Robustness and Kalman-Filtering

P. Ruckdeschel[1],H. Rieder[1]

[1] Bayreuth University, Universitätsstraße 30, D-95447 Bayreuth, Germany

Keywords: Robustness, Kalman–Filtering, Minimax Procedures, Substitutive Outliers, EM–Algorithm

1 Optimally Robust Kalman–Filtering

1.1 Robustness Problem in State Space Models

State–space models form a wide–spread and flexible class in modelling time dependent phenomena; we consider the filtering and prediction problem in the case of a linear, finite–dimensional and time–discrete model with an Euklidean state space, i.e. estimation of an unobservable state β_t by means of the observations $Y_1, \ldots, Y_{t[-1]}$.
Allowing for linear estimators only, one comes up with the Kalman-type filters and predictors as classically MSE–optimal solutions, which are recursive, but all based on second moments of the underlying distributions, however. This is clearly a robustness problem, i.e. small deviations from the model assumptions will cause large effects on the quality of the filter /predictor.
Following Fox (1972), these deviations may essentially be classified into two typical sorts of outliers — IO's and AO's. For our purposes, we will concentrate ourselves to a variant of AO's, subsitutive outliers or SO's.

1.2 Optimality of the rLS–Filter

Still insisting on strict recursivity for computability reasons, we define a new procedure, the *rLS–filter*, using a Huberized correction–step.
To better understand the excellent behaviour of this procedure when used to simulated, ideal and contaminated data, we reduce the state space model to a simpler form.
In this setup, we derive optimal robust filters under SO–contamination — both in a "Lemma 5" approach [c.f. Hampel (1968)] and in a minimax approach, the latter generalizing a result of Birmiwal and Shen (1993).
As in the location case, both solutions coincide, and, curiously enough yield the rLS–filter, if all inputs are Gaussian. Unfortunately, working with a past already treated with the rLS–filter, normality is lost.
Extending the SO–contamination neighborhood a little, however, the minimax and "Lemma 5"–solution of the original SO–neighborhood remain valid, and we are able to show [numerically] that the process of filters / predictions generated by the rLS–filter stays in this extended [e]SO–neighborhood about some fictive Gaussian ideal process, thus is eSO–optimal.

2 Unknown Hyper–Parameters

A second part of this talk will be devoted to the situation where we have to estimate hyper–parameters, i.e. the system matrices, from the data. Following Shumway and Stoffer (1982), we interprete the unobservable states as missing observations. Applying a general result on the preservation of L_2–differentiability under information loss — suitably adopted to our time–dependent situation — we are able to show L_2–differentiability of this parametric model, thus providing the LAN–property.
With this theoretic background, influence curves and as. linear estimators similar to H. Rieder (1994)

are available, and we are able to characterize the classically (Cramér–Rao)–optimal estimator for the hyper–parameters. Using a one–step construction we give a simple non–iterative alternative to the EM–Algorithm used by Shumway and Stoffer, which moreover achieves this optimality.
In order to robustly estimate the hyper–parameters, eventually, we replace the classically optimal influence curve in this setup by a bounded one, thus giving a non–iterative robust EM–Algorithm.

References

K. Birmiwal and J. Shen (1993). Optimal robust filtering. *Stat. Decis.*, 11(2), 101–119.

A.J. Fox (1972). Outliers in time series. *J. R. Soc. Ser. B*, 34, 350–363.

F.R. Hampel (1986). *Contributions to the theory of robust estimation*, PhD Thesis, University of California, Berkely, CA.

P.J. Huber (1981). *Robust Statistics.* Wiley & Sons, New York.

H. Rieder (1994). *Robust asymptotic statistics.* Springer, New York.

P. Ruckdeschel (2001). *Ansätze zur Robustifizierung des Kalman–Filters.* PhD Thesis, Bayreuther Mathematische Schriften, Bayreuth.

R.H. Shumway and D.S. Stoffer (1982). An approach to time series smoothing and forecasting using the EM algorithm. *J. Time Ser. Anal.*, 3, 253–264.

Please fill in this form and mail it together with your abstract.

My abstract fits best to topic number 21

(**"PLEASE INDICATE HERE THE NUMBER FROM THE LIST OF TOPICS BELOW WHICH BEST FITS TO YOUR ABSTRACT"**.).

List of Topics:

1. Algorithms
2. Applications
3. Biostatistics
4. Computing and graphics
5. Data analysis
6. Data mining
7. Economics, finance
8. Efficiency and robustness
9. Functionals and bias
10. Fuzzy statistics
11. Geostatistics
12. Inference for robust methods, model testing
13. Location depth and regression depth
14. Multivariate methods
15. Neural networks
16. Rank-based methods
17. Regression quantiles, trimming
18. Robust covariance
19. Robust designs
20. Robust regression
21. Time series analysis
22. Wavelets
23. Other (please specify)