Dae's approach in the solution of optimal control problems

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An optimal control problem (OCP) is a dynamic optimization problem subject to equality and inequality constraints in the state and control variables which is usually solved by either the variational approach or its transformation into a nonlinear optimization problem. In the first case, the resulting BVP is solved by iterative methods or by employing quasi-optimal conditions. In the second case, a sequential strategy of optimization and integration is often used. The optimization technique, called Differential-Algebraic Optimization, eliminates this distinguished treatment for purely differential or algebraic equations. The differential-algebraic approach proposes the numerical solution of a system of algebraic and differential equations simultaneously, thus avoiding the algebraic and differential manipulations necessary for obtaining a purely differential model. Besides eliminating a laborious, error-subject stage, the differential-algebraic approach favors the simulation of control systems guaranteeing that the trajectory defined by the controller will actually be followed. Also the verification of the performance of optimal control strategies is facilitated, since it merely requires the addition of a new set of equations to the process model. The purpose of this work is to solve the differential algebraic boundary value problems resulting from the application of the Pontryagin's principle through the differential-algebraic approach. In order to achieve this goal, the computer code COLDAE, implemented in Fortran Language by Ascher and Spiteri (1995) was chosen. This code displays as a main feature the capability of solving semi-implicit, nonlinear boundary value DAE systems with index at most 2 and totally implicit boundary value DAE systems with index 1. An additional and essential task to be carried out is the characterization of the DAE system regarding the shape of the equations, the index and the consistency of the initial conditions. The hard task of obtaining the conditions necessary for the optimum motivated the development of a code named OTIMA, implemented in Maple VR4, which is capable of performing the symbolic calculus of the optimum conditions and generating the output file in Fortran language utilized by COLDAE. The results obtained have proved the direct solution of the DAEs to be feasible, ensuring that the problem constraints have been fully respected. The subroutine COLDAE has been shown to be suitable for the solution of the problems selected, surpassing different degrees and sorts of difficulties associated with the definition of the boundary conditions definition, the initialization of the variables profiles and the convergence of the Newton's

method with or without damping throughout the nonlinear iteration. New investigations into the treatment of inequality constraints must be lead, especially for cases in which the problem presents more inequality constraints on the state variables than on the control variables. The strategy to identify bound points between regions with active constraints and systems of greater dimensions are also the object of future studies.
Keywords: index, differential-algebraic equations, nonlinear dynamic optimization, COLDAE, boundary value problem, optimal control, inequality constraints.