Convergence and long-term behaviour of time discretizations for fully nonlinear parabolic problems

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We study Runge-Kutta and linear multistep methods for discretizing a fully nonlinear parabolic evolution equation of the form $u'(t) = F(t, u(t)), t > 0$ in time. Our analysis is based on the framework of analytic semigroups and Banach space valued Hölder continuous functions. The interest in such an abstract differential equation stems from the fact that a wide range of parabolic initial-boundary value problems modelling nonlinear diffusion or heat conduction processes are of this form.

We present convergence results for variable stepsizes, and we show that the geometric properties of a hyperbolic structure are well captured by the time discretization. Numerical examples illustrate the theoretical results.