Digital Filters in Adaptive Time-Stepping

Gustaf A Soderlind
Gustaf.Soderlind@na.lu.se
Center for Mathematical Sciences, Lund University, Sweden

Locally adaptive time-stepping based on linear digital control theory has several advantages: the algortihms can be analyzed with respect to stability an adaptivity, and they can be designed to produce smoother stepsize sequences resulting in improved regularity and computational stability. The approach taken in this talk is to consider the transfer function $H_{\hat{\varphi}} : \log \hat{\varphi} \mapsto \log h$, where $\hat{\varphi}$ is the norm of the principal error function and h is the time-step size, as a *digital filter*, processing the signal $\log \hat{\varphi}$ in the (discrete-time) frequency domain.

We construct first and second order locally adaptive controllers implementing digital filters that completely quench $(-1)^n$ oscillations in the stepsize sequence. At no extra computational expense, the stepsize output is highly regular, and high-frequency error content is strongly suppressed even in the presence of a considerable "noise" in $\log \hat{\varphi}$. The controllers are suitable for ODE/DAE methods in general but may be of particular interest in non-smooth problems such as SDEs. The theory covers all control structures previously considered for locally adaptive time-stepping in the asymptotic stepsize-error regime.