On the attainable order of collocation methods for pantograph integro-differential equations

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Abstract

Consider the following *pantograph integro-differential equation* (PIDE) :

$$y'(t) = ay(t) + \int_0^1 y(\sigma(q)t)d\mu(q) + \int_0^1 y'(\rho(q)t)d\nu(q) + f(t), \text{ with } y(0) = 1,$$

and the delay *Volterra integro-differential equation* (DVIDE) :

$$y(t) = 1 + \int_0^t ay(s)ds + \int_0^1 \int_0^t y(\sigma(q)s)dsd\mu(q) + \int_0^1 \int_0^t y'(\rho(q)\tau)d\tau d\nu(q) + \int_0^t f(s)ds,$$

with proportional delays $\sigma(q)t$ and $\rho(q)t,\ 0 < q \leq 1$, and an entire complex function $f(t)$, a complex number a and complex-valued functions $\mu(q), \nu(q)$ of bounded variation on $[0,1]$, and the integrals under considerations are Riemann-Stieltjes type.

In this paper, on the attainable order of m-stage implicit (collocation-based) Runge-Kutta methods at the first mesh point $t = h$, we give conditions on the existence of the collocation polynomials $M_m(t)$ of $v(th),\ t \in [0,1]$ such that to the solution $y(t)$ and the collocation solution $v(t)$ of the PIDE, it satisfies $|v(h) - y(h)| = O(h^{2m+1})$. If $f(t)$ is a polynomial of t whose degree is equal to or less than m, then such conditions of $M_m(t)$ are simplified. If $f(t)$ is not, then to get such $M_m(t)$, we need to solve nonlinear equations, to which we propose an iteration method which use the homotopy method with two initial approximations.

Under the condition that $f(t)$ is a polynomial of t whose degree is equal to or less than $m-1$, we give the condition between $M_m(t)$ of $v(th)$ and $\hat{M}_m(t)$ of $u_{it}(th)$ to satisfy $v(t) = u_{it}(t),\ 0 \leq t \leq h$, where $u_{it}(t)$ is the 'iterated collocation solution' of the DVIDE and $\hat{M}_m(t)$ is the collocation polynomial of $u_{it}(th),\ t \in [0,1]$.