Techniques for solving indefinite linear systems

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In this talk we will discuss solution techniques for solving 2×2 block indefinite linear systems whose $(2,2)$ block is zero:

$$\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix} .$$

The matrix A is typically square, say $n \times n$, and B is $n \times p$, where $p \leq n$.

Many applications lead to linear systems with such a structure: problems arising from constrained optimization, least squares problems, the linearized Navier-Stokes equations, inverse problems in geophysics, and numerous other problems.

Schur Complement techniques, null-space methods, and the properties of preconditioning techniques for Krylov subspace solvers will be discussed. Some attention will be given to unusual cases such as a singular $(1,1)$ block or a nearly rank deficient constraint matrix.