The Polynomial Numerical Hulls of Jordan Blocks and Related Matrices

The polynomial numerical hull of degree k for a square matrix A is a set designed to give useful information about the norms of functions of the matrix; it is defined as

$$\{z \in \mathbf{C} : \ \|p(A)\| \geq |p(z)| \ \text{ for all } p \text{ of degree } k \text{ or less}\}.$$

While these sets have been computed numerically for a number of matrices, the computations have not been verified analytically in most cases.

We have recently shown analytically that the 2-norm polynomial numerical hulls of degrees 1 through $n-1$ for an n by n Jordan block are disks about the eigenvalue with radii approaching 1 as $n \to \infty$, and that the radius $r_{n-1,n}$ of the hull of degree $n-1$ is the positive root of $2r^n + r - 1 = 0$ when n is even, and satisfies a similar formula when n is odd. This turns out to be equivalent to a classical result in complex approximation theory. I will discuss a little of the history of this problem, both ancient and recent. I will also discuss what one can deduce about the behavior of functions of a Jordan block, such as $\|J^k\|$ or $\|e^{tJ}\|$, from knowledge of the polynomial numerical hulls of J. Finally, I will derive fairly tight bounds on the polynomial numerical hulls of banded triangular Toeplitz matrices.

This is joint work with **Vance Faber** and **Don Marshall**.