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The Role of Maximal L_p Bounds in Quantum Information Theory

Abstract: In quantum information theory, one works on a vector space formed from tensor products of $\mathbf{C}^2$ rather than $\mathbf{Z}^2$. Mixed states are described by density matrices, i.e., positive semi-definite operators with trace one. They can be regarded as the result of noise, which is modeled by the action of a completely positive, trace-preserving (stochastic) map. The concepts of von Neumann entropy, relative entropy and accessible information are used in the extension of Shannon's information theory to this non-commutative setting. Both the entropy and the L_p norm of a density matrix provide measures of the purity of a mixed state. The maximal L_p norm of a stochastic map measures the optimal purity of a state after noise. It has been conjectured that the maximal L_p norm is multiplicative. This implies that the minimal entropy and, in some cases, the channel capacity is additive. This talk will be primarily tutorial, concluding with a summary of what is known about the conjectures and important open questions.