Conference on Convexity and Asymptotic Theory of Normed Spaces

July 1-5

Abstracts

Juan Carlos Alvarez (Polytechnic University, USA)

Title: Dual mixed volumes and averaging of Hamiltonian systems

Abstract: A natural extension of the theory of mixed volumes to cotangent bundles generalizes Loewner's area-length method and can be used to prove Hamiltonian versions of the isosystolic inequality.

Keith Ball (University College London, UK)

Title: Convolution inequalities in convex geometry

Abstract: The talk explains how sharp inequalities for L_p norms of convolutions solve a number of problems that can be regarded as converse to the usual isoperimetric inequality.

Franck Barthe (Université de Marne la Vallée, France)

Title: Extremal properties of sections or projections of B_p^n

Abstract: We present a few results about extremal properties of the canonical sections or projections of B_p^n (related to e.g ℓ-norms, volume). The techniques involve different stochastic orderings and probabilistic representations of measures on B_p^n.

Károly Böröczky (Renyi Institute of Mathematics, Hungary)

Title: The stability of the Rogers–Shepard and the Zhang projection inequalities

Abstract: The Rogers–Shepard and the Zhang projection inequalities are two basic volume estimates, which were linked by a family of inequalities by Gardner and Zhang. The cases of equality is always characterized by simplices. In the lecture, a method is sketched how the stability of all these inequalities can be established. This work was done jointly with R. Gardner.

Andrea Colesanti (University of Florence, Italy)

Title: Geometric properties of capacity of convex bodies

Abstract: Capacity is a notion appearently far from convexity. It arises naturally in the theory of Sobolev spaces and in the calculus of variations; there are various equivalent definitions based on either minumum problems for integral functionals, or solutions of elliptic partial differential equations. On the other hand, capacity restricted to the class of n-dimensional convex bodies shares some peculiar properties with the n-dimensional volume. Examples are: isoperimetric inequalities, existence and uniqueness of the solution to the Minkowski problem, Brunn-Minkowski inequality. These results suggest to search for further evidences of the analogy between capacity and volume.

Dario Cordero-Erausquin (Université de Marne la Vallée, France)

Title: Santaló's inequality on C^n by complex interpolation

Abstract: Volume estimates for interpolated balls of C^n are given by using a generalization of Prékopa's inequality recently obtained by Berndtsson.

Matthieu Fradelizi (Université de Marne la Vallée, France)

Title: The extreme points of subsets of s-concave probabilities and a geometric localization theorem.

Abstract: We prove that the extreme points of the set of s-concave probability measures satisfying a linear constraint are some Dirac measures and some s-affine probabilities suppported by a segment. ¿From this we deduce that the constrained maximization of a convex functional on the s-concave probability measures is reduced to this small set of extreme points. This gives a new approach to a localization theorem due to Kannan, Lovász and Simonovits which happens to be very useful in geometry to obtain inequalities for integrals like concentration and isoperimetric inequalities. Roughly speaking, the study of such inequalities is reduced to these extreme points. Joint work with O. Guédon.

Fuchang Gao (University of Idaho, USA)

Title: Small ball probability via covering numbers

Abstract: Some associated measure spaces will be introduced for Gaussian processes; the relation between the covering numbers in the measure space and the small ball probability of the Gaussian process will be discussed; some applications will be presented.

Mohammad Ghomi (University of South Carolina, USA)

Title: A survey of some recent convexity results and problems in classical differential geometry

Abstract:

Apostolos Giannopoulos (University of Crete, Crete)

Title: Ψ_2-estimates for linear functionals and the slicing problem

Abstract:
Let K be a convex body in $\mathbb{R}^n$ with volume 1 and centre of mass at the origin. Let $1 \leq \alpha \leq 2$ and $y \neq 0$ in $\mathbb{R}^n$. We say that K satisfies a ψ_α-estimate with constant b_α in the direction of y if

$$(*) \qquad \|\langle \cdot, y\rangle\|_{\psi_\alpha} \leq b_\alpha \|\langle \cdot, y\rangle\|_1.$$

We say that K is a ψ_α-body with constant b_α if $(*)$ holds for every $y \neq 0$. In this talk we review the "ψ_2-approach" to the slicing problem. More precisely, we discuss:

1. The bound $\sqrt[4]{n}$ for the isotropic constant (Bourgain).
2. The existence of ψ_2-directions for certain classes of bodies: zonoids and 1-unconditional bodies (Paouris, Bobkov-Nazarov).
3. The question of characterizing ψ_2-bodies.
4. Bourgain's recent bound for the isotropic constant of ψ_2-bodies.
5. Related open questions.

Eric Grinberg (The Hebrew University of Jerusalem, Israel)

Title: Inversion of the Radon transform via Gårding-Gindikin fractional integrals.

Abstract: The Funk-Radon transform has figured prominently in convex geometry during the past decade and inversion formulas have played an important role. There are many standard inversions, some involving differential operators and others involving fractional integrals. Here we consider a natural generalization of the Radon transform, which acts on functions on Grassmannians to produce functions on other Grassmannians. These transforms also occur in convex geometry and have been studied and inverted before, e.g., by I.M. Gelfand and his collaborators. Here we establish a connection between the Radon transform and Gårding-Gindikin fractional integrals associated to the cone of positive definite matrices, and use it to obtain Abel type representations and inversion formulae on various function spaces, including L^p. Joint work with Boris Rubin.

Paolo Gronchi (Consiglio Nazionale delle Ricerche, Italy)

Title: Continuous movements and volume product

Abstract: I shall report on a joint work (in progress) with Stefano Campi. The main object is to use the continuous movements method and the Prekopa-Leindler inequality to recover Reisner's inequality for the volume product in the class of zonoids.

Peter Gruber (Vienna University of Technoloy, Austria)

Title: Optimal Quantization

Abstract: If we assign to each point from a given Jordan measurable set set in Euclidean space the nearest point from a fixed finite set we make an 'error'. The lecture deals with the questions, how should the finite set be chosen in oder to minimize the 'error' and what is the value of the 'minimum error'. The results obtained then are applied to the following problems: (i) Minimum distorsion in data transmission.
(ii) Approximation of probability measures by discrete measures.
(iii) Minimum error of numerical integration formulae.
(iv) Approximation of convex bodies by polytopes.
(v) The isoperimertic problem in Minkowski spaces.

Daniel Hug (Mathematisches Institut, Germany)

Title: On the limit shape of the zero cell of a Poisson hyperplane tessellation

Abstract: In the early 1940s David G. Kendall conjectured that the shape of the zero cell C of the random tessellation generated by a standard (stationary and isotropic) Poisson line process in the plane tends to circularity given that the area of $C \to \infty$ (or given the area of C is essentially fixed and the intensity of the line process $\to \infty$). Recently, R.E. Miles has offered alternative formulations as well as a heuristic proof of the original conjecture. In the talk, we consider the extension of Kendall's problem, to higher dimensions and to anisotropic hyperplane processes. Joint work with Matthias Reitzner and Rolf Schneider.

Alexander Iosevich (University of Missouri, USA)

Title: Combinatorial approach to orthogonal exponentials

Abstract: Abstract: We prove that a convex symmetric body in d dimensions whose boundary has everywhere non-vanishing curvature has only finitely many orthogonal exponentials if d is not congruent to 1 modulo 4. If d is congruent to 1 modulo 4, the number of orthogonal exponentials may be infinite, but they must be contained in a line. The proof is a mixture of Fourier analysis and combinatorial geometry. We also mention some related problems in the theory of orthogonal Fourier bases.

Boris Kashin (Steklov Mathematical Institute, Russia)

Title: Estimates for n-term approximation in Hilbert space and related problems

Abstract: The talk is devoted to the lower estimates of the best n-term approximations of certain sets in Hilbert spaces with respect to orthonormal systems (o.n.s.). In particular, it is proved that for any o.n.s. in $L^2(I^2)$, where $I^2 = (0,1)^2$ is a unit square, and for all $n = 1, 2, ...$ there exists a convex subset of unit square such that the best n-term approximation of its characteristic function with respect to this system is greater then c/n , where c is an absolute positive constant. In relation to the estimates of best n-term approximation, we have introduced and studied one characteristic of massivness of a given subset of a Hilbert space.

Boaz Klartag (Tel Aviv University, Israel)

Title: Isomorphic Steiner Symmetrization

Abstract: We investigate the minimal number of Steiner symmetrizations that can transform an arbitrary convex body into a new body which is uniformly isomorphic to a Euclidean ball. We prove that this number is proportional to the dimension, with small proportion constant, thus improving a previous estimate by a logarithmic factor.

Alexander Koldobski (University of Missouri-Columbia, USA)

Title: Fourier analytic tools in the solution of the Busemann-Petty problem

Abstract: We present a short proof of the affirmative part of the Busemann-Petty problem using the Funk-Hecke formula for spherical harmonics. The negative part will involve positive definite distributions and will be proved in the generalized form. The talk will include short introductions to spherical harmonics, fractional derivatives and positive definite distributions.

Hermann König (Universität Kiel, Germany)

Title: Some examples with extremal projection constants

Abstract: It is shown that spaces with maximal projection constants not only imbed into ℓ_∞ but also exist as subspaces of ℓ_1 . In general, these spaces are not uniquely determined, except in two dimensions. We give a method how to construct non-isometric spaces with extremal projection constants, an interesting case being real dimension 3. This is joint work with N. Tomczak-Jaegermann.

Alexander Litvak (University of Alberta, Canada)

Title: Randomized Isomorphic Dvoretzky Theorem

Abstract: Let K be a symmetric convex body in $\mathcal{R}^N$ for which B_2^N is the ellipsoid of minimal volume. We provide estimates for the geometric distance of a "typical" rank n projection of K to B_2^n, for $1 \leq n < N$. Known examples show that the resulting estimates are optimal (up to numerical constants) even for the Banach–Mazur distance. Joint work with P. Mankiewicz and N. Tomczak-Jaegermann.

Erwin Lutwak (Polytechnic University, USA)

Title: L_p-curvature

Abstract: L_p-curvature arises naturally in the L_p extension of the classical Brunn-Minkowski theory. The notion of L_p-curvature has already led directly to the discovery of the dual of the Legendre ellipsoid (Ludwig, Lutwak, Yang, Zhang) as well as L_p extensions of:
1. the John ellipsoid (Lutwak, Yang, Zhang),
2. the Petty projection inequality (Campi, Gronchi, Lutwak, Yang, Zhang),
3. the Busemann-Petty centroid inequality (Campi, Gronchi, Lutwak, Yang, Zhang),
4. the classical Minkowski problem (Chen, Lutwak, Oliker, Stancu, Umanskiy, Yang, Zhang),
5. affine surface area (Hug, Ludwig, Lutwak, Meyer, Schuett, Werner, Yang, Zhang),
6. geominimal surface area (Hug, Lutwak),
7. classical stability results (Hugg, Schneider),
8. the affine Sobolev-Zhang inequality (Lutwak, Yang, Zhang).
After being with us for eight years, L_p-curvature still has more questions surrounding it than answers.

Monika Ludwig (Technische Universität Wien, Austria)

Title: Affinely associated Ellipsoids and Matrix valued Valuations

Abstract: A central problem when studying convex bodies is to choose the right position ϕK of a convex body K where $\phi \in GL(n)$. The right position depends on the question we have about K and the most important questions are of the following type: for a given functional μ on the space of convex bodies we ask for the maximum or minimum of $\mu(\phi K)$ over all $\phi \in GL(n)$. For this class of questions, the problem to find the right positions of convex bodies K is equivalent to the following problem: to find ellipsoids $E(K)$ that are naturally connected with our question and have the property that

$$E(\phi K) = \phi E(K) \quad \text{for} \quad \phi \in GL(n).$$

We say that these ellipsoids are affinely associated to K. The convex body K is now in the right position if and only if the ellipsoid $E(K)$ is the unit ball of $\mathbb{R}^n$.

We give a classification of affinely associated ellipsoids defined by symmetric matrices that are Borel measurable valuations on the space $\mathcal{P}_o^n$ of n-dimensional convex polytopes containing the origin in their interiors. Here we say that a positive-definite symmetric matrix M defines an ellipsoid E if

$$E = \{x \in \mathbb{R}^n : x \cdot Mx \leq 1\}$$

where $x \cdot Mx$ denotes the standard scalar product of x and Mx. And we say that a function M on $\mathcal{P}_o^n$ is a valuation, if

$$MP_1 + MP_2 = M(P_1 \cup P_2) + M(P_1 \cap P_2)$$

whenever $P_1, P_2, P_1 \cup P_2 \in \mathcal{P}_o^n$. The only examples of such ellipsoids in $\mathbb{R}^n$, $n \geq 3$, are the Legendre ellipsoid Γ_2 and the ellipsoid Γ_{-2} recently defined by Lutwak, Yang, and Zhang.

Piotr Mankiewicz (Institute of Mathematics, Polish Academy of Sciences, Poland)

Title: On the geometry of proportional quotients of l_1^m

Abstract: Some recent results concerning the geometry of random n-dimensional quotients of $l_1^{(1+\delta)n}$ will be presented. It turns out that "vast majority" (with respect to the Haar measure on related Grassmann manifold) of such quotients X satisfies the property:
for every sufficiently nontrivial operator $T \in L(X)$ (i.e. for so called $(\kappa n, 1)$-mixing operators) the inequality $\|Tx\|_X \geq c\sqrt{n}$ is satisfied by at least $\delta n/2$ extreme points of the unit ball of X, where $c = c(\kappa, \delta) > 0$ depends on κ and δ only.
In particular, this implies that the basis constant (and some other "geometric constants" as well) of such a quotient is grater than or equal to $c\sqrt{n}$. Joint work with S. Szarek.

Shahar Mendelson (The Australian National University, Australia)

Title: Geometric parameters in Learning Theory

Abstract: The main question investigated in Learning Theory concerns the ability to find a "good" approximation of a target function T from a given class of functions F using empirical data. The learner is given a random sample $(X_i, T(X_i))_{i=1}^n$ where (X_i) is sampled according to an unknown probability measure μ. The question is "how large" must the sample be if one

wishes to produce a function f which is, with sufficiently high probability, a good approximation of T with respect to the $L_2(\mu)$ norm. It turns out that the geometry of F plays the key role in the analysis of this question. In fact, geometric parameters such as the empirical covering numbers, the shattering dimension and gaussian averages $\ell_n = \mathbb{E}_\mu n^{-1/2} \mathbb{E} \sup_{f \in F} |\sum_{i=1}^n g_i f(X_i)|$, control the "complexity" of the learning problem. We will present several problems which appear naturally in the context of Learning Theory and also seem to be of an independent interest as pure questions in the Local Theory of Banach spaces.

Vitali Milman (Tel Aviv University, Israel)

Title: Are randomizing properties of any two convex bodies similar ?

Shlomo Reisner (University of Haifa, Israel)

Title: An application of convex geometry to approximation theory: approximation by ridge functions

Abstract: We consider best approximation of some function classes by the manifold M_n consisting of sums of n arbitrary ridge functions. It is proved that the deviation of the Sobolev class $W_p^{r,d}$ from the manifold M_n in the space L_q for any $2 \le q \le p \le \infty$ behaves asymptotically as $n^{-\frac{r}{d-1}}$. In particular we obtain this asymptotic estimate for the uniform norm $p = q = \infty$. Joint work with Yehoram Gordon, Vitaly Maiorov and Mathieu Meyer

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Matthias Reitzner (University of Freiburg, Germany)

Title: Random polytopes and the Efron-Stein jackknife inequality

Abstract: Let K be a smooth convex body in $\mathcal{R}^d$, denote by P_n a random polytope in K (which is the convex hull of n random points in K), and by P_N^{best} a best–approximating polytope (which minimizes the difference between $V_d(K)$ and $V_d(P)$ among all polytopes P in K with at most N vertices).
As for the random polytope it was proved by Rényi and Sulanke, Wieacker, Bárány, and Schütt that the difference $V_d(K) - \mathbb{E}V_d(P_n)$ is of order $n^{-2/(d+1)}$ as n tends to infinity. On the other hand, the rate of convergence of the volume of the polytope to the volume of K increases from $n^{-2/(d+1)}$ to $N^{-2/(d-1)}$ if the random polytope is replaced by the best–approximating polytope: As was proved by Gruber, $V_d(K) - V_d(P_N^{\text{best}})$ is of order $N^{-2/(d-1)}$.

Inspired by the fact, that N denotes the number of vertices of the best–approximating polytope, whereas n denotes the number of random points generating P_n, Bárány suggested to measure the difference $V_d(K) - V_d(P_n)$ by using the number of vertices $N(P_n)$ of the random polytope instead of using n. He proved that for $d = 2, 3$

$$\mathbb{E}\left((V_d(K) - V_d(P_n))N(P_n)^{\frac{2}{d-1}}\right) \sim c_d\,\Omega(K)^{\frac{d+1}{d-1}}$$

(the the same order of magnitude as for best–approximating polytopes) which means that "Approximation by random polytopes is almost optimal". In our talk we show that for the remaining cases $d \geq 4$

$$\lim_{n\to\infty}(V_d(K) - V_d(P_n))N(P_n)^{\frac{2}{d-1}} = c_d\,\Omega(K)^{\frac{d+1}{d-1}}$$

holds with probability one.
The main tool is the Efron-Stein jackknife inequality for the variance of symmetric statistics which yields an estimate for the variance of $V_d(P_n)$ and $N(P_n)$. This leads to an estimate for the deviation of $V_d(P_n)$ and $N(P_n)$ from their mean values.

Boris Rubin (Hebrew University of Jerusalem, Israel)

Title: Analytic families associated to Radon transforms in integral geometry

Abstract: Various Radon transforms (in euclidean, elliptic, and hyperbolic spaces) give rise to the corresponding analytic families of integral operators. Numerous important operators of integral geometry (like the hemispherical Funk transform, spherical section transforms, p-cosine transforms and their higher rank generalizations) are members of these families. This way of thinking enables one to obtain a series of interesting results by making use of the relevant tools of harmonic analysis and number theory.
The lecture concerns the following topics: Fractional analogues of euclidean Radon transforms; geodesic transforms on the sphere and associated analytic families of intertwining operators; the Radon transform on Grassmann manifolds; the generalized Minkowski-Funk transform for non-central spherical sections, and small divisors for spherical harmonic expansions; the Busemann-Petty problem; cosine transforms in the real hyperbolic space.

Mark Rudelson (University of Missouri, USA)

Title: Asymptotic geometry of non-symmetric convex bodies

Abstract: The talk will discuss the recent progress in study of non-symmetric convex bodies. The theory of symmetric convex bodies is a very rich subject containing many deep theorems. This is due to the fact that such bodies may be considered as unit balls of finite-dimensional Banach spaces. However, the most powerful technical tools, such as K-convexity, are not available without the symmetry assumption. During the last few years several new methods applicable to general convex bodies were developed. These methods allowed to obtain fundamental results of Geometric Functional Analysis like General Dvoretzky Theorem, Quotient Subspace Theorem etc. for convex bodies, which are not necessary symmetric. The talk will survey this new approach as well as the new phenomena, which cannot be observed in the symmetric setting.

Gideon Schechtmann (The Weizmann Institute, Israel)

Title: Non Linear Type and Pisier's Inequality.

Abstract: We prove that for the class of UMD Banach spaces, the $\log n$ term in an inequality of Pisier can be replaced by a constant independent of n. This is applied to show that for UMD Banach spaces, various non-linear notions of type p are implied by (Rademacher) type p. (Joint work with Assaf Naor.)

Rolf Schneider (University of Freiburg, Germany)

Title: A spherical analogue of the Blaschke-Santalo inequality.

Abstract: Let K be a spherically convex body in the sphere S^n, and let K^* be its spherical polar body. If K has given (positive) volume, then the volume of K^* is maximal if and only if K is a ball (in S^n).

Carsten Schütt (Case Western Reserve University, USA)

Title: Surface bodies

Abstract: We prove an asymptotic formula for the volume difference of a convex body and its surface body.

Alina Stancu (Polytechnic University, USA)

Title: An application of curvature flows to the L_0-Minkowski problem

Abstract: Asymptotic shapes of compact bodies whose boundaries are evolving by curvature flows are often convex bodies with particular properties. By using an appropriate planar curvature flow, we discuss the existence and uniqueness of polygons which are solutions to the discrete L_0-Minkowski problem. This problem proposed, in a larger generality, by E. Lutwak extends a classical question posed by Minkowski: Let $\mathcal{U} = \{\vec{u}_1, ..., \vec{u}_N\}$ be an ordered family of directions in $\mathbf{S}^1$, not all in a half-disk, and let $\Gamma = \{\gamma_1, ..., \gamma_N\}$ be an ordered set of strictly positive numbers. Does there exist a convex N-gon such that the side i has outer normal $\vec{u}_i$ and the triangle formed by this side and the origin has area γ_i?

Rick Vitale (University of Connecticut, USA)

Title: Two Gaussian Notes

Abstract: We discuss (i) a Gaussian version of the Weil/Lindquist condition [1,3] for the generating density of a generalized zonoid and (ii) a Gaussian approximation scheme [2] for convex bodies.

1. Lindquist, N.F. (1975). Support functions of centrally symmetric convex bodies. *Portugal. Math.* **34**, 241-252.
2. Vitale, R.A. (1994). Stochastic smoothing of convex bodies: two examples. Supplement to *Rend. Circ. Mat. Palermo* **35**, 315–322.
3. Weil, W. (1982). Zonoide und verwandte Klassen konvexer Körper. *Monatsh. Math.* **94**, 73-84.

Wolfgang Weil (University of Karlsruhe, Germany)

Title: Directed Projection Functions

Abstract: In a paper from 1997, Groemer introduced the semi-girth of a convex body K in $\mathbb{R}^3$ as a function on pairs (L, u), where $L \subset \mathbb{R}^3$ is a plane and $u \in L$ is a unit vector. He then showed a stability result which implied that this function determines K uniquely (up to translations). We generalize his results in two directions. First we consider general directed projection functions $v_{i,j}(K; L, u)$, for convex bodies K in $\mathbb{R}^d$, where L is a j-dimensional subspace, $2 \leq j \leq d-1$, $u \in L$, and where the girth is replaced by the ith intrinsic volume, $1 \leq i \leq j$, and show a corresponding uniqueness result

for $v_{i,j}(K;\cdot)$. Then, we consider the averages of $v_{1,j}(K;L,u)$ (for $d=3$, $v_{1,2}(K;L,u)$ is the semi-girth) over all L that contain u and show that the resulting functions $\overline{v}_{1,j}(K;\cdot)$ on the unit sphere for certain values of d and j suffice to determine K, whereas for other values this is not the case. For example, $\overline{v}_{1,2}(K;\cdot)$ determines K in all dimensions, whereas $\overline{v}_{1,2i+1}(K;\cdot)$ does not determine K in dimension $d=3i+1$, $i=1,2,\ldots$. Joint work with Paul Goodey, University of Oklahoma.

Elisabeth Werner (Case Western Reserve University, USA)

Title: p-affine surface areas and random polytopes

Abstract: We investigate connections between random polytopes and p-affine surface areas.

Deane Yang (Polytechnic University, USA)

Title: L_p affine Sobolev-Zhang inequalities

Abstract: The sharp L_p Euclidean Sobolev inequality proved by Aubin and Talenti plays a crucial role in differential geometry and partial differential equations. It is proved using rearrangement and the Euclidean isoperimetric inequality. Recently, Zhang used the Petty projection inequality to prove a sharp L_1 affine Sobolev inequality that is stronger than the Euclidean version. We show that a recently proved L_p projection inequality can be used to prove a sharp L_p affine Sobolev inequality that is stronger than the Euclidean version.
