

The symbiosis of direct and iterative methods for solving large sparse systems

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Abstract: We start the talk by discussing the size of problems that can currently be solved by sparse direct methods. We then consider the limitations of such methods, where current research is going in moving these limitations, and how far we might expect to go with direct solvers in the near future.

This leads us to the conclusion that very large systems, by which we mean three dimensional problems in more than a million degrees of freedom, require the assistance of iterative methods in their solution. However, even the strongest advocates and developers of iterative methods recognize their limitations when solving difficult problems, that is problems that are poorly conditioned and/or very unstructured. It is now universally accepted that sophisticated preconditioners must be used in such instances.

A very standard and sometimes successful class of preconditioners are based on incomplete factorizations or sparse approximate inverses, but we very much want to exploit the powerful software that we have developed for sparse direct methods over a period of more than thirty years. We thus discuss various ways in which a symbiotic relationship can be developed between direct and iterative methods in order to solve problems that would be intractable for one class of methods alone. In these approaches, we will use a direct factorization on a “nearby” problem or on a subproblem.

We examine the use of a symmetric indefinite solver in constrained optimization problems that is used in the first way, and the use of a direct solver within domain decomposition techniques in the second.