# High-order convergent deferred correction schemes based on Runge-Kutta(-Nystrom) methods

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Deferred correction, which looks like,

$$\phi(\eta) = 0 \tag{1}$$
$$\phi(\bar{\eta}) = \psi(\eta) \tag{2}$$

is a widely used technique for the solution of systems of nonlinear two-point boundary value problems.

In an influential paper, Skeel has proven the following result. Consider the approximate numerical solution of the problem on a mesh $\pi : a = x_1 < x_2 < \ldots < x_{N+1} = b$. Denote by $\Delta y$ the restriction of the continuous solution $y(x)$ to the finite grid $\pi$. Let $\phi$ be a stable numerical method and assume that the following conditions hold for the deferred correction scheme (**??**), (**??**) : (i) $\|\eta - \Delta y\| = O(h^p)$, (ii) $\|\psi(\Delta y) - \phi(\Delta y)\| = O(h^{r+p})$ and (iii) $\psi(\Delta w) = O(h^r)$ for arbitrary functions $w$ having at least $r$ continuous derivatives. If $\phi(\bar{\eta}) = \psi(\eta)$ then $\|\bar{\eta} - \Delta y\| = O(h^{r+p})$. In the context of two-point BVPs for first order, $\phi$ can be chosen to be a Runge-Kutta methods of order $p$ while $\psi = \phi - \phi^*$ where $\phi^*$ is a Runge-Kutta method of order $p + r$. For second-order problems Runge-Kutta-Nystrom methods can be chosen.

The feature that is common to all of the deferred correction schemes that have been implemented so far is that $r = 2$ and for schemes of this type the order of accuracy is increased by 2. A sufficient condition to achieve this increase in accuracy is basically that the Runge-Kutta(-Nystrom) formulae $\phi$ and $\phi^*$ should be symmetric and that they should be written in a special way that is appropriate for boundary value problems. The main reason why it is hard to get more than two orders of accuracy improvement per iteration is the difficulty in satisfying condition (iii) for $r > 2$. Such deferred correction schemes with $r > 2$ are called high-order convergent or superconvergent.

The present talk mainly addresses the question whether it is possible to choose $\phi$ and $\phi^*$ so that we can achieve $r > 2$. To obtain the algebraic conditions, the B-series and P-series approaches are used.