

Alexandru Nica (Waterloo):

2-point functions for multi-matrix models, and non-crossing partitions in an annulus

Abstract: Let X_1, \dots, X_s be a family of selfadjoint Gaussian random matrices with independent entries. For every word $w = (r_1, r_2, \dots, r_p)$ over the alphabet $\{1, \dots, s\}$, we denote $X_w := X_{r_1} X_{r_2} \cdots X_{r_p}$. We consider expressions of the form $(E \circ tr)(X_v \otimes X_w)$, with v, w words over $\{1, \dots, s\}$ (and where “ tr ” is the normalized trace on the appropriate space of matrices, and “ E ” stands for expectation). We find a precise combinatorial formula for such an expression; the formula is best put into perspective when placed in the framework of the Cayley graph of the symmetric group $S_{|v|+|w|}$, where $|v|$ and $|w|$ denote the lengths of the words v and w . As a consequence, we obtain that the asymptotics for an expression of the form

$$(E \circ tr)(X_v \otimes X_w) - (E \circ tr)(X_v) \cdot (E \circ tr)(X_w)$$

exists, and has a combinatorial description involving summation over non-crossing partitions in an annulus.

We observe that an analogous phenomenon takes place when we consider certain examples of Wishart matrices (instead of selfadjoint Gaussian matrices with independent entries).

This is joint work with James Mingo.