On perturbations of linear systems of differential equations with periodic coefficients

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Consider the following linear system of differential equations

$$\frac{dy}{dt} = A(t)y, \quad t \geq 0, \tag{1}$$

where $A(t)$ is an $(N \times N)$-matrix with continuous T-periodic entries. It is well-known that the zero solution of system (1) with periodic coefficients is asymptotically stable if and only if all eigenvalues of the monodromy matrix for (1) belong to the unit disk $\{|\lambda| < 1\}$. However the problem of localizing the spectra of matrices is poorly conditioned. Therefore it is necessary to have criteria that would be applicable for numerical research of asymptotic stability. In the paper [1], using the differential Lyapunov equation, the authors established the following criterion:

I. *If all eigenvalues of the monodromy matrix for (1) lie in the unit disk* $\{|\lambda| < 1\}$ *then, there is a unique solution* $H(t)$ *of the boundary value problem*

$$\begin{array}{l} \frac{dH}{dt} + HA(t) + A^*(t)H = -I, \quad 0 < t < T, \\ H(0) = H(T); \end{array} \tag{2}$$

moreover, $H(t) = H^*(t) > 0$*,* $t \in [0, T]$*.*

II. *If the boundary value problem (2) has a Hermitian solution* $H(t)$ *such that* $H(0) > 0$ *then all eigenvalues of the monodromy matrix of (1) lie in the unit disk* $\{|\lambda| < 1\}$*.*

Using the matrix $H(t)$, G.V.Demidenko [1] obtained an estimate for solutions of system (1), enabling us to indicate the decay rate of solutions as $t \to +\infty$.

In the present paper we discuss the affect of periodic perturbations. Thus, we indicate conditions on the matrix $A_1(t)$ under which the zero solution of the system

$$\frac{dy}{dt} = [A(t) + A_1(t)]y, \quad t \geq 0,$$

$$A(t+T) = A(t), \quad A_1(t+T) = A_1(t),$$

is asymptotically stable if all eigenvalues of the monodromy matrix of (1) belong to the unit disk. We establish also an estimate for the norm of the difference $\|H(t) - \widetilde{H}(t)\|$, $0 \leq t \leq T$, where $\widetilde{H}(t)$ is a T-periodic solution of the Lyapunov differential equation with perturbation

$$\frac{d}{dt}\widetilde{H} + \widetilde{H}(A(t) + A_1(t)) + (A^*(t) + A_1^*(t))\widetilde{H} = -I.$$

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References.

1. G.V.Demidenko, I.I.Matveeva. On stability of solutions to linear systems with periodic coefficients. (Russian) Sib. Mat. Zh. 42, No. 2, 332–348 (2001); English transl. in Sib. Math. J. 42, No. 2, 282–296 (2001).