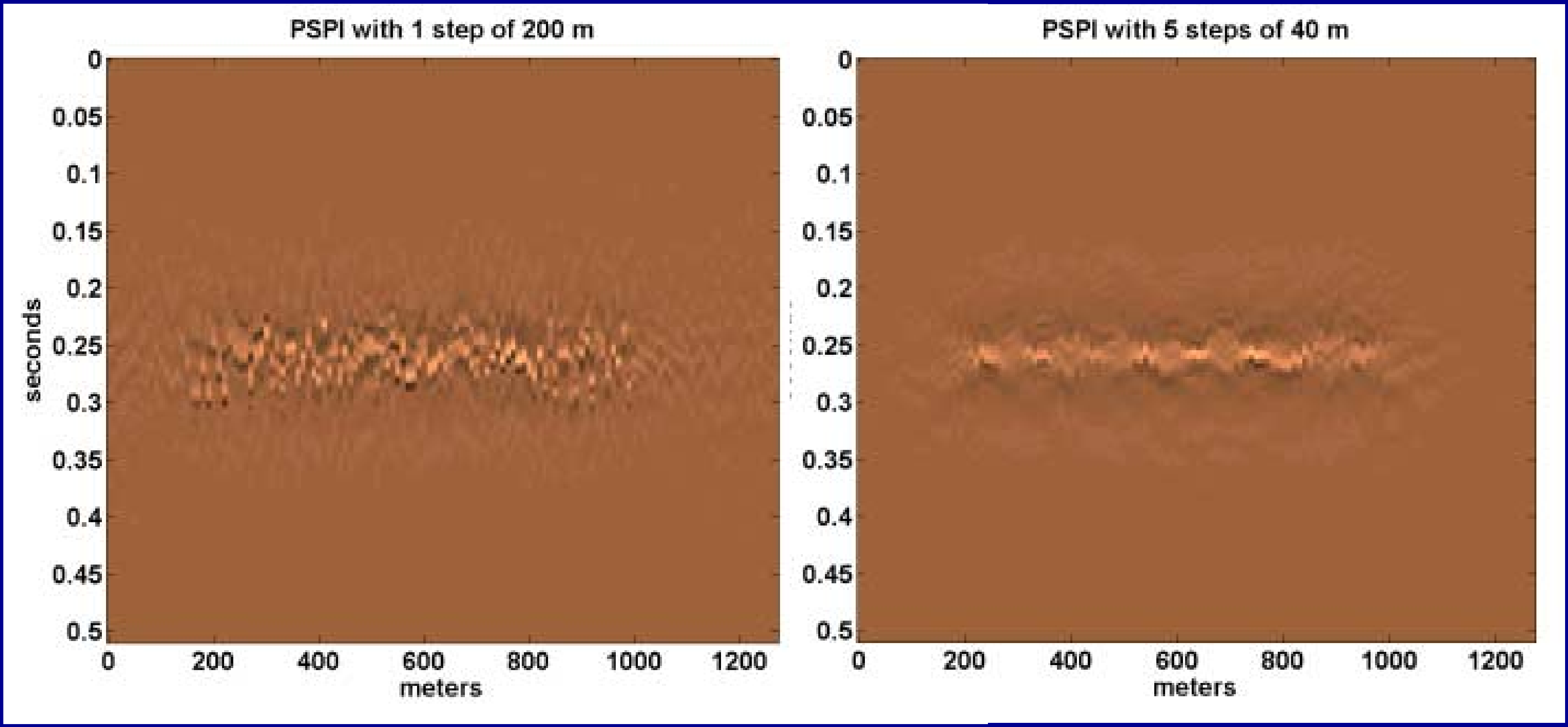
Bandlimited impulses



Exact extrapolator

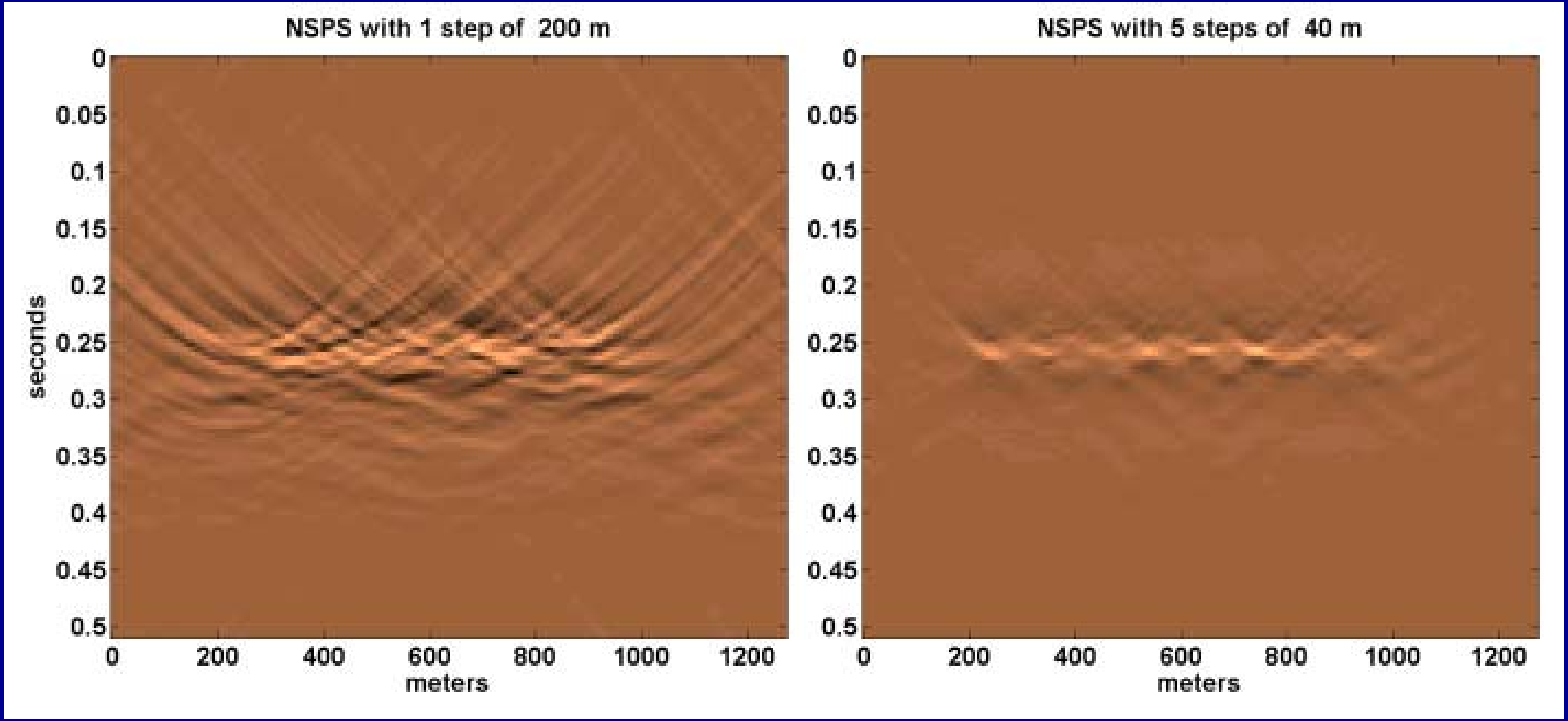


PSPI



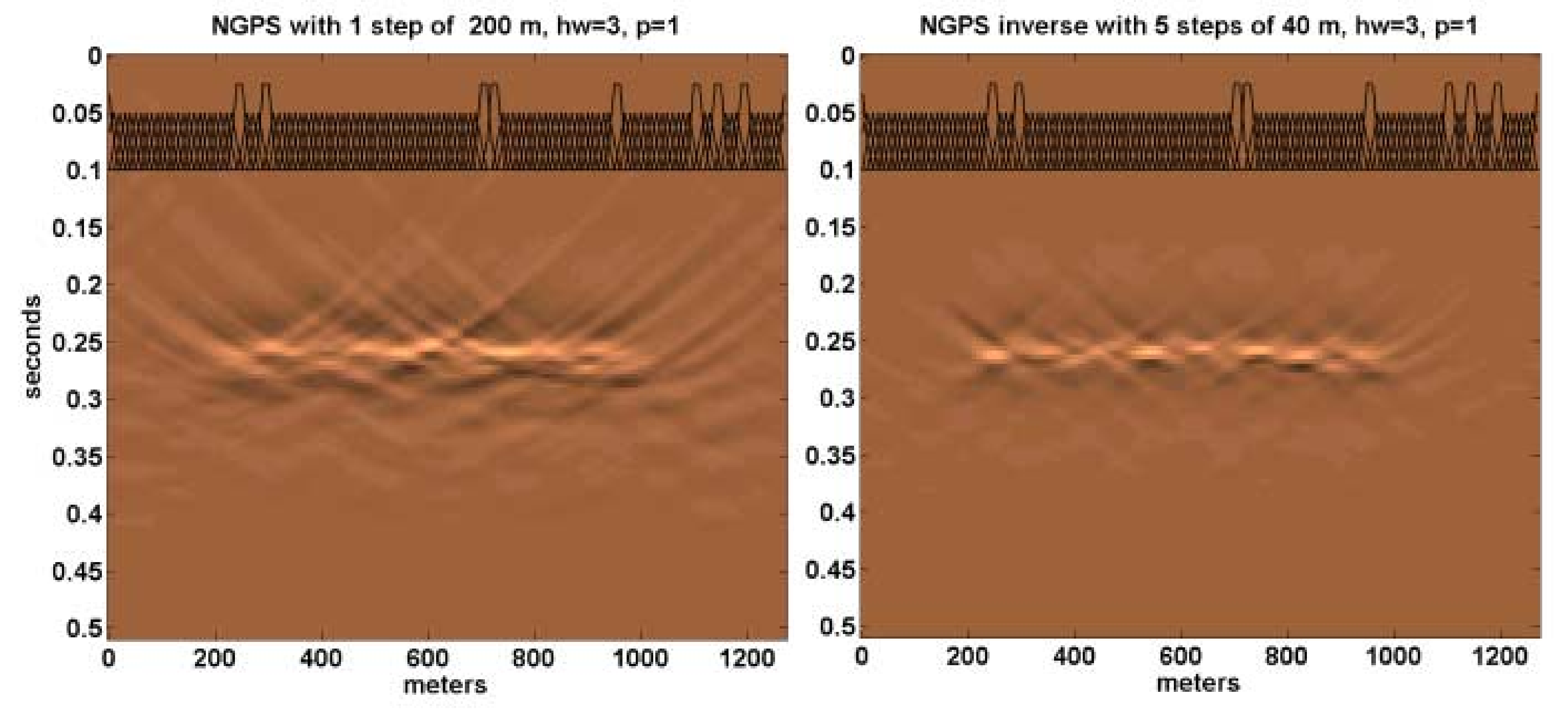
PSPI with 1 step of 200 m
PSPI with 5 steps of 40 m
seconds
meters
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200

NSPS



NSPS with 1 step of 200 m
NSPS with 5 steps of 40 m
seconds
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200
meters

AGPS



NGPS with 1 step of 200 m, hw=3, p=1
NGPS inverse with 5 steps of 40 m, hw=3, p=1
seconds
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
200
400
600
800
1000
1200
meters

Extrapolation algorithm	Absolute cost for step/random velocities (sec)	Relative cost for step/random velocities (% of Exact)
Exact	47.067/46.317	100/100
NSPS - *1 step*	3.946/3.675	8.2/7.9
NSPS - *5 steps*	20.580/17.545	42.5/37.9
PSPI - *1 step*	3.995/3.746	8.2/8.1
PSPI - *5 steps*	20.365/19.217	41.9/41.5
AGPS - *1 step*	0.601/3.916	1.2/8.5
AGPS - *5 steps*	3.245/19.438	6.4/42.0

Summary

Adaptive Gabor extrapolation…

- has accuracy comparable to NSPS/PSPI
- reduces to NSPS/PSPI in limiting cases
- has cost proportional to complexity of v(x)
- is typically much faster than NSPS/PSPI
- doesn’t separate waves along jumps in v(x)
- doesn’t generate reflections at interfaces

Acknowledgements

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- POTSI and its sponsors