ARRANGEMENTS OF SIGNS AND REARRANGEMENTS OF VECTORS IN $\mathbb{R}^n$

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Let $x_1, \dots, x_m \in \mathbb{R}^n$ be arbitrary vectors with $\|x_k\|_2 \le 1$. It is proved that there exist signs $\varepsilon_1, \dots, \varepsilon_m = \pm 1$ and a permutation π of $\{1, \dots, m\}$ such that

$$\| \, \varepsilon_1 x_{\pi(1)} + \dots + \varepsilon_k x_{\pi(k)} \, \|_2 \le C\sqrt{n} \qquad (k = 1, \dots, m),$$

where C is some numerical constant. Let γ_n be the n-dimensional standard Gaussian measure on $\mathbb{R}^n$ with density $(2\pi)^{-n/2} e^{-\|x\|_2^2/2}$ and let U be a symmetric convex body in $\mathbb{R}^n$ such that $1 - \gamma_n(U) \le (2m)^{-1}$. Then there exist signs $\varepsilon_1, \dots, \varepsilon_m = \pm 1$ such that

$$\| \, \varepsilon_1 x_1 + \dots + \varepsilon_k x_k \, \|_U \le C' \qquad (k = 1, \dots, m),$$

where C' is some other numerical constant; it follows that there exist $\varepsilon_1, \dots, \varepsilon_m = \pm 1$ such that

$$\| \, \varepsilon_1 x_1 + \dots + \varepsilon_k x_k \, \|_2 \le C'\sqrt{n} + C'\sqrt{2 \log 2m} \qquad (k = 1, \dots, m).$$