General linear methods with inherent Runge-Kutta stability

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Until recently practical general linear methods which were neither Runge-Kutta methods, linear multistep methods or slight modifications of these traditional methods did not really exist. The main reasons for this are, the complexity of the order conditions, makes constructing these methods very difficult, unless clever simplifying assumptions are used. Constructing the starting procedure of a general linear method which effectively requires r independent numerical methods can be a demanding task, especially if the starting procedure has no particular pattern.

A class of potentially practical general linear methods with the property known as "inherent Runge-Kutta stability" will be described. These methods use certain simplifying assumptions and require the starting procedure to be of Nordsieck type. This allows methods with stability regions identical to those of Runge-Kutta methods which have high stage order and the additional property that they can be diagonally implicit, to be constructed. This makes these methods suitable candidates for the solution of stiff and non-stiff differential equations. These methods will be derived in a sufficiently general way such that the ESIRK and DESIRE methods become special cases.

The implementation of this class of methods in a variable stepsize, variable order code will also be discussed.