WESTERN CANADA LINEAR ALGEBRA MEETING
Program - Abstracts - Participants

University of Regina

May 10-11, 2002

Organizing Committee

Shaun Fallat, Hadi Kharaghani, Steve Kirkland, Peter Lancaster, Dale Olesky, Michael Tsatsomeros, Pauline van den Driessche

Local Organizers:
Shaun Fallat and Steve Kirkland

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W-CLAM 2002 is organized with the support of the
National Program Committee of the Institutes (CRM, Fields, PIMS)
and the
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Invited Speakers

Prof. Ludwig Elsner (Universitaet Bielefeld)
Prof. Jane Day (San Jose State University)
Prof. Chris Godsil (University of Waterloo)

Location

Room 407, Classroom Bldg., University of Regina

1 Meeting Program (Room 407, Classroom Bldg.)

Friday, May 10, 2002

08:00-08:45 **Registration**

08:45-09:00 **Welcome & Information**

Chair: Pauline van den Driessche

09:00-09:50 **L. Elsner**, *Convergent Infinite Products of Matrices*

10:00-10:25 **P. Lancaster**, *Irreversible Markov Processes for Phylogenetic Models*

10:30-10:55 **Break**

Chair: Shaun Fallat

11:00-11:25 **D. Olesky**, *Sums of Matrices from Positivity Classes*

11:30-11:55 **H. Kharaghani**, *On a Decomposition of Complete Graphs*

12:00-12:25 **F. Barioli**, *Completely Positive Matrices of Rank Three*

12:30-14:00 **Lunch**

Chair: Steve Kirkland

14:00-14:25 **P. van den Driessche**, *M-matrices in Some Compartmental Epidemic Models*

14:30-14:55 **D. Barnes**, *Finding Optimal and Near-Optimal Partitions of a Data Set Using Bayesian Blocks: Algorithms to Improve Local Search Results*

15:00-15:25 **P. Zizler**, *Spectral Radius of a Sampling Operator*

15:30-15:55 **Break**

Chair: Peter Lancaster

16:00-16:50 **J. Day**, *R.C. Thompson and Alfred Horn's Conjecture*

Saturday, May 11, 2002

Chair: Dale Olesky

09:00-09:50	**C. Godsil**, *Bounds on Independent Sets*
10:00-10:25	**R. Craigen**, *Permutations, Transpose, and Symmetry*
10:30-11:00	**Break and meeting photo**

Chair: Hadi Kharaghani

11:00-11:25	**S. Fallat**, *Principal-Minor Inequalities for Sign-symmetric Tridiagonal P-matrices*
11:30-11:55	**Y. Tian**, *How to Express a Parallel Sum of k Matrices*
12:00-12:25	**S. Narayan**, *Spectrally Stable Matrices*
12:30-14:00	**Lunch**

Chair: Shaun Fallat

14:00-14:25	**J. Stuart**, *Ray Patterns: k-Potent and Powerful*
14:30-14:55	**J. Molitierno**, *The Algebraic Connectivity of two Trees Connected by an Edge of Infinite Weight*
15:00-15:25	**S. Kirkland**, *Conditioning Properties of the Stationary Distribution for a Markov Chain*
15:30-15:55	**Break**

Chair: Steve Kirkland

16:00-16:25	**D. Farenick**, *Young's Inequality*
16:30-	**Closing remarks**

2 Abstracts (alphabetical by speaker)

Completely Positive Matrices of Rank Three
Francesco Barioli

A nonnegative symmetric n-by-n matrix is called *completely positive* if it is the Gram matrix of a set of nonnegative vectors with respect to the usual inner product, namely if $A = V^t V$ for some nonnegative t-by-n matrix V. The minimal t that such a matrix V exists for is called the *Cp-rank* of A. If $\text{rank}(A) = 2$, then complete positivity is equivalent to double nonnegativity, a condition which can be checked easily. For matrices of rank three no definitive test is known to determine the complete positivity of a given matrix. We present a tool which provides an easy test for the complete positivity of rank three matrices with at least one zero entry occurring in the pattern.

Finding Optimal and Near-Optimal Partitions of a Data Set Using Bayesian Blocks: Algorithms to Improve Local Search Results
David Barnes

I will introduce the results of a project supported by San Jose State University and NASA Ames Research Center. The aim is to find the optimal subdivision, with respect to a given objective function motivated by Bayesian statistics, of a data set. Finding the best, or most probable, partition amounts to maximizing this function. Algorithms that provide maximal, or near maximal values will be explored. Time tagged photon event data motivated this approach and it has been used to analyze various other types of astronomical occurrences. Results from the analysis of some such data will be presented.

Permutations, Transpose, and Symmetry
Robert Craigen

Here is a simple matrix question motivated by some examples in the study of symmetric block designs.

Two matrices are *permutation equivalent* if one can be transformed into the other by a permutation of its rows and columns. Suppose a matrix is permutation equivalent to its transpose. Must it be permutation equivalent to a symmetric matrix? Put another way: Permutation equivalence is an equivalence relation; think of the equivalence classes. An equivalence class is *symmetric* if it is invariant under transpose. Does a symmetric equivalence class necessarily contain a symmetric matrix?

Although no obvious counterexamples are forthcoming, the answer is ”no”. The object of this talk is to construct examples, particularly examples that could be used in the setting of block designs, of symmetric classes not containing symmetric matrices. The method will involve a complete characterization of matrices permutation equivalent to their transposes.

R.C. Thompson and Alfred Horn's Conjecture
Jane Day

Throughout his career, Bob Thompson was fascinated by Horn's 1962 conjecture describing the spectral set for a Hermitian sum. He fully appreciated this beautiful and difficult problem and thought deeply about it. He was certain it was true, and many of his lovely results on inequalities were motivated by it. He would have been delighted to see the complete proof which others finally produced a few years after his death. I will discuss some of the history and Bob's ideas for proofs, in particular an outline based on Horn's approach, which would yield an elementary proof if it could be completed.

Convergent Infinite Products of Matrices
Ludwig Elsner

It is well known that there are finite sets $\Sigma = \{A_1, \ldots, A_k\}$ of real or complex n-by-n matrices such that all left infinite products

$$...A_{j_r} A_{j_{r-1}} ... A_{j_1}$$

are convergent. Examples are orthogonal projections.

It was known for some time that *paracontractivity* of Σ is sufficient for this property. Here paracontractivity is the existence of a vector norm in C^n such that for each $x \in C^n$ and each $A \in \Sigma$ either $Ax = x$ or $||Ax|| < ||x||$. Recently it was proved that this is also a necessary condition. In this talk a string of other equivalent conditions are discussed. They involve properties of trajectories constructed with the matrices in Σ, such as boundedness und uniform boundedness of the length of the trajectories.

Finally we discuss the connections between the convergence of all *left* infinite and all *right* infinite products. We report here joint results with A. Vladimirov and W.-J. Beyn.

Principal-Minor Inequalities for Sign-symmetric Tridiagonal P-matrices
Shaun Fallat

The question of which ratios of products of principal minors are bounded over all matrices in a given class has been of interest historically. This question is settled for the class of tridiagonal sign-symmetric P-matrices, which essentially lie in each of the classes: positive definite; invertible totally nonnegative and M-matrices. It happens that all bounded ratios are bounded by one.

Young's Inequality
Doug Farenick

A variant of the arithmetic–geometric mean inequality is Young's inequality: if p, q are positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$, then $|\alpha\beta| \leq \frac{1}{p}|\alpha|^p + \frac{1}{q}|\beta|^q$, for all complex numbers α and β. In this lecture, I will survey recent formulations of Young's inequality at the level of compact operators and operator norms.

Bounds on Independent Sets
Chris Godsil

An independent set in a graph is a set of vertices, no two of which are adjacent. Cvetkovic discovered an upper bound on the size of an independent set in terms of the numbers of positive and negative eigenvalues of its adjacency matrix. I will outline a simple derivation of this bound and give some of its applications; I will also discuss some difficulties that arise in the process.

On a Decomposition of Complete Graphs
Hadi Kharaghani

Using simple properties of finite fields we show that for a prime power q, the complete graph on $1+q+q^2+q^3$ vertices can be decomposed into a union of $1+q$ Siamese strongly regular graphs;

$$srg(1+q+q^2+q^3, q+q^2, q-1, q+1)$$

sharing $1+q^2$ disjoint cliques of size $1+q$. An extension to more complicated parameters will be discussed.

Conditioning Properties of the Stationary Distribution for a Markov Chain
Steve Kirkland

An irreducible stochastic matrix T can be thought of as the transition matrix for a Markov chain. One of the principal quantities of interest for the chain is its stationary distribution, i.e. the left Perron vector of T, normalized so that the entries sum to 1. If we perturb T to produce another transition matrix $\tilde{T}$, how large can the change be in the perturbed stationary vector? Several condition numbers have been proposed to help study that question, and in this talk we discuss the derivative of the stationary vector with respect to a given perturbation matrix, and its connection with one of those condition numbers. The results will be applied to the class of Markov chains arising from a random walk on a tree.

Irreversible Markov Processes for Phylogenetic Models

Peter Lancaster

A widely used model from phylogenetics will be discussed with emphasis on the role of matrix theory and numerical analysis in our work. This is a report on collaboration with E. Bohl of the University of Konstanz.

The Algebraic Connectivity of two Trees Connected by an Edge of Infinite Weight

Jason Molitierno

In graph theory, we can represent a graph on n vertices in terms of an associated $n \times n$ *Laplacian matrix*. Laplacian matrices are singular positive semidefinte matrices and thus we can order the eigenvalues as follows: $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. The eigenvalue λ_2 is of special interest because it is a measure of the connectivity of the graph. Thus, λ_2 is termed the *algebraic connectivity* of the graph. In the past, many results have been proven about the algebraic connectivity of graphs. For example, a graph has algebraic connectivity of zero if and only if it is disconnected.

In this presentation, we will consider the algebraic connectivity of the tree $\hat{T}$ which is created by joining two trees, T_1 and T_2, by an edge whose weight is allowed to tend to infinity. By interlacing properties of eigenvalues of symmetric matrices we know that $\lambda_2(\hat{T}) \leq \min\{\lambda_2(T_1), \lambda_2(T_2)\}$. Here, we show precisely when this inequality is sharp. This requires the computation of the spectral radii of inverse prinicpal submatricies of Laplacian matrices.

Spectrally Stable Matrices

Sivaram K.Narayan

Let $P = \{\text{diag}(d(1), ..., d(n))|$ every $d(i)$ is in $[-1, 1]\}$. We investigate the $n \times n$ complex matrices A such that every eigenvalue of every member of AP is contained in the convex hull of the eigenvalues of the extreme points of AP.

Sums of Matrices from Positivity Classes

Dale Olesky

Let A be an $m \times n$ entrywise positive matrix. A partition of A into k $m_i \times n_i$ disjoint submatrices A_i such that each A_i is totally positive (i.e., all minors are positive) is called a TP k-partition of A. It is shown that any matrix A with a TP k-partition is the sum of k TP matrices. Since any $m \times n$ positive matrix has a TP k-partition with $k = \min(m, n)$, it follows that any such positive matrix can be written as a sum of $\min(m, n)$ TP matrices. When m=n=3, an example is given to show that there exist positive matrices that cannot be written as a sum of fewer than 3 TP matrices. Analogous results using sums of inverse M-matrices will be given if time permits.

Ray Patterns: k-Potent and Powerful
Jeff Stuart

An $n \times n$ *ray pattern* is the collection of all $n \times n$ complex matrices with a specified zero-nonzero pattern such that the argument of each nonzero entry is also specified (modulo 2π). If A and B are each $n \times n$ ray patterns, then AB is defined to be the collection of all matrices of the form MN where M is in A and N is in B. Under certain circumstances, AB is itself a ray pattern. A ray pattern B is called *pattern k-potent* if there exists a smallest positive integer k such that $B^{k+1} = B$. A ray pattern B is called *powerful* if B^h is again a ray pattern for all positive integers h. Ray patterns are natural extensions of the much studied real sign patterns. Powerful sign patterns and k-potent sign patterns have been extensively investigated by Eschenbach, Hall, Kirkland, Li and Stuart, among others. We present some recent results that highlight the similarities and differences between sign patterns and ray patterns. In particular, we will discuss results for powerful and k-potent ray patterns due to Beasley, Hall, Li, Shader and Stuart.

How to Express a Parallel Sum of k Matrices
Yongge Tian

The parallel sum of two nonnegative definite matrices of the same order is

$$A : B = A(A + B)^{\dagger}B,$$

which also is nonnegative definite, where $(A + B)^{\dagger}$ is the Moore-Penrose inverse of $A + B$. In my talk, I shall present how to express the parallel sum of k matrices $A_1, \ldots, A_k$ by the Schur complement of a block matrix generated by $A_1, \ldots, A_k$. In addition, some matrix inequalities for parallel sums of matrices will be discussed.

M-matrices in Some Compartmental Epidemic Models

Pauline van den Driessche

A general compartmental disease transmission model is formulated as a system of ordinary differential equations. The basic reproduction number R_0 is defined as the spectral radius of the product of a nonnegative matrix and an inverse M-matrix. Using properties of M-matrices, this number is shown to act as a threshold, with the disease-free equilibrium being locally stable if $R_0 < 1$, but unstable if $R_0 > 1$. The results are illustrated by some specific examples including a treatment model for tuberculosis.

Spectral Radius of a Sampling Operator

Peter Zizler

Let h be a Laurent polynomial on the unit circle $\mathbf{T} = \{z \in \mathbf{C} \mid |z| = 1\}$. Consider a sampling operator $S = S_h(1,2)$ on the Hilbert space $L^2(\mathbf{T})$ defined as

$$S \;: f(z) \mapsto h(z)f(z^2)$$

where $f \in L^2(\mathbf{T})$. The sampling operator S is obtained from the bi-infinite matrix representation of the multiplication operator L_h by removing every second column.

Under certain conditions, which are quite often met in relevant applications, we provide an upper bound and lower bound on the spectral radius of the operator S. In particular we prove

$$\rho(S)^2 \leq \left|\sum_{n\neq 0} p_n\right| + p_0$$

where p_n is the nth Fourier coefficient of the nonnegative Laurent polynomial $|h|^2$. In the case of a nonnegative Laurent polynomial $h(z) = \sum_{n=1-N}^{N-1} h_n z^{-n}$ we have

$$\rho(S_h) \leq \left|\sum_{n\neq 0} h_n\right| + h_0.$$

We prove a result regarding a lower bound, in particular,

$$\ln(\rho(S)) \geq \frac{1}{4\pi}\int_0^{2\pi} \ln(|h|^2(\theta))\, d\theta.$$

Sampling operators appear naturally in wavelet analysis and their spectral radii are connected to the smoothness of the scaling functions. We show how our results provide sharper bounds on the smoothness of scaling functions. We also mention applications to ergodic theory.

3 Participants

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