

**Yoshiharu Kohayakawa** (University of San Paulo, Brazil)

Title: The Regularity Lemma for Sparse Graphs

Abstract: One of the fundamental tools in asymptotic graph theory is the well-known regularity lemma of Szemerédi. In essence, the regularity lemma tells us that any large graph may be decomposed into a bounded number of quasi-random, induced bipartite graphs. Thus, this lemma is a powerful tool for detecting and making transparent the random-like behaviour of large deterministic graphs. Furthermore, in general, the quasi-random structure that the lemma provides is amenable to deep analysis, and this makes the lemma a very important tool.

The quasi-random bipartite graphs that Szemerédi's lemma uses in its decomposition are certain graphs in which the edges are uniformly distributed. The measure of uniformity is such that this concept becomes trivial for graphs of vanishing density. To manage sparse graphs, one may adjust this notion of uniform edge distribution in a natural way, and it is a routine matter to check that the original proof extends to this notion, provided we restrict ourselves to graphs of vanishing density that do not contain 'dense patches'. However, the quasi-random structure that the lemma reveals in this case is not too informative, and this puts into question the applicability of this variant of the lemma for 'sparse graphs'. Nevertheless, there have been some successful applications of the lemma in this context. In this talk, we shall concentrate on the difficulties one faces and how one can overcome them in certain situations.