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Title: How to Compute a Norm?

Abstract: Let us fix a univariate polynomial $p(n)$. For any finite-dimensional normed space V, we want to construct an algorithm, which, for any given point x in V approximates the norm of x within a certain factor $c(dimV)$ in time bounded by $p(dimV)$. How small can $c(n)$ be? We show that for any $\epsilon > 0$ there is a p so that $c(n) = \epsilon n^{1/2}$. We show further that for some particularly symmetric norms (for which the polar of the unit ball in V is the convex hull of an orbit of a compact group), we can do better. For example, there is a polynomial time approximation scheme to compute the maximum absolute value of a fewnomial (= a polynomial having a fixed number of monomial terms) on the unit sphere in Euclidean space. The algorithms are based on approximating the norm by a root of a lower-degree or otherwise easily computable polynomial. Time permitting, I am planning to discuss related results on approximating an n-dimensional convex body by the projection of a polytope whose dimension and the number of facets are bounded by a polynomial in n.