Application of Langlands' functorial lift of SO(2n+1) to GL(2n)

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Together with Cogdell, Piatetski-Shapiro and Shahidi, we proved earlier the existence of a weak functorial lift of a generic cuspidal representation of SO_{2n+1} to GL_{2n}. Recently, Ginzburg, Rallis and Soudry obtained a more precise form of the lift using their integral representation technique, namely, the lift is an isobaric sum of cuspidal representations of GL_{n_i} (more precisely, cuspidal representations of GL_{2n_i} such that the exterior square L-functions have a pole at $s = 1$). I want to show that I can prove the same result (strong lift) using Langlands-Shahidi method, if we know the criterion for the pole of the L-function at $s = 1$. It is a major unresolved question of how Langlands-Shahid method can be used to give the criterion for the pole of the L-functions. I will describe some conjecture. I give several applications of the functorial lift: First, we parametrize square integrable representations with generic supercuspidal support, which have been classified by Moeglin and Tadic. Second, I give a criterion for cuspidal reducibility of supercuspidal representations of $GL_m \times SO_{2n+1}$.