On Introducing Robust Statistics in an Undergraduate Mathematical Statistics Course: A Case Study Approach With a Redescending M-estimator

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Abstract

Robust statistics tends to be very mathematical and needs to be its own course to be taught thoroughly. However, its applications are flexible and are worthy of introduction into an existing undergraduate mathematical-statistics course. To make robust statistics accessible yet thorough, I propose a case study approach to the study of M-estimators. A recent article by Stefanski and Boos (2002) does an excellent job of accomplishing this at the graduate level. To make the study of M-estimators accessible to undergraduates, I propose a specific comparison between a classic estimator and a particularly smooth M-estimator for the location problem. The novelty in my method is that the robust estimator I use to introduce the concepts can be used to cover many of the main concepts of M-estimation at a mathematical level appropriate for undergraduate statistics majors. The approach requires only the expectation of a function of a normal variable in both the uncontaminated data case and contaminated data case. I simplify the contaminated case by studying robustness with a mixture of normal distributions. While less general than a more thorough approach, it allows the students to appreciate M-estimators at the mathematical level while also delving into more difficult concepts.

In this approach, students learn that M-estimators are broader than simply the mean and the median, and they can be extended beyond the univariate case. I also include a tuning parameter to bridge the gap between the median and the mean.

The purpose of my talk is to provide an outline to teach undergraduate students about M-estimators using this case study approach. My approach includes an intuitive understanding of M-estimators, a basic theoretical appreciation of M-estimators and finally provides an accessible computer program for use in real world problems.

I describe robustness using an intuitive definition of a sensitivity curve, which provides a building block for defining a specific smooth M-estimator. I compare the smooth M-estimator to the maximum likelihood estimator (MLE) using asymptotic variances assuming both a normal distribution and a contaminated normal distribution. This allows students with basic knowledge of expectations of a function of normal random variables to become acquainted with the theory. Finally, I explain how undergraduates can use MATLAB to calculate any smooth M-estimate (this has obvious parallels to code in S+, R and other software packages). I also provide a list of homework questions. I make concluding remarks about assessing this approach on undergraduate statistics majors taking my mathematical-statistics course at St. Cloud State University.

References

Bibliography

[1] Stefanski, L.A. and Boos, D.D.(2002). The Calculus of M-Estimation, *The American Statistician, 56(1)*, pp. 29–38.