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Title: A survey of some recent convexity results and problems in classical differential geometry

Abstract: In this talk we discuss a number of recent results and problems which lie in the intersection of classical convexity with differential geometry and topology of curves and surfaces in Euclidean space.
For instance, we prove that ellipsoids are the only closed surfaces on which every closed curve has a pair of parallel tangent lines. This result, which uses some previous works of Blaschke, arose unexpectedly from the study of shadows (or the shaded regions) cast on illuminated surfaces. If a convex body is illuminated from any given direction, then the shadow cast on the surface forms a connected subset. We will prove that this property characterizes convex bodies among all simply connected surfaces which are smoothly immersed in Euclidean space. There are nonsimply connected surfaces, however, which have connected shadows in all directions, and thus behave similar to a convex body. These findings were originally motivated by the study of soap films (surfaces of constant mean curvature), but are also of interest in computer vision (the "shape from shading" applications).