Sheaves on adic spaces for p-adic group representation theory

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Let K^{nr} be the unramified closure of a p-adic field with ring of integers $\mathfrak{O}^{nr}$ and residue field $\overline{k}$, let $G_{K^{nr}}$ be a smooth group K^{nr}-scheme and let $\mathfrak{H}$ be a family of $\mathfrak{O}^{nr}$-schemes H such that H is a smooth group scheme, the generic fibre H_η is isomorphic to $G_{K^{nr}}$ (as group K^{nr}-schemes), the special fibre H_s is a reduced group scheme of finite type over $\overline{k}$ and $H(\mathfrak{O}^{nr})$ is a compact open subgroup in $H_\eta(K^{nr}) = G(K^{nr})$. Then each $H \in \mathfrak{H}$ determines a formal $\mathfrak{O}^{nr}$-scheme $\hat{H}$ and a rational affinoid analytic adic space H^{ad} with reduced special fibre H_s. Using the inclusion j_H of H^{ad} into the adic space $G^{ad}_{K^{nr}}$ associated to $G_{K^{nr}}$ and the morphism of sites $\nu_H\colon (H^{ad})_{et} \to (H_s)_{et}$, we define a restriction functor Res_{H_s} from the bounded derived category of constructible ℓ-adic sheaves on $G^{ad}_{K^{nr}}$ to the bounded derived category of constructible ℓ-adic sheaves on H_s as $\mathrm{Res}_{H_s} := R\nu_{H!}\, Rj_H{}^*$. The restriction functor can be used to defined a notion of $\mathfrak{H}$-perversity: a sheaf $\mathcal{F}$ on $(G^{ad}_{K^{nr}})_{et}$ will be called $\mathfrak{H}$-perverse if $\mathrm{Res}_{H_s}\mathcal{F}$ is perverse, for every $H \in \mathfrak{H}$. We will give examples of such situations using the Moy-Prasad filtrations.

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