Robust estimation for GARCH models

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Keywords: GARCH models, robust estimates

Abstract

One way to model heteroskedasticity for a time series $x_1, ..., x_T$ is by using GARCH models. The GARCH(p, q) model assumes that conditionally on $x_1, ...x_{T-1}$, the distribution of x_t is $F(x/\sigma_t)$, where F is a specified distribution with mean zero and variance one, and the conditional variance σ_t^2 is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + ... + \alpha_p x_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2.$$

In particular, when $q = 0$, we get the family of ARCH models. Usually, the parameters of the GARCH models are estimated by conditional maximum likelihood (CML) assuming normal F. These estimates are very sensitive to outliers: just one outlier, may have an unbounded effect on them. To get robust estimates we make two changes in the definition of the CML estimates. First, we bound the normal likelihood in a suitable way, so that the resulting estimates are still Fisher consistent. This modification guarantees robustness for ARCH models, but not for GARCH models. The lack of robustness in the GARCH case is due to the fact that a single large outlier has much influence on the conditional variance of an undetermined and large number of subsequent observations. To obtain robustness for GARCH models, the proposed estimates include an additional mechanism which restrict the propagation of the outlier effect. This requires embedding the GARCH model in a larger class of models of the form

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + ... + \alpha_p x_{t-p}^2 + \beta_1 g(\sigma_{t-1}^2, \gamma) + ... + \beta_q g(\sigma_{t-q}^2, \gamma)$$

where $g(., \gamma)$ is a bounded function so that $g(u, \gamma) = u$ for $u \leq \gamma$. We prove the consistency and asymptotic normality of the proposed estimates. A Monte Carlo study shows that the maximum likelihood estimate practically collapses in the presence of a small percentage of outlier contamination, while the proposed robust estimates have a much better robust behavior.

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