Robust estimation for time series models based on infinitesimal neighborhoods.

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We consider parametric time series models which are LAN with a stationary scores function that is the product of the innovation scores and a function of the past; in view of the work by Drost, Klaassen, Wercker (1997), these models include ARMA, TAR, ARCH, and GARCH. Based on this kind of LAN, we define influence curves of asymptotically linear estimators by stationary and ergodic martingale differences that satisfy a Fisher consistency condition with respect to the model scores. Employing similar LAN-expansions of the loglikelihoods, infinitesimal neighborhoods of transition probabilities may be introduced, their radius depending on the past of the process. The neighborhoods allow for IO-outliers which are nonstationary and depend on the past, but may cover also AO- and SO-outliers if the radius function is chosen suitably.

Optimally robust influence curves can then be derived by minimizing the maximum asymptotic MSE of asymptotically linear estimators over such infinitesimal neigborhoods, and corresponding least favorable radius curves are obtained.

The construction problem has been solved at least for the smallest neighborhood model: conditional contamination and total variation neighborhoods with bounded radius curve.

Our work unifies and extends the previous approaches to robust time series estimation by Künsch (1984), Staab (1984), and Martin and Yohai (1986).