Geometric integration methods for nonlinear diffusion

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In this paper geometric integration methods are applied to the nonlinear diffusion equation with extinction,

$$u_t = \frac{\nabla^2 (u^m)}{m} - \beta u^q,$$

where ∇^2 is the radial Laplacian in arbitrary dimension, $m > 1, q > 0$ and $\beta \geq 0$. This equation models several important physical processes and has the feature of being invariant under changes of scale. Singular solutions exist which may have interfaces or tend to zero in a finite time. By considering asymptotic solutions scale-invariant moving mesh methods are constructed which have the same scaling invariance properties as the original equation. These methods use equidistribution of the spatial mesh and are based on the monitor function

$$M = |v_{xx}|^p,$$

where v is the pressure, $v = \dfrac{u^{(m-1)}}{m-1}$. We show that this formulation permits the accurate resolution of interfaces between regions where the solution is large and where it is small. We also consider the effects of the solution scaling on such issues as accuracy, stability and stiffness.