Refinements in the exponentially small estimates for backward error analysis for ODEs

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Abstract

It is well known that the numerical solution provided by a one-step integrator applied to an ODEs can be formally interpreted as the exact solution of a modified ODE. A new approach is presented to explicitly obtain the modified ODE of a given one-step integrator. The expression for the modified ODE thus obtained is also valid for integrators for ODEs on manifolds. For a rigourous backward error analysis, the modified ODEs must be suitably truncated, since the (formal) modified ODEs do not in general converge. It is well known that when appropriately truncating the modified ODE, the difference between the numerical solution and the exact solution of the truncated modified ODE is sponentially small with respect to the time-step h. Different proofs of this result can be found in the literature. Using our explicit representation of the modified equations, we simplify the proof while better estimates for exponentially small error are obtained.