Numerical Solutions to Free Boundary Problems

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Many physically interesting problems involve propagation of free surfaces. Vortex sheet roll-up in hydrodynamic instability, wave interactions on the ocean's free surface, solidification problem for crystal growth, Hele-Shaw cells for pattern formation are some of the significant examples. These problems present a great challenge to physicists and applied mathematicians because the underlying problem is very singular. The physical solution is sensitive to small perturbations. Naive discretizations may lead to numerical instabilities. Other numerical difficulties include singularity formation and possible change of topology in the moving free surfaces, and the severe time stepping stability constraint due to the stiffness of high order regularization effects, such as surface tension.

In this talk, I will review some of the recent advances in developing stable and efficient numerical methods for solving free boundary value problems arising from fluid dynamics and materials science. In particular, we will consider boundary integral methods and the level set approach for water waves, general multi-fluid interfaces, Hele-Shaw cells, phase transition in shape memory materials. The issue of numerical stability and convergence will be discussed. We then use these highly accurate and stable methods to investigate the peculiar regularization effect of small surface tension in overturning water waves, unstably stratified two-fluid interfaces, Hele-Shaw flows, and the dynamic generation of small scales in phase transition of shape memory materials. Several interesting and surprising phenomena are revealed.