

# Asymptotic distributions of principal components based on robust dispersions

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## Abstract

Principal components are routinely used in multivariate data analysis. Algebraically, they can be defined as the eigenvalues and eigenvectors of a covariance or correlation matrix, but they are statistically meaningful as successive projections of the multivariate data in the direction of maximal variability. An attractive alternative in the robust principal component analysis is to replace the classical variability measure, i.e. variance, by a robust dispersion measure. This projection-pursuit approach was first proposed in Li and Chen (1985) as a method of constructing a robust scatter matrix. Recent work of Croux and Ruiz-Gazen (2001) provided the influence functions of the resulting principal components. The present paper focuses on the asymptotic distributions of robust principal components. In particular, we obtain the asymptotic normality of the principal components that maximize a robust dispersion measure. We also explain the need to use a dispersion functional with a continuous influence function.

*Some key words:* Asymptotic normality; Dispersion; Principal component; Projection pursuit; Robustness.

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23. Other (Robust principal components)

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