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Title: Random polytopes and the Efron-Stein jackknife inequality

Abstract: Let K be a smooth convex body in $\mathcal{R}^d$, denote by P_n a random polytope in K (which is the convex hull of n random points in K), and by P_N^{best} a best–approximating polytope (which minimizes the difference between $V_d(K)$ and $V_d(P)$ among all polytopes P in K with at most N vertices).
As for the random polytope it was proved by Rényi and Sulanke, Wieacker, Bárány, and Schütt that the difference $V_d(K) - \mathbb{E}V_d(P_n)$ is of order $n^{-2/(d+1)}$ as n tends to infinity. On the other hand, the rate of convergence of the volume of the polytope to the volume of K increases from $n^{-2/(d+1)}$ to $N^{-2/(d-1)}$ if the random polytope is replaced by the best–approximating polytope: As was proved by Gruber, $V_d(K) - V_d(P_N^{\text{best}})$ is of order $N^{-2/(d-1)}$.
Inspired by the fact, that N denotes the number of vertices of the best–approximating polytope, whereas n denotes the number of random points generating P_n, Bárány suggested to measure the difference $V_d(K) - V_d(P_n)$ by using the number of vertices $N(P_n)$ of the random polytope instead of using n. He proved that for $d = 2,\ 3$

$$\mathbb{E}\left((V_d(K) - V_d(P_n))N(P_n)^{\frac{2}{d-1}}\right) \sim c_d\, \Omega(K)^{\frac{d+1}{d-1}}$$

(the the same order of magnitude as for best–approximating polytopes) which means that "Approximation by random polytopes is almost optimal". In our talk we show that for the remaining cases $d \geq 4$

$$\lim_{n\to\infty}(V_d(K) - V_d(P_n))N(P_n)^{\frac{2}{d-1}} = c_d\, \Omega(K)^{\frac{d+1}{d-1}}$$

holds with probability one.
The main tool is the Efron-Stein jackknife inequality for the variance of symmetric statistics which yields an estimate for the variance of $V_d(P_n)$ and $N(P_n)$. This leads to an estimate for the deviation of $V_d(P_n)$ and $N(P_n)$ from their mean values.