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Title: Affinely associated Ellipsoids and Matrix valued Valuations

Abstract: A central problem when studying convex bodies is to choose the right position ϕK of a convex body K where $\phi \in GL(n)$. The right position depends on the question we have about K and the most important questions are of the following type: for a given functional μ on the space of convex bodies we ask for the maximum or minimum of $\mu(\phi K)$ over all $\phi \in GL(n)$. For this class of questions, the problem to find the right positions of convex bodies K is equivalent to the following problem: to find ellipsoids $E(K)$ that are naturally connected with our question and have the property that

$$E(\phi K) = \phi E(K) \quad \text{for} \quad \phi \in GL(n).$$

We say that these ellipsoids are affinely associated to K. The convex body K is now in the right position if and only if the ellipsoid $E(K)$ is the unit ball of $\mathbb{R}^n$.

We give a classification of affinely associated ellipsoids defined by symmetric matrices that are Borel measurable valuations on the space $\mathcal{P}_o^n$ of n-dimensional convex polytopes containing the origin in their interiors. Here we say that a positive-definite symmetric matrix M defines an ellipsoid E if

$$E = \{x \in \mathbb{R}^n : x \cdot Mx \leq 1\}$$

where $x \cdot Mx$ denotes the standard scalar product of x and Mx. And we say that a function M on $\mathcal{P}_o^n$ is a valuation, if

$$MP_1 + MP_2 = M(P_1 \cup P_2) + M(P_1 \cap P_2)$$

whenever $P_1, P_2, P_1 \cup P_2 \in \mathcal{P}_o^n$. The only examples of such ellipsoids in $\mathbb{R}^n$, $n \geq 3$, are the Legendre ellipsoid Γ_2 and the ellipsoid Γ_{-2} recently defined by Lutwak, Yang, and Zhang.