Fast convolution for non-reflecting boundary conditions

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Non-reflecting boundary conditions for problems of wave propagation are non-local in space and time. While the non-locality in space can be efficiently handled by Fourier or spherical expansions in special geometries, the arising temporal convolutions still form a computational bottleneck. In this talk, a new algorithm for the evaluation of these convolution integrals is presented. To compute a temporal convolution over N_t successive time steps, the algorithm requires $O(N_t \log N_t)$ operations and $O(\log N_t)$ active memory. The key idea is to reduce the problem to locally solving $O(\log N_t)$ different sets of scalar linear ordinary differential equations. The computational work and the memory requirements for accurately implementing non-reflecting boundary conditions become thus less than those for the interior domain, and are asymptotically negligible as the spatial and temporal grid sizes tend to zero.

In numerical examples, the convolution algorithm is used to discretize the Dirichlet-to-Neumann and Neumann-to-Dirichlet operators arising from the formulation of non-reflecting boundary conditions in rectangular geometries for Schrödinger and wave equations.

The talk is based on joint work with Achim Schaedle (Tuebingen).