Numerical Methods for Ill-Posed Problems

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Abstract: Ill-posed problems occur frequently in the physical sciences. Image deblurring is commonly used as an example of such a problem. The difficulty in solving discretized ill-posed problems is the presence of noise in the data; that is,

$$b = A(x_{\text{true}}) + e,$$

where $A : \mathbb{R}^n \to \mathbb{R}^m$, x_{true} is the true solution, and e denotes noise. The least squares problem $b \approx A(x)$ may not have a unique solution, and even if it does, the ill-posed nature of the problem ensures that this solution will not approximate the desired solution x_{true}.

In this talk we give an introduction to numerical techniques for approximating x_{true} in discretized ill-posed problems, primarily considering the linear case for ease of discussion. Our approximate solutions are solutions to *regularized problems*, whose quality as an approximation to x_{true} is dependent on the value of a *regularization parameter*. We focus on iterative regularization methods and discuss both well-known and new parameter selection techniques for such methods. Illustrations of these methods and techniques on imaging applications will be given.