Titles and Abstracts of Talks

Sunday, August 4

9:00 a.m. : Franz Gähler, ITAP, Universität Stuttgart

Modelling Aperiodic Solids: Concepts and Properties of Tilings and their Physical Interpretation

In this talk I shall review how aperiodic solids like quasicrystals can be modelled by suitable tilings or Delone sets, and how this is motivated by experiment and other physical considerations. Several important concepts and properties of tilings will be introduced and discussed from a physical point of view, among them also equivalence concepts like local isomorphism or mutual local derivability. Of particular interest for physics are tilings admitting perfect matching rules. The local isomorphism class of such tilings is completely characterized by the local patterns of the tiling up to a fixed size. The existence of such a local characterization may be used to explain the stability of quasicrystals. A particularly efficient and physically appealing formulation of the matching rules can often be obtained for tilings admitting a covering by overlapping copies of a single cluster. In fortunate cases, the internal structure of the covering cluster imposes constraints on the possible cluster overlaps which effectively imply the matching rules. In this way, a low energy covering cluster can enforce a perfectly ordered aperiodic structure.

10:30 a.m. : Charles Radin, University of Texas, Austin

Aperiodicity: lessons from various generalizations

Most research (and therefore intuition) on aperiodic tiling comes from planar examples such as the kite and dart. Generalizations such as Peter Schmitt's 3-dimensional tiling, and his planar packings, and the hyperbolic tilings of Roger Penrose et al, are therefore useful in isolating the "essential" aspects of the structure of the subject, those aspects which will make fruitful contact with other parts of mathematics.

1:30 p.m. : Robert Williams, University of Texas, Austin

Tiling spaces as Cantor set fiber bundles

I will discuss some recent computations of tiling spaces via the Anderson Putnam inverse limit technique. Especially some examples that are (perhaps) new but simpler than many that have been in print. Why the base space must be of "polynomial growth" in area as a counterpart to "flatness".

3:00 p.m. : Jean-Marc Gambaudo, Université de Bourgogne, Dijon

Delone sets, tilings and solenoids: from finite translation type to finite isometry type

4:10 p.m. : John Hunton, University of Leicester

New Models and Methods for Tiling Spaces

The *Tiling Space* or *Continuous Hull* Ω_P of an aperiodic tiling or pattern P has proved to be an important topological construction in the analysis of such objects. There are already a number of models and descriptions of this space, but here we give a new description of it, as a classifying space, and a new machine for describing the K-theoretic invariants for the non-commutative space $C(\Omega_P) \times R^d$. This can be seen as the non-commutative analogue of an old spectral sequence due to Atiyah.

Monday, August 5

9:00 a.m. : Claire Anantharaman-Delaroche, Université d'Orléans

Amenable groupoids. Examples and applications

This talk aims to give an overview on the notion of amenability for groupoids, mainly in the topological framework, with a look at the measured case. We discuss several examples and give some recent applications to the theory of C^*-algebras.

10:30 a.m. : Thierry Fack, Université de Lyons I

Introduction to cyclic cohomology

Cyclic cohomology was introduced by A. Connes to describe the homology of quantum spaces. In particular, cyclic cocycles allow to construct maps from the K-theory of quantum spaces to the scalars, and hence to write down index formulae. We shall explain how to get such index formulae and their use in the resolution of several conjectures in topology (Novikov's conjecture) and physics (quantum Hall effect, gap labelling).

1:30 p.m. : Michael Baake, Universität Greifswald

Mathematical diffraction theory and model sets

(Joint work with Robert Moody.) Mathematical diffraction theory is concerned with the Fourier analysis of the autocorrelation of translation bounded measures on locally compact Abelian groups, with n-dimensional Euclidean space being the most important example. Of special interest are measures with pure point diffraction, which is intimately related with the cut and project formalism. Of particular interest are geometric properties of a set which can be inferred from pure point diffractivity. Several results in this direction will be indicated, as well as open questions.

3:00 p.m. : Jeong-Yup Lee, University of Alberta

Consequences of Pure Point Diffraction Spectra for Discrete Point Sets

We will uncover equivalent criteria for discrete point sets to have pure point diffraction spectrum in substitution. In the case that the point sets are on a lattice, we have some easy way to check if they are pure point diffractive. This checking method is parallel with Dekking's coincidence criterion, which says that point sets coming from equal-length substitution satisfy coincidence if and only if they have pure point discrete spectrum. But our method is more general in the sense that it works for arbitrary n-dimension not only 1-dimension point sets. We will also talk about when pure point dynamical spectrum is equivalent to pure point diffraction spectrum.

4:30 p.m. : Laurent Bartholdi, University of California, Berkeley

Tilings and Groupoids Acting on Rooted Trees

(Joint work with Volodymyr Nekrashevych.)

We will discuss:

(a) the definition of recurrent action on a rooted tree; examples: Z^n, $GL(n, Z)$, Grigorchuk group; any group associated to a self-covering of a topological space; a natural extension to groupoids, in the case of Penrose tilings.

(b) the limit space of such a groupoid, constructed as an "inverse" or "dual" construction to the usual orbit equivalence relation. the limit solenoid, tiled by translates of the limit space.

(c) equivalence of the original topological orbi-spaces and the limit space.

(d) Penrose tilings as the limit space of a groupoid with free automorphism groups.

Tuesday, August 6

9:00 a.m. : Thierry Giordano, University of Ottawa

Affable equivalence relations and orbit structure of Cantor minimal systems

10:30 a.m. : N. Christopher Phillips, University of Oregon

The structure of the C*-algebras of free minimal actions of $\mathbf{Z}^d$ on the Cantor set

Let $\mathbf{Z}^d$ act freely and minimally on the Cantor set X, and let $A = C^*(\mathbf{Z}^d, X)$ be the crossed product C*-algebra. We prove that A has stable rank one and real rank zero, and that the order on $K_0(A)$ is determined by traces. The conclusions mean that the invertible elements of A are dense in A, that every selfadjoint element of A can be approximated by selfadjoint elements with finite spectrum, and that a K-theory class in $K_0(A)$ is represented by an actual projection whenever its images under the maps determined by all the normalized traces are strictly positive.

The C*-algebras of various kinds of aperiodic tilings are closely related to such crossed product C*-algebras. In particular, the theorem from which we obtain the results on crossed products also generalizes a recent paper of Ian Putnam in Commun. Math. Phys., which considered the C*-algebras arising from substitution tiling systems satisfying certain mild conditions. Our results also show that the C*-algebras of some other kinds of aperiodic tilings have stable rank one and real rank zero, and that the order on the K_0-group is determined by traces.

This talk is intended to be accessible to a broad audience. We will explain the conditions in the conclusion, and we will sketch the connection with groupoid C*-algebras, but there will be very little in the way of proof.

Afternoon free

Wednesday, August 7

9:00 a.m. : Lorenzo Sadun, University of Texas

When size matters: the effect of geometry on 1-D tiling dynamics

If two tiling spaces have identical combinatorics, do they necessarily have identical dynamical properties? For tilings based on 1-D substitution subshifts, the answer depends on the second eigenvalue of the substitution matrix. If $|\lambda_2| \geq 1$, then the dynamics are sensitive to tile lengths, and a generic choice of tile lengths gives a weakly mixing R-action. However, if $|\lambda_2| < 1$, then all choices of tile lengths give topologically conjugate dynamics, up to an overall scale. In particular, there is a topological conjugacy between the self-similar tiling space (as studied by Solomyak) and the ordinary suspension of the subshift. Several recent results on Pisot substitution subshifts then follow as easy corollaries. This is joint work with Alex Clark.

10:30 a.m. : Marcy Barge, Montana State University

The topology of one-dimensional tiling spaces

Joint work with Beverly Diamond, Jarek Kwapicz.

Each primitive, non-periodic substitution gives rise to a collection of aperiodic tilings of the real line. With a natural topology, this collection is the tiling space associated with the substitution. These spaces all have a finite and positive number of asymptotic arc components. In certain simple cases, gluing together these asymptotic arc components turns the tiling space into a torus. This is a special case of *geometric realization.* If the substitution is unimodular and Pisot, there is a finite-to-one map of its tiling space onto the torus of dimension equal to the size of the alphabet. The map semiconjugates the natural flow on the tiling space with irrational flow on the torus and the inflation and substitution homeomorphism on the tiling space with an Anosov diffeomorphism of the torus. We prove that geometric realization is almost everywhere one-to-one in dimension 2 and report on progress in the higher dimensional case. The conjecture that geometric realization is a.e. one-to-one in all dimensions is a strong version of what has come to be called the *coincidence conjecture* and is closely related to the conjecture that all Pisot substitutive systems have pure discrete spectrum.

1:30 p.m. : Klaus Schmidt, University of Vienna

T.B.A.

3:00 p.m. : Jerry Kaminker, I.U.P.U.I.

Index theory on foliated spaces and applications.

Noncommutative geometry had its origins in the index theorem for foliated manifolds due to A. Connes. This was extended to foliated spaces by C. Moore and C. Schochet. We will first try to give a very elementary survey of index theory in this setting and indicate how the added generality of foliated spaces can be useful for problems in dynamics and why index theory in general can provide information. We will then describe joint work with Ian Putnam which gives one of the proofs of Bellissard's Gap Labelling Conjecture and also some other types of applications in dynamics.

4:30 p.m. : Franz Gähler, ITAP, Universität Stuttgart

Cohomology of Quasiperiodic Tilings

The hull of a repetitive tiling carries a natural topology, making it a compact topological space. Its topology can be characterized by certain topological invariants. Among these, cohomology groups are relatively easy to compute. They depend only on the mutual local derivability class (MLD class) of the tiling. To compute the cohomology of the hull, a sequence of simpler spaces is constructed from larger and larger local patches of the tiling, and it is shown that the hull is homeomorphic to the inverse limit of this sequence. The cohomology of the hull is therefore equal to the direct limit of the cohomologies of these simpler spaces. For many quasiperiodic tilings, it can be shown that the limit of these cohomologies is reached already after a finite number of steps, so that the limit can actually be computed.

The criteria for this to happen are discussed and elucidated with the help of several examples. The mechanism at work is similar to the one which makes matching rules be finite range. Our approach is inspired by that of Anderson and Putnam for substitution tilings, but does not use or need selfsimilarity.

Thursday, August 8

9:00 a.m. : Daniel Lenz, TU- Chemnitz

Uniform ergodic theorems on Delone dynamical systems and applications

We study strictly ergodic Delone dynamical systems and prove a uniform ergodic theorem for certain Banach-space valued functions on the associated set of pattern classes. As an application we show existence of the integrated density of states in the sense of uniform convergence in distribution for associated random operators. This, in particular, allows us to characterize the points of discontinuity of the integrated density of states in terms of existence of locally supported eigenfunctions. (Joint work with P. Stollmann and S. Klassert).

10:30 a.m. : Bob Burton, Oregon State University

A dynamical approach to constructing sequences in the unit cube which are well dispersed

(Joint with Aimee Johnson.) It has become important to find sequences of points in the unit cube which are good sample points to integrate functions that belong to a reasonable class. In applications to financial problems the dimension of the cube may be as large as 360. There is a classical number theory of this (sequences with low discrepancy). We frame this into the language of spatial dynamical systems, especially those with aperiodic behavior and give some preliminary results.

1:30 p.m. : Claude Schochet, Wayne State University

Life After K-theory

Topological K-theory has proven to be an extraordinarily powerful tool for the study of C^*-algebras. By adding on order structure, coefficients, and Kasparov's two-variable KK-theory, we have been able to make remarkable progress on the classification of simple C^*-algebras, the Novikov conjecture, the Gromov-Lawson-Rosenberg conjecture, the Baum-Connes conjecture, and many related problems.

Joe Taylor's paper ("Banach algebras and topology", pp. 118-186 in *Algebras in analysis* (Birmingham, 1973), edited by J. H. Williamson, Academic Press, London, 1975) served as one of the main entry points to K-theory for functional analysts in the early 1970's. Taylor was interested in the following question. Suppose that A is a commutative Banach algebra with maximal ideal space $\Delta(A)$. Is it possible to read off the various topological invariants of $\Delta(A)$ from the structure of A? Taylor pointed out that it was possible to retrieve $H^n(\Delta(A);\mathbb{Z})$ for $n = 0, 1,$ and 2 using results of Shilov, Arens-Royden, and Forster, respectively. He then comments:

> *It is reasonable to assume that further information connecting the topology of $\Delta(A)$ and the structure of A will have important consequences. However, . . . the relation between the structure of A and the higher Čech cohomology groups seems not very well understood.*

He then switched over to showing that the K-theory groups $K^*(X)$ developed by Atiyah, Wood, and Karoubi may be obtained as a special case of $K_*(A)$. Since the Chern character induces an isomorphism

$$K^0(X) \otimes \mathbb{Q} \longrightarrow H^{even}(X;\mathbb{Q})$$

and similarly for K_1, we may say that the functors $H^{even}(-;\mathbb{Q})$ and $H^{odd}(-;\mathbb{Q})$ have been retrieved. However since we are retrieving only the *sum* of the rational cohomology groups, this functor is unable to determine that $C(S^2)$ and $C(S^4)$ are not isomorphic, or, for that matter, that $M_2(\mathbb{C})$ and $M_3(\mathbb{C})$ are not isomorphic. (It is possible to retrieve the individual groups $H^n(-;\mathbb{Q})$ from $H^{**}(-;\mathbb{Q})$ using Adams operations, but these operations do not extend to K-theory for non-commutative C^*-algebras.)

The original problem was never solved. Leaving out the word "commutative"and restricting to C^*-algebras, the problem becomes more complex and at the same time more focused. If A is simple and unital then Δ is trivial. However, Taylor's original question may be restated as follows: Which of the various standard functors in algebraic topology (homology, cohomology, cobordism, . . .) may be extended usefully to a large category of C^*-algebras?

In this talk we shall demonstrate how to extend the functor $H^n(X;\mathbb{Q})$ to a computable functor on C^*-algebras which *does* distinguish between the C^*-algebras mentioned above and we shall suggest new directions for understanding the topological structure of spaces related to A.

3:00 p.m. : Chaim Goodman-Strauss, University of Arkansas

Triangle Tilings and Regular Productions

We conjecture precisely which triangles do, and which don't, admit tilings of the hyperbolic plane, Euclidean plane and sphere (Typically, these tilings are weakly aperiodic). We prove the conjecture on a measure 1 set in the space of triangles. Our main interest is to illustrate the use of "regular production systems" as a means of analyzing tilings in general. Such systems are a certain generalization of symbolic substitution systems but tend to be quite subtle in practice. In essence, the combinatorics of any given set of tiles can be captured by such a system, and under many circumstances, this corresponding system can then be analyzed (though in general, for an arbitrary regular production system, certain basic questions are undecidable and analysis impossible.

4:30 p.m. : Alex Kumjian, University of Nevada, Reno

Actions of $\mathbb{Z}^k$ associated to higher rank graphs

(Joint work with D. Pask.) An action of $\mathbb{Z}^k$ is associated to a higher rank graph Λ satisfying a mild assumption. This generalises the construction of a topological Markov shift arising from a nonnegative integer matrix. We show that the stable Ruelle algebra of Λ is strongly Morita equivalent to $C^*(\Lambda)$.