

Guillaume Aubrun (ENS Paris):

A small deviation inequality for the largest eigenvalue of a random matrix

Abstract: We prove that the largest eigenvalue λ_1 of a matrix from the $n \times n$ Gaussian Unitary Ensemble satisfies a “small deviation” inequality: $\mathbf{P}(\lambda_1 \geq 2 + t) \leq Ce^{-cnt^{3/2}}$, where $C, c > 0$ are universal constants. It completes the usual concentration inequality, which is sharper for “large deviations”. Following the approach of Tracy and Widom, we reduce the problem to an analysis of several integral operators which converge, in a proper sense, to an operator whose determinant can be estimated.